HL IB Physics

Standing Waves & Resonance

Contents

Standing Waves

Standing Waves

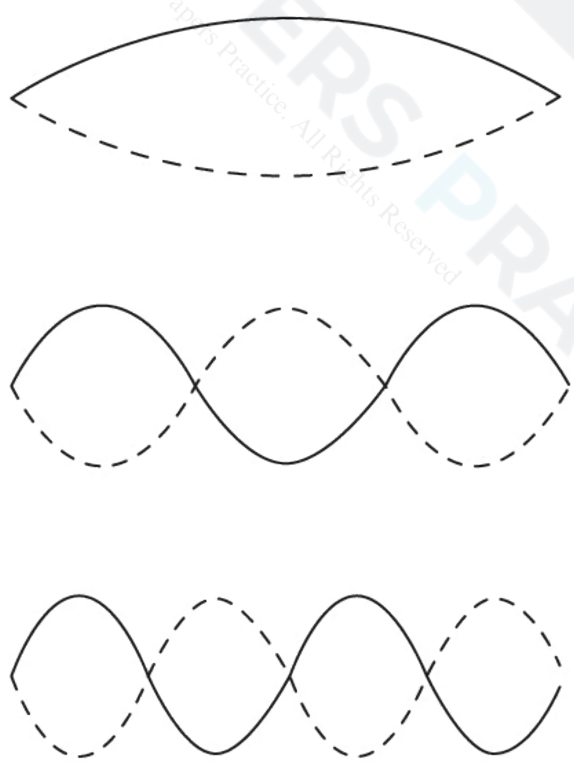
- **Standing waves** are produced by two waves as they travel in **opposite directions**
- This is usually achieved when a travelling wave superimposes its reflection
  - The superposition produces a wave pattern where the crests and troughs only move vertically

Formation of Standing Waves

- Standing waves are formed from the principle of superposition. This is when:

  Two waves travelling in opposite directions along the same line with the same frequency superpose

- The principle of superposition applies to **all** types of waves i.e. transverse and longitudinal, progressive and stationary
- The waves must have:
  - The same wavelength
  - A similar amplitude
- As a result of superposition, a resultant wave is produced



Standing waves produced at varying frequencies

Comparing Progressive and Standing Waves

- Standing waves (or stationary waves) **store** energy
- **Progressive waves** (or travelling waves) **transfer** energy
- The table below outlines the main differences between progressive and stationary waves

Table of Differences Between Progressive and Stationary Waves

Feature	Progressive Waves	Stationary Waves
Energy Transfer	Transfer energy from one point to another.	Do not transfer energy; energy is confined within the wave.
Wave Motion	Propagate through a medium, moving in a particular direction.	Appears stationary, with particles vibrating in place.
Amplitude	All points within a wavelength have the same amplitude.	Different points have different amplitudes (nodes: 0, antinodes: maximum).
Phase	All points within a wavelength are not in phase (except at the crest and trough).	All points between nodes are in phase (oscillating together).
Formation	Created by a vibrating source and propagate through a medium.	Formed by the superposition of two progressive waves traveling in opposite directions.
Examples	Sound waves traveling through air, light waves, water waves.	Vibrations of a guitar string, standing waves in a pipe.

Nodes & Antinodes

Nodes & Antinodes

- A standing wave is made up **nodes** and **antinodes**
  - **Nodes** are locations of **zero amplitude** and they are **separated by half a wavelength** ($\lambda/2$)
  - **Antinodes** are locations of **maximum amplitude**
- The nodes and antinodes **do not** move along the wave
  - Nodes are fixed and antinodes only oscillate in the vertical direction

The Formation of Nodes and Antinodes

- At the **nodes**:
  - The waves are in anti-phase meaning **destructive** interference occurs
  - The crest of one wave meets the trough of another
  - This causes the two waves to **cancel** each other out
- At the **antinodes**:
  - The waves are in phase meaning **constructive** interference occurs
  - The crest of one wave meets the crest of another (same for troughs)
  - This causes the waves to **add** together

Phase on a Standing Wave

- Two points on a standing wave are either in phase or in anti-phase
  - Points that have an **odd** number of nodes between them are in **anti-phase**
  - Points that have an **even** number of nodes between them are **in phase**
  - All points within a **"loop"** are **in phase**

Boundary Conditions for Standing Waves

Boundary Conditions

- Stationary waves can form on strings or in pipes
- In both cases, progressive waves travel in a medium (i.e. the string or air) and superimpose with their reflections
- The number of nodes and antinodes that fit within the available length of medium depends on:
  - The frequency of the progressive waves
  - The **boundary conditions** (i.e. whether the progressive waves travel between two fixed ends, two free ends or a fixed and a free end)

Standing Waves on Stretched Strings

- When guitar strings are plucked, they can vibrate with different frequencies
- The frequency with which a string vibrates depends on:
  - The tension, which is adjusted using rotating 'tuning pegs'
  - The mass per unit length, which is the reason why a guitar has strings of different thicknesses

- For a string, the boundary condition can be
  - Fixed at both ends
  - Free at both ends
  - One end fixed, the other free

- At specific frequencies, known as **natural frequencies**, an **integer number of half wavelengths** will fit on the length of the string
  - As progressive waves of different natural frequencies are sent along the string, standing waves with different numbers of nodes and antinodes form

Standing Waves in Pipes

- When the air within a pipe vibrates, longitudinal waves travel along the pipe
- Simply blowing across the open end of a pipe can produce a standing wave in the pipe
- For a pipe, there is more than one possible boundary condition, theses are pipes that are:
  - Closed at both ends
  - Open at both ends
  - Closed at one end and open on the other

Nodes & Antinodes

- When a progressive wave travels towards a free end for a string, or open end for a pipe:
  - The **reflected wave** is **in phase** with the incident wave
  - The amplitudes of the incident and reflected waves add up
  - A free end is a location of maximum displacement - i.e. an **antinode**

- When a progressive wave travels towards a fixed end for a string, or closed end for a pipe:
  - The **reflected wave** is in **anti-phase** with the incident wave
  - The two waves cancel out
  - A **fixed** end is a location of zero displacement - i.e. a **node**
  - The **open** end is therefore a location of maximum displacement - i.e. an **antinode**

Harmonics in Strings & Pipes

Harmonics

- Stationary waves can have different wave patterns, known as **harmonics**
  - These depend on the **frequency** of the vibration and the **boundary conditions** (i.e. fixed and/or free ends)
- The harmonics are the only frequencies and wavelengths that will form standing waves on strings or in pipes

Harmonics on Strings

- The **boundary condition** is that **both ends are fixed**
- The simplest wave pattern is a single loop made up of two nodes (i.e. the two fixed ends) and an antinode
  - This is called the **first harmonic**
  - The wavelength of this harmonic is $\lambda_1 = 2L$
  - Using the wave equation, the frequency is $f_1 = \frac{v}{2L}$, where *v* is the wave speed of the travelling waves on the string (i.e. the incident wave and the reflected wave)
- As the vibrating **frequency** increases, more complex patterns arise
  - The **second** harmonic has three nodes and two antinodes
  - The **third** harmonic has four nodes and three antinodes

- The *n*th harmonic will have (*n* + 1) nodes and *n* antinodes
- The general expression for the **wavelength of the *n*th harmonic** on a string that is fixed at both ends is:

$$\lambda_n = \frac{2L}{n}$$

- Where:
  - λ_n = wavelength in metres (m)
  - L = length of the string in metres (m)
  - n = integer number greater than zero - i.e. 1, 2, 3...

- Knowing the wavelength λ_n of the standing wave and the speed v of the travelling waves (i.e. incident and reflected), the **natural frequency** f_n of any harmonic can be calculated using the wave equation $v = f\lambda_n$, so that:

$$f_n = \frac{nv}{2L}$$

Harmonics in Pipes

- The **boundary conditions** vary, since pipes can have:
  - **two open ends**
  - only **one open end**

- For a pipe that is **open at both ends**:
  - The simplest wave pattern is one central node and two antinodes
  - The second harmonic consists of two nodes and three antinodes
  - The nth harmonic will have $(n + 1)$ antinodes and n nodes
  - The expression for the wavelength of the nth harmonic in a pipe of length L is the same as that given above for nth harmonic on a string

- For a pipe that is **open at one end**:
  - The lowest harmonic is a "**half-loop**" with one node and one antinode
  - The next possible harmonic will have two nodes and two antinodes
    - This is the **third** harmonic, **not** the second one
    - Since **only odd harmonics can exist** under this boundary condition
- The expression for the wavelength of the nth harmonic in a pipe of length L is:

$$\lambda_n = \frac{4L}{n}$$

- Where this time, n is an odd number - i.e. 1, 3, 5...
- Under both boundary conditions, the natural frequencies are once again calculated from the **wavelength** of the standing wave and the **speed** v of the travelling waves using the wave equation

Worked example

Transverse waves travel along a stretched wire 100 cm long. The speed of the waves is 250 m s^{-1}.

Determine the maximum harmonic detectable by a person who can hear up to 15 kHz.

Answer:

Step 1: Write down the known quantities

- Length of the wire, L = 100 cm = 1.00 m
- Speed of the waves, v = 250 m s^{-1}
- Maximum frequency of human hearing, f_n = 15 kHz = 15 000 Hz

Step 2: Write down the equation for the frequency of the *n*th harmonic and rearrange for n

$$f_n = \frac{nv}{2L} \Rightarrow n = \frac{2Lf_n}{v}$$

Step 3: Substitute the numbers into the above equation

$$n = \frac{2 \times 1.00 \times 15\,000}{250} = 120$$

- The person can hear up to the 120th harmonic

The Nature of Resonance

Free & Forced Oscillations

Free Oscillations

- Free oscillations occur when there is no transfer of energy to or from the surroundings
  - This happens when an oscillating system is displaced and then left to oscillate
- In practice, this only happens in a vacuum. However, anything vibrating in air is still considered a free vibration as long as there are **no external forces** acting upon it
- Therefore, a **free oscillation** is defined as:

  An oscillation where there are only internal forces (and no external forces) acting and there is no energy input

- A free vibration always oscillates at its **resonant frequency**

Forced Oscillations

- In order to sustain oscillations in a simple harmonic system, a periodic force must be applied to replace the energy lost in damping
  - This periodic force **does work** on **resistive** forces (i.e. the force that decreases the amplitude of the oscillations), such as air resistance
  - It is sometimes known as an **external driving** force
- This period force creates **forced oscillations** (or vibrations) is are defined as:

  Oscillations which are produced by a periodic external force

- Forced oscillations are made to oscillate at the same frequency as the external oscillator creating the external, periodic driving force
  - This means the driving force can change the frequency of the oscillator

Resonance

- The frequency of the forced oscillations on a system is referred to as the **driving frequency** f
- All oscillating systems have a **natural frequency** f_0 which is defined as

  The frequency of an oscillation when the oscillating system is allowed to oscillate freely

- Oscillating systems can exhibit a property known as **resonance** when

  driving frequency f **=** **natural frequency** f_0

- When the driving frequency approaches the natural frequency of an oscillator, the system **gains more energy** from the driving force
  - Eventually, when they are equal, the oscillator vibrates with its **maximum amplitude**
  - This is **resonance**
- Resonance is defined as:

When the frequency of the applied force to an oscillating system is equal to its natural frequency, the amplitude of the resulting oscillations is at its maximum

Example of resonance: a child pushed on a swing

- Every system (in this case, the swing and the child) has a **fixed natural frequency**

- A small push (the **driving force**) after each cycle increases the **amplitude** of the oscillations, resulting in the swing's motion back and forth
- The frequency at which the swing is pushed is the **driving frequency**

- When the **driving** frequency is **equal** to the **natural** frequency of the swing, **resonance** occurs
- If the driving frequency is slightly lower or higher than the natural frequency, the amplitude will increase but to a **lesser** extent than if they were equal
- This is because, at resonance, **energy** is transferred from the **driver** to the **oscillating** system **most efficiently**
  - Therefore, at resonance, the driving force transfers the **maximum kinetic energy** to the system
  - In this case, the child will swing the highest when resonance occurs

Worked example

State and explain whether the following scenarios are examples of free or forced oscillations:

(a) Striking a tuning fork
(b) Breaking a glass from a high pitched sound
(c) The interior of a car vibrating when travelling at a high speed
(d) Playing the clarinet

Answer:

(a) Striking a tuning fork

- This is a free vibration
- When a tuning fork is struck, it will vibrate at its natural frequency and there are no other external forces

(b) Breaking a glass from a high-pitched sound

- This is a forced vibration
- The glass is forced to vibrate at the same frequency as the sound until it breaks (when it equals the natural frequency of the glass)
- The frequency of the high-pitched sound is the external driving frequency

(c) The interior of a car vibrating when travelling at a particular speed

- This is a forced vibration
- The interior of the car vibrates at the same frequency as the wheels travelling over a rough surface at a high speed

(d) Playing the clarinet

- This is a forced vibration
- The air from the player's lungs is used to sustain the vibration in the air column in a clarinet to create and hold a sound
- The air column inside the clarinet mimics the vibrations at the same frequency as the air forced into the mouthpiece of the clarinet (the reed). This creates the sound

The Effect of Damping

Types of Damping

- In practice, all oscillators eventually stop oscillating
  - Their amplitudes decrease rapidly, or gradually
- This happens due to **resistive forces**, such as friction or air resistance, which act in the **opposite** direction to the motion of an oscillator
- Resistive forces acting on an oscillating simple harmonic system cause **damping**
  - These are known as **damped** oscillations
- Damping is defined as:

> ***The reduction in energy and amplitude of oscillations due to resistive forces on the oscillating system***

- Damping continues until the oscillator comes to rest at the equilibrium position
- A key feature of simple harmonic motion is that the **frequency** of damped oscillations **does not change** as the amplitude decreases
  - For example, a child on a swing can oscillate back and forth once every second, but this time remains the same regardless of the amplitude

Types of Damping

- There are three degrees of damping depending on how quickly the amplitude of the oscillations decreases:
  - **Light** damping
  - **Critical** damping
  - **Heavy** damping

Light Damping

- When oscillations are lightly damped, the amplitude **does not decrease linearly**
  - It decays exponentially with time
- When a lightly damped oscillator is displaced from the equilibrium, it will oscillate with **gradually decreasing** amplitude
  - For example, a swinging pendulum decreasing in amplitude until it comes to a stop
- **Key features of a displacement-time graph for a lightly damped system:**
  - There are many oscillations represented by a sine or cosine curve with gradually decreasing amplitude over time
  - This is shown by the height of the curve decreasing in both the positive and negative displacement values
  - The amplitude decreases **exponentially**
  - The frequency of the oscillations remains constant, this means the time period of oscillations must stay the same and each peak and trough are equally spaced

Critical Damping

- When a critically damped oscillator is displaced from the equilibrium, it will **return** to rest at its equilibrium position in the shortest possible time **without** oscillating
  - For example, car suspension systems prevent the car from oscillating after travelling over a bump in the road
- **Key features of a displacement-time graph for a critically damped system:**
  - This system does **not** oscillate, meaning the displacement falls to 0 straight away
  - The graph has a fast decreasing gradient when the oscillator is first displaced until it reaches the x axis
  - When the oscillator reaches the equilibrium position ($x = 0$), the graph is a horizontal line at $x = 0$ for the remaining time

Heavy Damping

- When a heavily damped oscillator is displaced from the equilibrium, it will take a **long time** to return to its equilibrium position **without** oscillating
- The system returns to equilibrium more slowly than the critical damping case
  - For example, door dampers to prevent them from slamming shut

- **Key features of a displacement-time graph for a heavily damped system:**
  - There are no oscillations. This means the displacement does not pass 0
  - The graph has a **slow decreasing gradient** from when the oscillator is first displaced until it reaches the x-axis
  - The oscillator reaches the equilibrium position (x = 0) after a **long** period of time, after which the graph remains a **horizontal** line for the remaining time

Worked example

A mechanical weighing scale consists of a needle which moves to a position on a numerical scale depending on the weight applied.

Sometimes the needle moves to the equilibrium position after oscillating slightly, making it difficult to read the number on the scale to which it is pointing.

Suggest, with a reason, whether light, critical or heavy damping should be applied to the mechanical weighing scale to read the scale more easily.

Answer:

- Ideally, the needle should not oscillate before settling - this means the scale should have either **critical** or **heavy damping**
- Since the scale is read straight away after a weight is applied, ideally the needle should settle as quickly as possible
- Heavy damping would mean the needle will take some time to settle on the scale
- Therefore, **critical damping** should be applied to the weighing scale so the **needle can settle as quickly as possible** to read from the scale

Effects of Damping

- The effects of damping can be seen on a **resonance curve**
- This is a graph of driving frequency *f* against amplitude *A* of oscillations
- It has the following key features:
  - When $f < f_0$, the amplitude of oscillations **increases**
  - At the peak where $f = f_0$, the amplitude is at its **maximum**
    - This is **resonance**
  - When $f > f_0$, the amplitude of oscillations starts to **decrease**

- Damping **reduces** the amplitude of resonance vibrations
- The height and shape of the resonance curve will therefore change slightly depending on the degree of damping
  - **Note:** the natural frequency f_0 of the oscillator will remain the same

- As the degree of damping is increased, the resonance graph is altered in the following ways:
  - The amplitude of resonance vibrations **decrease**, meaning the peak of the curve lowers
  - The resonance peak **broadens**
  - The resonance peak moves slightly to the **left** of the natural frequency when heavily damped

- Therefore, damping reduced the sharpness of resonance and reduces the amplitude at resonant frequency