HL IB Physics

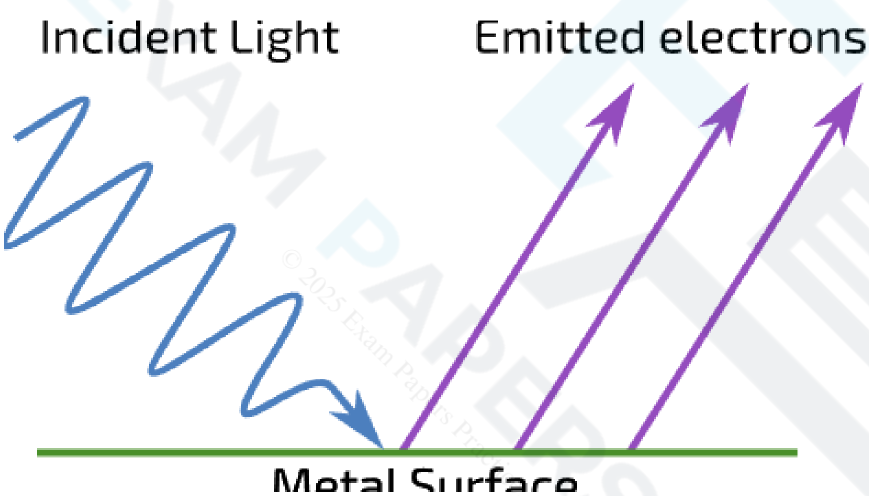
Quantum Physics

Contents

The Photoelectric Effect (HL)

The Photoelectric Effect

- The **photoelectric effect** is a phenomenon in which electrons are emitted from the surface of a metal upon the **absorption** of electromagnetic radiation
  - Electrons removed from the surface of a metal in this manner are known as **photoelectrons**
- The photoelectric effect provides important evidence that light behaves as a **particle** i.e. it is quantised, or carried in discrete packets
  - This is shown by the fact each electron can absorb only a single photon



Photoelectrons are emitted from the surface of metal when light shines onto it

Threshold Frequency

- Photoelectrons are emitted from the surface of a metal when light of sufficient energy shines on it
- The frequency of the photons required for the photoelectric effect to occur is called the **threshold frequency**
- The threshold frequency of a metal is defined as:

  The minimum frequency of incident electromagnetic radiation required to remove a photoelectron from the surface of a metal

- Threshold frequency and wavelength are properties of a material and vary from metal to metal

Threshold frequencies and wavelengths for different metals

Metal	Threshold Frequency f_0 / Hz	Threshold Wavelength λ_0 / nm
sodium	4.40×10^{14}	682

potassium	5.56×10^{14}	540
zinc	1.02×10^{15}	294
iron	1.04×10^{15}	289
copper	1.13×10^{15}	266
gold	1.23×10^{15}	244
silver	9.71×10^{15}	30.9

The Work Function

- The work function Φ, or threshold energy, of a material is defined as:

  The minimum energy required to release a photoelectron from the surface of a metal

- Consider the electrons in a metal as trapped inside an 'energy well' where the energy between the surface and the top of the well is equal to the work function Φ
  - **One** electron absorbs **one** photon
  - Therefore, an electron can only escape from the surface of the metal if it absorbs a photon which has an energy equal to the work function Φ or higher
- Different metals have different threshold frequencies and hence different work functions
- Using the well analogy:
  - A more tightly bound electron requires **more** energy to reach the top of the well
  - A less tightly bound electron requires **less** energy to reach the top of the well
- Alkali metals, such as sodium and potassium, have threshold frequencies in the **visible light region**
  - This is because the attractive forces between the surface electrons and positive metal ions are relatively weak
- Transition metals, such as zinc and iron, have threshold frequencies in the **ultraviolet region**
  - This is because the attractive forces between the surface electrons and positive metal ions are much stronger

The Photoelectric Equation (HL)

The Photoelectric Equation

- The energy of a photon is equal to:

$$E = hf$$

- This equation shows that:
  - Photon energy (E) and frequency (f) are directly proportional
  - Therefore, a photon which has a **greater** frequency than the **threshold frequency** of the metal will also have a **greater** energy than the **work function** of the metal
- When a photon is incident on the surface of a metal, its energy is divided as follows:
  - The energy equal to the work function is used to liberate a **photoelectron** from the metal
  - The remaining energy will be transferred to the photoelectron as **kinetic energy**
- This can be described using the **photoelectric equation**:

$$hf = \phi + E_{k\ max}$$

- The **maximum kinetic energy** a photoelectron can have is therefore:

$$E_{k\ max} = hf - \phi$$

- Where:
  - *h* = Planck's constant (J s)
  - *f* = frequency of the incident radiation (Hz)
  - Φ = work function of the metal (J)
  - $E_{K\ max}$ = maximum kinetic energy of a photoelectron (J)
- The photoelectric equation shows that **incident photons**:
  - Which do **not** have enough energy to **overcome** the work function (Φ) will not liberate any photoelectrons
  - Which have a frequency equal to the **threshold frequency** (f_0) will be **just able** to liberate photoelectrons from the surface of the metal
  - These photons have energy equal to:

$$E = hf_0 = \Phi$$

- The photoelectric equation shows that for **photoelectrons**:
  - Those that are **just able** to escape the surface of the metal will have **zero** kinetic energy
  - The majority will have kinetic energies **less** than $E_{k\ max}$
  - The maximum kinetic energy $E_{k\ max}$ depends only on the **frequency** of the incident photon and **not** the intensity of the radiation

Graphical Representation of Work Function

- The photoelectric equation can be rearranged into the equation of a straight-line:

$$y = mx + c$$

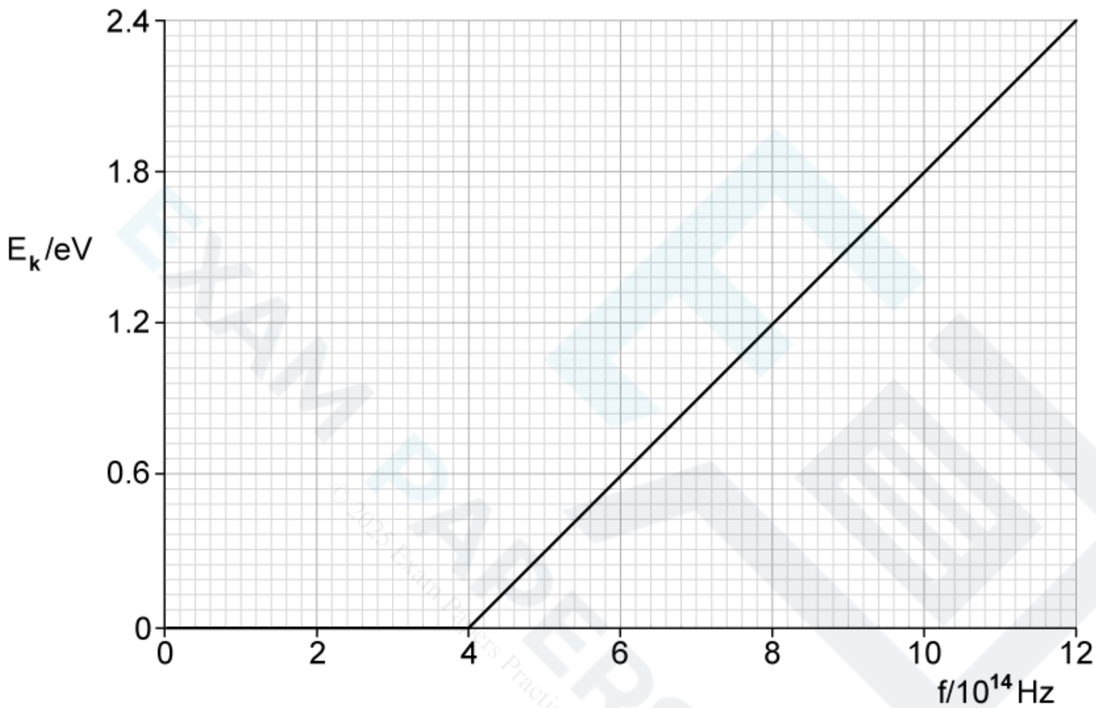
- Comparing this to the photoelectric equation:

$$E_{K\ max} = hf - \Phi$$

- A graph of maximum kinetic energy $E_{K\ max}$ against frequency f can be obtained

Worked example

The graph below shows how the maximum kinetic energy E_k of electrons emitted from the surface of sodium metal varies with the frequency *f* of the incident radiation.



Calculate the work function of sodium in eV.

Answer:

Step 1: Write out the photoelectric equation and rearrange it to fit the equation of a straight line

$$hf = \Phi + E_{K\,max}$$

$$E_{K\,max} = hf - \Phi$$

$$y = mx + c$$

- Therefore, when $E_K = 0$, $hf = \Phi$ and $f = f_0$

Step 2: Identify the threshold frequency from the x-axis of the graph

- From the graph:
  - When $E_K = 0$, threshold frequency: $f = f_0 = 4 \times 10^{14}$ Hz

Step 3: Calculate the work function

$$\Phi = hf_0 = (6.63 \times 10^{-34}) \times (4 \times 10^{14})$$

Work function: $\Phi = 2.652 \times 10^{-19}$ J

Step 4: Convert the work function into eV

- To convert from J to eV: divide by 1.6×10^{-19} J

$$E = \frac{2.652 \times 10^{-19}}{1.6 \times 10^{-19}} = 1.66 \text{ eV}$$

Intensity & Photoelectric Current

Kinetic Energy & Intensity

- The **kinetic energy** of the photoelectrons is **independent of the intensity** of the incident radiation
  - This is because **each electron can only absorb one photon**
  - Kinetic energy is only dependent on the **frequency** of the incident radiation
- **Intensity** is the **rate of energy transferred per unit area** and is related to the number of incident photons striking the metal plate
- Increasing the number of photons striking the metal plate will not increase the kinetic energy of the photoelectrons; it will increase the number of photoelectrons emitted

Why Kinetic Energy is a Maximum

- Each electron in the metal acquires the **same** amount of energy from the photons in the incident monochromatic radiation.
- However, the energy required to remove an electron from the metal varies because some electrons are on the surface whilst others are deeper in the metal
  - The photoelectrons with the maximum kinetic energy will be those on the **surface** of the metal since they do not require as much energy to leave the metal
  - The photoelectrons with lower kinetic energy are those deeper within the metal since some of the energy absorbed from the photon is used to approach the metal surface (and overcome the work function)
  - There is less kinetic energy available for these photoelectrons once they have left the metal

Photoelectric Current

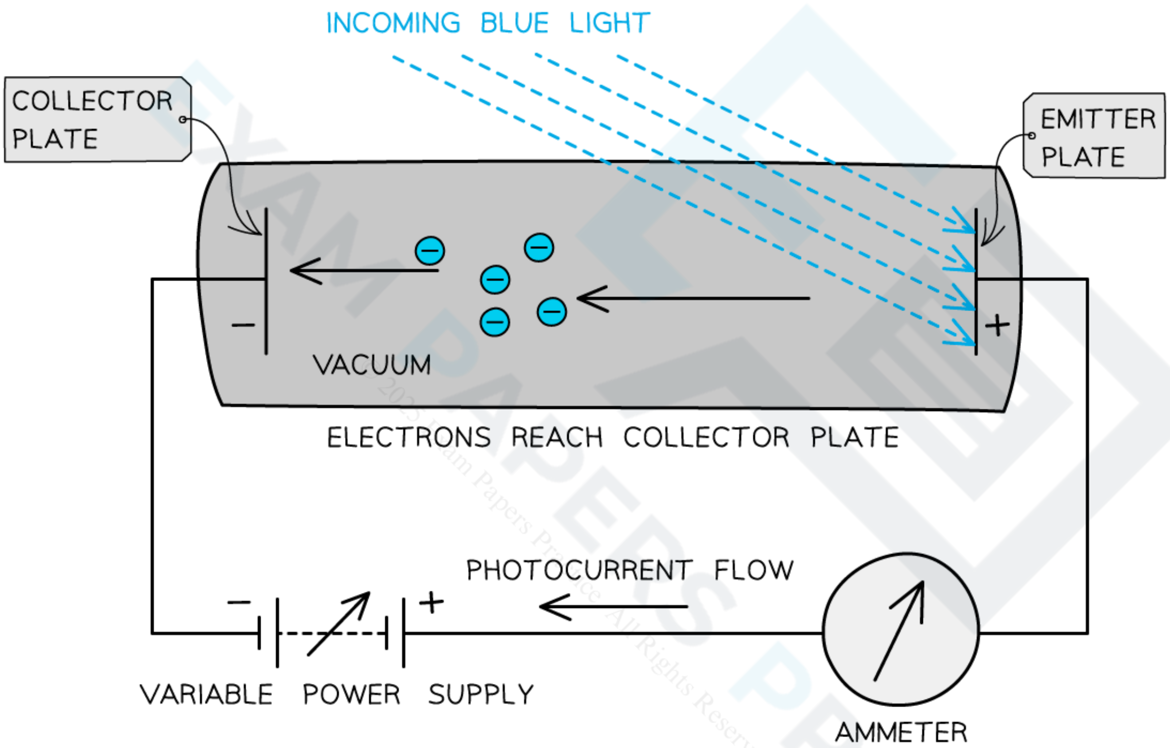
- The photoelectric current is a measure of the number of photoelectrons **emitted** per second
  - The value of the photoelectric current is calculated by the number of electrons emitted multiplied by the charge on one electron
- **Photoelectric current** is proportional to the **intensity** of the radiation incident on the surface of the metal
- This is because intensity is proportional to the number of photons striking the metal per second
- Since each photoelectron absorbs a single photon, the photoelectric current must be proportional to the intensity of the incident radiation

Stopping Potential

- Stopping potential V_s is defined as:

The potential difference required to stop photoelectron emission from occurring

- The photons arriving at the metal plate cause photoelectrons to be emitted
  - This is called the **emitter plate**
- The electrons that cross the gap are collected at the other metal plate
  - This is called the **collector plate**



This setup can be used to determine the maximum kinetic energy of the emitted photoelectrons

- The flow of electrons across the gap sets up an e.m.f. between the plates that allows a current to flow around the rest of the circuit
  - Effectively, it becomes a photoelectric cell which produces a **photoelectric current**
- If the e.m.f. of the variable power supply is initially **zero**, the circuit operates **only** on the photoelectric current
- As the supply is turned up, the emitter plate becomes more **positive**
  - This is because it is connected to the positive terminal of the supply
- As a result, electrons leaving the emitter plate are **attracted** back towards it
  - This is because the p.d. across the tube **opposes** the motion of the electrons between the plates
- If any electrons escape with high enough kinetic energy, they can **overcome** this attraction and cross to the collector plate

- And if they don't have enough energy, they can't cross the gap
- By **increasing** the e.m.f. of the supply, eventually, a p.d. will be reached at which no electrons will be able to cross the gap
  - This value of e.m.f. is equal to the stopping potential V_S
- At this point, the energy needed to cross the gap is equal to the maximum kinetic energy $E_{K\ max}$ of the electrons

$$V_s = \frac{W}{Q} = \frac{E}{Q} = \frac{E_{k\ max}}{e}$$

- Therefore:

$$E_{K\ max} = eV_S$$

- Where:
  - $E_{K\ max}$ = maximum kinetic energy of the electrons (J)
  - e = elementary charge (C)
  - V_S = stopping voltage (V)

Intensity and Stopping Potential

- Increasing the **intensity** of the incident radiation on the plate increases
  - The **number of photons** incident on the metal plate
  - The number of photoelectrons emitted from the plate, i.e. the **photoelectric current**
- For a given potential difference, increasing the intensity increases the photoelectric current but the stopping potential remains the **same**
  - This shows that the **intensity** does **not** affect the **kinetic energy** of the photoelectrons
- The **maximum kinetic energy** of the photons (and photoelectrons) depends only on
  - The frequency (or wavelength) of the incident photons
  - The work function of the metal
- However, if the frequency or wavelength is changed whilst keeping the intensity constant, the photoelectric current will **not** be constant
- For example, **increasing** the frequency of the incident radiation whilst keeping the intensity **constant** will cause the photoelectric current to **decrease**. This is because:
  - Increasing the frequency of a source means the energy of each photon increases
  - Keeping intensity the same means the energy transferred per unit area in a given time is constant
  - So, a higher frequency source must emit **fewer** photons per unit area in a given time than a lower frequency source (of the same intensity)
  - If there are fewer photons incident on a given area each second, the number of electrons emitted each second must **decrease**

Worked example

Monochromatic light of wavelength λ_1 is incident on the surface of a metal. The stopping potential for this light is V_1.

When another monochromatic light of wavelength λ_2 is incident on the same surface, the stopping potential is V_2.

What is the quantity $\dfrac{(V_2 - V_1)}{\left(\frac{1}{\lambda_2} - \frac{1}{\lambda_1}\right)}$ equal to?

A. $\dfrac{h}{2\pi}$ B. $\dfrac{h}{c}$ C. $\dfrac{hc}{e}$ D. $\dfrac{h}{e}$

Answer: C

- The photoelectric equation for light 1 is

$$hf_1 = \Phi + E_{K\ max(1)}$$

Where $E_{K\ max(1)} = eV_1$ and $E_1 = hf_1 = \dfrac{hc}{\lambda_1}$

- The photoelectric equation for light 2 is

$$hf_2 = \Phi + E_{K\ max(2)}$$

Where $E_{K\ max(2)} = eV_2$ and $E_2 = hf_2 = \dfrac{hc}{\lambda_2}$

- Since the metal is the same, the work function Φ is the same for both, so:

$$\Phi = \frac{hc}{\lambda_1} - eV_1 = \frac{hc}{\lambda_2} - eV_2$$

- Collecting the terms together and simplifying gives

$$eV_2 - eV_1 = \frac{hc}{\lambda_2} - \frac{hc}{\lambda_1}$$

$$e(V_2 - V_1) = hc\left(\frac{1}{\lambda_2} - \frac{1}{\lambda_1}\right)$$

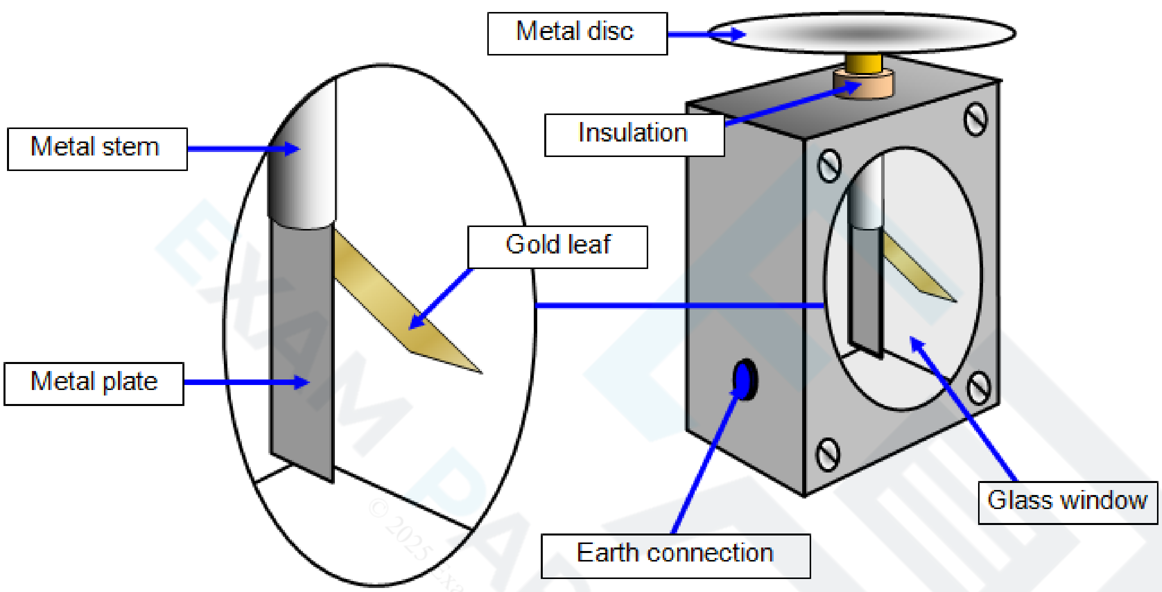
$$\frac{(V_2 - V_1)}{\left(\frac{1}{\lambda_2} - \frac{1}{\lambda_1}\right)} = \frac{hc}{e}$$

The Particle Nature of Light (HL)

Evidence for the Particle Nature of Light

- The photoelectric effect can be observed on a **gold leaf electroscope**

- A plate of metal, usually **zinc**, is attached to a rod and a strip of gold leaf
  - The plate is given a negative charge, causing the gold leaf to be repelled
- **UV light** is then shone onto the metal plate, leading to the **emission** of photoelectrons
- The gold leaf begins to fall back towards the central rod
  - This is because the plate, rod and gold leaf become less negatively charged and therefore repel less



Typical set-up of the gold leaf electroscope experiment

- Some notable observations from the gold leaf experiment:
  - The gold leaf begins to fall down instantly when illuminated with UV light
  - The gold leaf takes longer to fall at low intensities of UV light, although it still happens instantly
  - The gold leaf does not fall down when lower frequencies of light are used
  - The gold leaf does not fall down at lower frequencies, even if the intensity of the light is increased
- The photoelectric effect is **evidence** for the **particle nature** of light
- The observations cannot be explained by the **wave** model of light
  - The wave model predicts that photoelectrons would be released at any frequency since energy would be accumulated by the electron with each wavefront
  - Wave theory suggests that for a given frequency, the energy of the wave is proportional to the intensity of the beam of light; therefore, the kinetic energy of the emitted photoelectrons should be dependent on the intensity

Explanations of Photoelectric Emission

1. The gold leaf begins to fall down instantly when illuminated with UV light

- The fact that the gold leaf falls down at all indicates that electrons are emitted from the surface of the metal plate
  - As electrons are emitted, the net charge on the plate and gold leaf decreases

- The gold leaf is repelled less, so it falls down
- UV light has a high frequency, so the energy of a UV photon is sufficient to release an electron from the surface of the metal
    - This supports a threshold frequency and minimum energy required to release an electron from the surface of the metal (work function)

2. The gold leaf falls slower at lower intensities of UV light, although it still happens instantly

- The fact that the gold leaf still falls indicates that electrons are still emitted at low intensities
- The fact that the gold leaf falls more slowly at lower intensities indicates that the intensity does affect the rate of photoelectric emission
- The fact that electrons are released instantly supports the one-to-one quantised energy transfer from photon to electron

3. The gold leaf does not fall down when lower frequencies of light are used

- The fact that the gold leaf does not fall when lower frequencies (for example, visible light) are used indicates that no electrons are emitted at lower frequencies
    - Lower frequency photons have lower energies so they do not have sufficient energy to release an electron from the surface of the metal
    - This supports the threshold frequency and minimum energy required to release an electron (work function)
    - This also indicates that it is intensity and not frequency that affects the rate of photoelectric emission

4. The gold leaf does not fall down at lower frequencies, even if the intensity of the light is increased

- The fact that the gold leaf still does not fall if the intensity increases indicates that no electrons are emitted below a certain frequency
- Higher intensity means more photons are incident, but none of them have sufficient energy to release an electron from the surface of the metal
    - This supports the threshold frequency and minimum energy required to free an electron from the surface (work function)

Worked example

Describe how the photoelectric effect proves the particulate nature of light.

Answer:

Step 1: Outline what wave theory predicts about the photoelectric effect

- Wave theory predicts that...
  - Energy should be transferred to the electrons continuously until they have enough energy to be ejected
  - The kinetic energy of the emitted photoelectrons should depend on the intensity of the incident wave

Step 2: Outline the observations of the photoelectric effect experiment

- However, observations show that...
  - When $f < f_0$ (below threshold frequency): no electrons are emitted
  - When $f > f_0$ (above threshold frequency): as f increases, the max KE of the emitted electrons increases
  - Increasing the intensity of the radiation does not increase the kinetic energy of the emitted electron

Step 3: Suggest how the observations support the particulate nature of light

- These observations support the particle nature of light because...
  - One photon interacts with one electron, so each photon must have an energy greater than Φ
  - Photons carry energy which is proportional to the frequency of the radiation and not intensity

The de Broglie Wavelength (HL)

The de Broglie Wavelength

- Louis de Broglie thought that if waves can behave like particles, then perhaps particles can also behave like waves
- He proposed that electrons travel through space as waves
  - This would explain why they can exhibit wave-like behaviour such as **diffraction**
- De Broglie suggested that electrons must also hold wave properties, such as **wavelength**
  - This came to be known as the **de Broglie wavelength**
- However, he realised that **all particles** can show wave-like properties, not just electrons
  - He hypothesised that all moving particles have a **matter wave** associated with them
- The definition of a de Broglie wavelength is:

  The wavelength associated with a moving particle

- De Broglie suggested that the momentum *p* of a particle and its associated wavelength *λ* are related by the equation:

$$\lambda = \frac{h}{p}$$

- Since momentum *p* = *mv*, the de Broglie wavelength can be related to the speed of a moving particle *v* by the equation:

$$\lambda = \frac{h}{mv}$$

Kinetic Energy & de Broglie Wavelength

- Kinetic energy is defined as

$$E_K = \frac{1}{2}mv^2$$

$$E_K = \frac{p^2}{2m} \quad \Rightarrow \quad p = \sqrt{2mE_K}$$

- Combining this with the previous equation gives relationship between the de Broglie wavelength of a particle to its kinetic energy:

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE_K}}$$

- Where:
  - *λ* = the de Broglie wavelength (m)

- h = Planck's constant (J s)
- p = momentum of the particle (kg m s^{-1})
- E_K = kinetic energy of the particle (J)
- m = mass of the particle (kg)
- v = speed of the particle (m s^{-1})

Worked example

A proton and an electron are each accelerated from rest through the same potential difference.

Determine the ratio: $\frac{\text{de Broglie wavelength of the proton}}{\text{de Broglie wavelength of the electron}}$

- Mass of a proton = 1.67×10^{-27} kg
- Mass of an electron = 9.11×10^{-31} kg

Answer:

Step 1: Consider how the proton and electron can be related via their masses

The proton and electron are accelerated through the same p.d., therefore, they both have the same kinetic energy

Step 2: Write the equation relating the de Broglie wavelength of a particle to its kinetic energy

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE}}$$

$$\lambda \propto \frac{1}{\sqrt{m}}$$

Step 3: Calculate the ratio

$$\frac{\text{de Broglie wavelength of the proton}}{\text{de Broglie wavelength of the electron}} = \frac{1}{\sqrt{m_p}} \div \frac{1}{\sqrt{m_e}}$$

$$\sqrt{\frac{m_e}{m_p}} = \sqrt{\frac{9.11 \times 10^{-31}}{1.67 \times 10^{-27}}} = 2.3 \times 10^{-2}$$

This means the de Broglie wavelength of the proton is 0.023 times smaller than that of the electron **OR** the de Broglie wavelength of the electron is about 40 times larger than that of the proton

Evidence for the Wave Nature of Matter

- The majority of the time, and for everyday objects travelling at normal speeds, the de Broglie wavelength is far too small for any quantum effects to be observed
  - A typical electron in a metal has a de Broglie wavelength of about 10 nm
- Therefore, the quantum effects of diffraction will only be observable when the width of the aperture is of a similar size to the de Broglie wavelength

- The electron diffraction tube can be used to investigate how the de Broglie wavelength of electrons depends on their speed

- To observe the diffraction of electrons, they must be focused through a gap similar to their de Broglie wavelength, such as an atomic lattice
  - Graphite film is ideal for this purpose because of its crystalline structure
  - The gaps between neighbouring planes of the atoms in the crystals act as slits, allowing the electron waves to spread out and create a diffraction pattern
  - This phenomenon is similar to the diffraction pattern produced when light passes through a diffraction grating
  - If the electrons acted as particles, a pattern would not be observed, instead, the particles would be distributed uniformly across the screen. The diffraction pattern is observed on the screen as a series of concentric rings
- It is observed that:
  - a larger accelerating voltage reduces the diameter of a given ring

- a lower accelerating voltage increases the diameter of the rings

- As the voltage is increased:
  - The **speed** of the electrons is increased, therefore:
    - the **momentum** of the electrons is increased
    - the **kinetic energy** of the electrons is increased
    - the **de Broglie wavelength** is decreased
  - The **angle** of **diffraction** is decreased, therefore:
    - the **radius** of the diffraction **pattern** is decreased

- Electron diffraction was the first clear evidence that matter can behave like light and has wave properties

- The de Broglie wavelength can be used to calculate the angle of the first maximum in the diffraction pattern

$$n\lambda = d\sin\theta \Rightarrow \sin\theta = \frac{\lambda}{d}$$

- Where:
  - θ = angle of diffraction of the first maximum
  - λ = de Broglie wavelength
  - d = spacing between atoms used for diffraction

- The first minimum:
  - will be half the value of the first maximum
  - will not be zero
- Subsequent minima reduce in intensity until reaching zero

Worked example

Electrons are accelerated through a film of graphite. The electrons are accelerated through a potential difference of 40 V. The spacing between the graphite atoms is 2.1×10^{-10} m.

Calculate the angle of the first minimum of the diffraction pattern.

Answer:

Step 1: Determine the kinetic energy gained by an electron

- Kinetic energy gained through a potential difference of 40 V = 40 eV

$$E_k = 40 \times (1.6 \times 10^{-19}) = 6.4 \times 10^{-18} \text{ J}$$

Step 2: Determine the speed of the electron

$$E_k = \frac{1}{2}mv^2 \Rightarrow v = \sqrt{\frac{2E_k}{m}}$$

$$v = \sqrt{\frac{2(6.4 \times 10^{-18})}{9.11 \times 10^{-31}}} = 3.748 \times 10^6 \text{ m s}^{-1}$$

Step 3: Determine the de Broglie wavelength of the electron

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

$$\lambda = \frac{6.63 \times 10^{-34}}{(9.11 \times 10^{-31})(3.748 \times 10^6)} = 1.942 \times 10^{-10} \text{ m}$$

Step 4: Determine the angle of the first maximum

$$\sin\theta = \frac{\lambda}{D}$$

$$\sin\theta = \frac{1.942 \times 10^{-10}}{2.1 \times 10^{-10}} = 0.9248$$

$$\theta = \sin^{-1}(0.9248) = 68° \text{ (2 s.f.)}$$

Step 5: Determine the angle of the first minimum

$$\frac{68}{2} = 34°$$

Wave-Particle Duality (HL)

Wave-Particle Duality

- Light can behave as a particle (i.e. photons) **and** a wave
- This phenomenon is called the wave-particle nature of light or **wave-particle duality**
- Light interacts with matter, such as electrons, **as a particle**
  - The evidence for this is provided by the photoelectric effect
- Light propagates through space **as a wave**
  - The evidence for this comes from the diffraction and interference of light in Young's Double Slit experiment

Light as a Particle

- Einstein proposed that light can be described as a quanta of energy that behave as particles, called photons
- The photon model of light explains that:
  - Electromagnetic waves carry energy in discrete packets called photons
  - The energy of the photons are quantised according to the equation ***E = hf***
  - In the photoelectric effect, each electron can absorb only a single photon - this means only the frequencies of light above the threshold frequency will emit a photoelectron
- The wave theory of light does **not** support the idea of a threshold frequency
  - The wave theory suggests any frequency of light can give rise to photoelectric emission if the exposure time is long enough
  - This is because the wave theory suggests the energy absorbed by each electron will increase gradually with each wave
  - Furthermore, the kinetic energy of the emitted electrons should increase with radiation intensity
  - However, in the photoelectric effect, this is not what is observed
- If the frequency of the incident light is above the threshold and the intensity of the light is increased, **more photoelectrons are emitted per second**
- Although the wave theory provides good explanations for phenomena such as interference and diffraction, it fails to explain the photoelectric effect

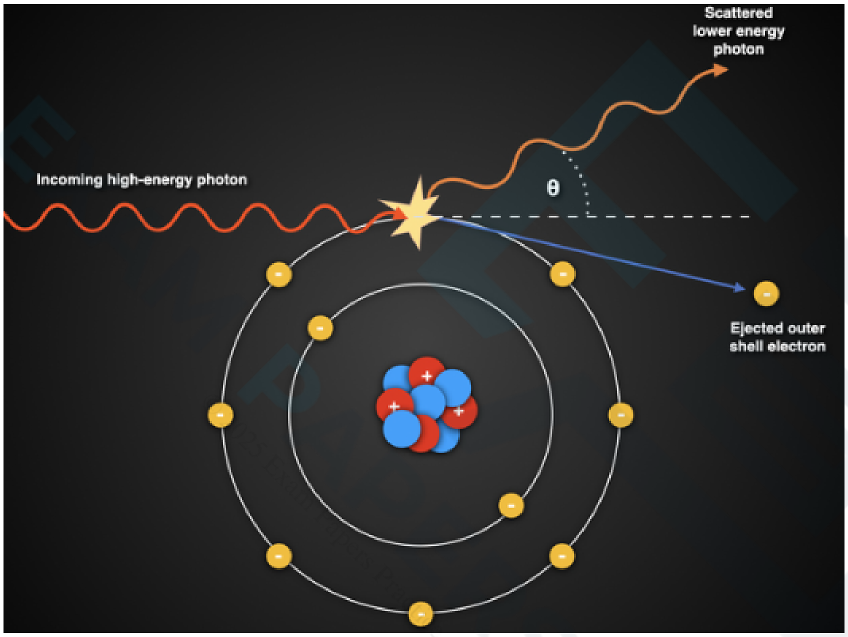
Development of the Theory of Wave-Particle Duality

- Ideas about the nature of light were contested by modern science for around 300 years
- The evidence to prove both theories was available
  - Some prominent scientists argued light was a wave
  - Others contested that light was a particle
- It was not until the early 20th century that scientists settled on a theory of duality

Compton Scattering (HL)

Compton Scattering

- Compton scattering can be observed when a high energy photon (typically X-ray or gamma) interacts with an orbital electron
  - This phenomenon is further **evidence** of the particle nature of light



- The Compton Effect is defined as:

  The interaction of a high-energy photon with an orbital electron which causes an increase in the wavelength of the photon and the ejection of the electron

- During the collision, the photon transfers some of its energy to the orbital electron
- Because of this transfer of energy:
  - The photon is **deflected** from its initial path
  - The photon's **wavelength** increases, as its energy decreases
  - The electron involved is **ejected** from the atom
- The electron and photon are deflected in different directions due to **conservation of momentum**

The Compton Formula

- The Compton scattering formula is given by

$$\Delta\lambda = \frac{h}{m_e c}(1 - \cos\theta)$$

- Where:
  - $\Delta\lambda$ = change in the wavelength of the photon $(\lambda_f - \lambda_i)$ (m)
  - h = Planck constant
  - m_e = mass of an electron (kg)
  - c = speed of light (m s^{-1})
  - θ = scattering angle of the photon (°)
- The constant $\frac{h}{m_e c}$ is known as the **Compton wavelength**
- The equation tells us:
  - The reduced wavelength of the photon depends on the scattering angle
  - The greater the scattering angle, the longer the wavelength
- This equation assumes that the electron is initially at rest before the interaction

Worked example

An X-ray photon collides with a stationary orbital electron. The scattered photon has an energy of 120 keV and the recoiling electron has an energy of 40 keV.

Determine

(a) the wavelength of the incident X-ray photon.
(b) the change in wavelength of the photon.
(c) the scattering angle of the photon.

Answer:

(a) Initial photon wavelength

- Photon energy and wavelength are related by

$$E_i = hf_i = \frac{hc}{\lambda_i} \quad \Rightarrow \quad \lambda_i = \frac{hc}{E_i}$$

- The energy of the incident photon, E_i = 120 + 40 = 160 keV

$$\lambda_i = \frac{(6.63 \times 10^{-34})(3.00 \times 10^8)}{(160 \times 10^3)(1.6 \times 10^{-19})}$$

$$\lambda_i = 7.77 \times 10^{-12} \text{ m} = 0.0078 \text{ nm}$$ (2 s.f.)

(b) Change in photon wavelength

- The energy of the scattered photon, E_f = 120 keV

$$\lambda_f = \frac{hc}{E_f}$$

$$\lambda_f = \frac{(6.63 \times 10^{-34})(3.00 \times 10^8)}{(120 \times 10^3)(1.6 \times 10^{-19})}$$

$$\lambda_f = 1.04 \times 10^{-11} \text{ m} = 0.0104 \text{ nm}$$

- Therefore, the change in wavelength is

$$\Delta\lambda = \lambda_f - \lambda_i$$

$$\Delta\lambda = 0.0104 - 0.00777 = 0.00263 \text{ nm} = 0.0026 \text{ nm (2 s.f.)}$$

(c) Photon scattering angle

- The Compton formula is

$$\Delta\lambda = \frac{h}{m_e c}(1 - \cos\theta) \quad \Rightarrow \quad \cos\theta = \left(1 - \frac{m_e c \Delta\lambda}{h}\right)$$

- The scattering angle is therefore:

$$\cos\theta = 1 - \frac{(9.11\times10^{-31})\times(3.00\times10^{8})\times(0.00263\times10^{-9})}{6.63\times10^{-34}} = -0.0841$$

$$\theta = \cos^{-1}(-0.0841) = 94.8° = 95° \text{ (2 s.f.)}$$

Worked example

Deduce the scattering angle at which

(a) no change in photon wavelength is observed
(b) the largest change in photon wavelength is observed

Answer:

(a) No change in photon wavelength

- From the Compton formula:

$$\Delta\lambda \propto (1 - \cos\theta)$$

- When $\theta = 0°$, $\cos\theta = 1$

So, $(1 - \cos\theta) = 0$ when $\theta = 0°$

- Therefore, when $\theta = 0°$, the change in photon wavelength will be zero

(b) Maximum change in photon wavelength

- When $\theta = 90°$, $\cos\theta = 0$

So, $(1 - \cos\theta) = 1$ when $\theta = 90°$

- When $\theta = 180°$, $\cos\theta = -1$

So, $(1 - \cos\theta) = 2$ when $\theta = 180°$

- Therefore, when $\theta = 180°$, the change in photon wavelength will be **twice** the Compton wavelength of the electron