**1.** The $n$th even number is $2n$.

The next even number after $2n$ is $2n + 2$

(a) Explain why.

Even numbers go up in 2s. The 2 times table:

$2 \xrightarrow{+2} 4 \xrightarrow{+2} 6 \xrightarrow{+2} 8 \xrightarrow{+2} 10$

**(1)**

(b) Write down an expression, in terms of $n$, for the next even number after $2n + 2$

$2n + 4$

**(1)**

(c) Show algebraically that the sum of any 3 consecutive even numbers is always a multiple of 6

$$2n + (2n+2) + (2n+4)$$

$$6n + 6$$

$$6(n+1)$$

Because we can factorise out 6, 6n+6 is always a multiple of 6.

**(3)**

**(5 marks)**

**2.** Prove that $(3n + 1)^2 - (3n - 1)^2$ is a multiple of 4, for all positive integer values of $n$.

$$(3n+1)(3n+1) - (3n-1)(3n-1)$$

$$(9n^2+3n+3n+1) - (9n^2-3n-3n+1)$$

$$(9n^2+6n+1) - (9n^2-6n+1)$$

$$9n^2+6n+1-9n^2+6n-1$$

$$12n$$

$$\underline{\underline{4(3n)}}$$

**(3 marks)**

**3.** Prove, using algebra, that the sum of two consecutive whole numbers is always an odd number.

$n + n+1$

$2n+1$

$2n$ is an even number. An even number plus one is always odd.

**(3 marks)**

**4.** Prove that

$$(2n + 3)^2 - (2n - 3)^2 \text{ is a multiple of } 8$$

for all positive integer values of $n$.

$$(2n+3)(2n+3) - (2n-3)(2n-3)$$

$$(4n^2 + 6n + 6n + 9) - (4n^2 - 6n - 6n + 9)$$

$$(4n^2 + 12n + 9) - (4n^2 - 12n + 9)$$

$$24n$$

$$\underline{\underline{8(3n)}}$$

**(3 marks)**

***5.** Prove algebraically that the difference between the squares of any two consecutive integers is equal to the sum of these two integers.

$n$ and $n+1$ (two consecutive integers)

$(n+1)^2 - n^2$ (diff. between squares)

$(n+1)(n+1) - n^2$

$n^2+n+n+1-n^2$

$\underline{2n+1}$

Sum of $n$ and $n+1$ = $\underline{2n+1}$

**(4 marks)**

**6.** Prove that $(5n + 1)^2 - (5n - 1)^2$ is a multiple of 5, for all positive integer values of $n$.

$(5n+1)(5n+1) - (5n-1)(5n-1)$

$(25n^2+5n+5n+1) - (25n^2-5n-5n+1)$

$(25n^2+10n+1) - (25n^2-10n+1)$

$20n$

$\underline{\underline{5(4n)}}$

**(3 marks)**

**7.** If $2n$ is always even for all positive integer values of $n$, prove algebraically that the sum of the squares of any two consecutive even numbers is always a multiple of 4.

$2n$ and $2n+2$

$(2n)^2 + (2n+2)^2$

$4n^2 + (2n+2)(2n+2)$

$4n^2 + 4n^2 + 4n + 4n + 4$

$8n^2 + 8n + 4$

$4(2n^2 + 2n + 1)$

**(3 marks)**

**8.** Prove that

$(n + 1)^2 - (n - 1)^2 + 1$ is always odd for all positive integer values of $n$.

$(n+1)(n+1) - (n-1)(n-1) + 1$

$(n^2+n+n+1) - (n^2-n-n+1) + 1$

$(n^2+2n+1) - (n^2-2n+1) + 1$

$n^2+2n+1-n^2+2n-1+1$

$4n+1$

The four times table is always even. An even plus one is odd.

**(3 marks)**

EXAM PAPERS PRACTICE

**9.** Prove algebraically that the sum of the squares of any two consecutive numbers always leaves a remainder of 1 when divided by 4.

$n$ and $n+1$

$n^2 + (n+1)^2$

$n^2 + (n+1)(n+1)$

$n^2 + n^2 + n + n + 1$

$2n^2 + 2n + 1$

$2n(n+1) + 1$

$2(n)(n+1) + 1$

2 × an even is always a multiple of 4, either n or n+1 is even. ∴ 2(n)(n+1) is divisible by 4. There is a remainder of 1.

**(4 marks)**