HL IB Physics

Processing Uncertainties

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Random & Systematic Errors

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- Measurements of quantities are made with the aim of finding the true value of that quantity
  - In reality, it is impossible to obtain the true value of any quantity as there will always be a degree of **uncertainty**
- The uncertainty is an estimate of the difference between a **measurement reading** and the **true value**
- The two types of **measurement errors** that lead to uncertainty are:
  - Random errors
  - Systematic errors

Random Errors

- Random errors cause unpredictable fluctuations in an instrument's readings as a result of uncontrollable factors, such as environmental conditions
- This affects the precision of the measurements taken, causing a wider spread of results about the mean value
- To **reduce** random error:
  - **Repeat** measurements several times and calculate an average from them

Reading Errors

- When measuring a quantity using an **analogue** device such as a ruler, the uncertainty in that measured quantity is **±0.5 the smallest measuring interval**
- When measuring a quantity using a **digital** device such as a digital scale or stopwatch, the uncertainty in that measured quantity is **±1 the smallest measuring interval**
- To **reduce** reading errors:
  - Use a **more precise** device with **smaller measuring intervals** and therefore less uncertainty

Systematic Errors

- Systematic errors arise from the use of faulty instruments or from flaws in the experimental method
- This type of error is repeated consistently every time the instrument is used or the method is followed, which affects the accuracy of all readings obtained
- To **reduce** systematic errors:
  - Instruments should be **recalibrated**, or different instruments should be used
  - Corrections or adjustments should be made to the technique

Zero Errors

- This is a type of systematic error which occurs when an instrument gives a reading when the **true reading is zero**
  - For example, a top-ban balance that starts at 2 g instead of 0 g
- To **account for** zero errors
  - Take the **difference** of the **offset** from each value
  - For example, if a scale starts at 2 g instead of 0 g, a measurement of 50 g would actually be 50 – 2 = 48 g
  - The offset could be positive or negative

Precision

- Precise measurements are ones in which there is **very little spread** about the **mean** value, in other words, **how close** the measured values are **to each other**
- If a measurement is repeated several times, it can be described as **precise** when the values are **very similar to, or the same** as, each other
  - Another way to describe this concept is if the **random uncertainty** of a measurement is **small**, then that measurement can be said to be **precise**
- The precision of a measurement is reflected in the values recorded – measurements to a greater number of decimal places are said to be more **precise** than those to a whole number

Accuracy

- A measurement is considered **accurate** if it is close to the true value
  - Another way to describe this concept is if the **systematic error** of a measurement is **small**, then that measurement can be said to be **accurate**
- The accuracy can be **increased by repeating measurements** and finding a mean of the results
- Repeating measurements also helps to identify anomalies that can be omitted from the final results

Reliability

- Reliability is defined as

  A measure of the ability of an experimental procedure to produce the expected results when using the same method and equipment

- A reliable **experiment** is one which produces consistent results when repeated many times
- Similarly, a reliable **measurement** is one which can be reproduced consistently when measured repeatedly
- When thinking about the reliability of an experiment, a **good question** to ask is
  - Would similar conclusions be reached if someone repeated this experiment?

Validity

- The validity of an experiment relates to the experimental method and the appropriate choice of variables
- Validity is defined as

  A measure of the suitability of an experimental procedure to measure what it is intended to measure

- It is essential that any variables that may affect the outcome of an experiment are **identified** and **controlled** in order for the results to be **valid**
- For example, when using Charles' law to determine absolute zero, **pressure** must be kept constant
- When thinking about the validity of an experiment, a **good question** to ask is
  - How relevant is this experiment to my original research question?

Calculating Uncertainties

Calculating Uncertainties

- There is always a degree of uncertainty when measurements are taken; the uncertainty can be thought of as the difference between the **actual** reading taken (caused by the equipment or techniques used) and the **true value**
- Uncertainties are **not** the same as errors
  - Errors can be thought of as issues with equipment or methodology that cause a reading to be different from the true value
  - The uncertainty is a range of values around a measurement within which the true value is expected to lie, and is an **estimate**

- For example, if the true value of the mass of a box is 950 g, but a systematic error with a balance gives an actual reading of 952 g, the uncertainty is ±2 g
- These uncertainties can be represented in a number of ways:
  - **Absolute Uncertainty:** where uncertainty is given as a fixed quantity
  - **Fractional Uncertainty:** where uncertainty is given as a fraction of the measurement
  - **Percentage Uncertainty:** where uncertainty is given as a percentage of the measurement

$$\text{percentage uncertainty} = \frac{\text{uncertainty}}{\text{measured value}} \times 100\%$$

- To find uncertainties in different situations:
  - **The uncertainty in a reading:** ± half the smallest division
  - **The uncertainty in a measurement:** at least ±1 smallest division
  - **The uncertainty in repeated data:** half the range i.e. ± ½ (largest - smallest value)
  - **The uncertainty in digital readings:** ± the last significant digit unless otherwise quoted
  - **The uncertainty in the natural log of a value:** absolute uncertainty in ln(x) = $\frac{\text{uncertainty in } x}{x}$

- Always make sure your absolute or percentage uncertainty is to the same number of **significant figures** as the reading

Combining Uncertainties

- When combining uncertainties, the rules are as follows:

Operation	Example	Propagation Rule
Addition & Subtraction	$y = a \pm b$	$\Delta y = \Delta a + \Delta b$ The sum of the absolute uncertainties
Multiplication & Division	$y = a \times b$ or $y = \frac{a}{b}$	$\frac{\Delta y}{y} = \frac{\Delta a}{a} + \frac{\Delta b}{b}$ The sum of the fractional uncertainties
Power	$y = a^{\pm n}$	$\frac{\Delta y}{y} = n\left(\frac{\Delta a}{a}\right)$ The magnitude of n times the fractional uncertainty

Adding / Subtracting Data

- **Add** together the absolute uncertainties

Multiplying / Dividing Data

- **Add** the percentage or fractional uncertainties

Raising to a Power

- **Multiply** the percentage uncertainty by the power

Determining Uncertainties from Graphs

Determining Uncertainties from Graphs

- The uncertainty in a measurement can be shown on a graph as an **error bar**
- This bar is drawn above and below the point (or from side to side) and shows the **uncertainty** in that measurement
- Error bars are plotted on graphs to show the **absolute uncertainty** of values plotted
- To calculate the **uncertainty in a gradient**, two lines of best fit should be drawn on the graph:
  - The 'best' line of best fit, which passes as **close** to the points **as possible**
  - The 'worst' line of best fit, either the **steepest possible** or the **shallowest possible** line which fits within all the error bars
- The percentage uncertainty in the **gradient** can be found using the magnitude of the 'best' and 'worst' gradients:

$$\text{percentage uncertainty} = \frac{best\ gradient - worst\ gradient}{best\ gradient} \times 100\%$$

- Either the steepest or shallowest line of best fit may have the 'worst' gradient on a case-by-case basis.
  - The 'worst' gradient will be the one with the **greatest difference** in magnitude from the 'best' line of best fit.
  - The equation **above** is for the case where the 'worst' gradient is the **shallowest**.
  - If the 'worst' gradient is the **steepest**, then the 'worst' gradient should be **subtracted** from the 'best' gradient and **then** divided by the best gradient and multiplied by 100
- Alternatively, the **average** of the two maximum and minimum lines can be used to calculate the percentage uncertainty:

$$\text{percentage uncertainty} = \frac{max.\ gradient - min.\ gradient}{2} \times 100\%$$

- The percentage uncertainty in the **y-intercept** can be found using:

$$\text{percentage uncertainty} = \frac{best\ y\ intercept - worst\ y\ intercept}{best\ y\ intercept} \times 100\%$$

$$\text{percentage uncertainty} = \frac{max.\ y\ intercept - min.\ y\ intercept}{2} \times 100\%$$

Percentage Difference

- The percentage difference gives an indication of how close the **experimental value** achieved from an experiment is to the **accepted value**
  - It is **not** a percentage uncertainty
- The percentage difference is defined by the equation:

$$\text{percentage difference} = \frac{experimental\ value - accepted\ value}{accepted\ value} \times 100\%$$

- The experimental value is sometimes referred to as the 'measured' value
- The accepted value is sometimes referred to as the 'true' value
  - This may be labelled on a component such as the capacitance of a capacitor or the resistance of a resistor
  - Or, from a reputable source such as a peer-reviewed data booklet
- For example, the acceleration due to gravity g is known to be 9.81 m s^{-2}. This is its **accepted value**
  - From an experiment, the value of g may be found to be 10.35 m s^{-2}
  - Its **percentage difference** would therefore be 5.5 %
- The **smaller** the percentage difference, the more **accurate** the results of the experiment