HL IB Physics

Motion in Electromagnetic Fields

Contents

Magnetic Force on a Current-Carrying Conductor

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- A current-carrying conductor produces its own magnetic field
  - When interacting with an external magnetic field, it will experience a **force**
- The force F on a conductor carrying current I at an angle θ to a magnetic field with flux density B is defined by the equation

$$F = BIL \sin \theta$$

- Where:
  - F = force on a current-carrying conductor in a B field (N)
  - B = magnetic flux density of applied B field (T)
  - I = current in the conductor (A)
  - L = length of the conductor (m)
  - θ = angle between the conductor and applied B field (degrees)
- This equation shows that the force on the conductor can be increased by:
  - Increasing the strength of the magnetic field
  - Increasing the current flowing through the conductor
  - Increasing the length of the conductor in the field
- **Note**: The length L represents the length of the conductor that is **within** the field

- A current-carrying conductor (e.g. a wire) will experience the **maximum** magnetic force if the current through it is **perpendicular** to the direction of the magnetic field lines
  - It experiences no force if it is parallel to magnetic field lines
- The **maximum** force occurs when sin θ = 1
  - This means θ = 90° and the conductor is **perpendicular** to the B field
- The equation for the magnetic force becomes:

$$F = BIL$$

- The **minimum** force, i.e. F = 0 N, is when sin θ = 0°
  - This means θ = 0° and the conductor is **parallel** to the B field
- It is important to note that a current-carrying conductor will experience **no** force if the current in the conductor is parallel to the field
  - This is because the *F*, *B* and *I* must be **perpendicular** to each other

Observing the Force on a Current-Carrying Conductor

- The force due to a magnetic field can be observed by
  - placing a copper rod in a uniform magnetic field
  - connecting the copper rod to a circuit
- When current is passed through the copper rod, it experiences a **force**
  - This causes it to accelerate in the direction of the force

Worked example

A current of 0.87 A flows in a wire of length 1.4 m placed at 30° to a magnetic field of flux density 80 mT.

Calculate the force on the wire.

Answer:

Step 1: Write down the known quantities

- Magnetic flux density, B = 80 mT = 80 × 10^{-3} T
- Current, I = 0.87 A
- Length of wire, L = 1.4 m
- Angle between the wire and the magnetic field, θ = 30°

Step 2: Write down the equation for force on a current-carrying conductor

$$F = BIL \sin \theta$$

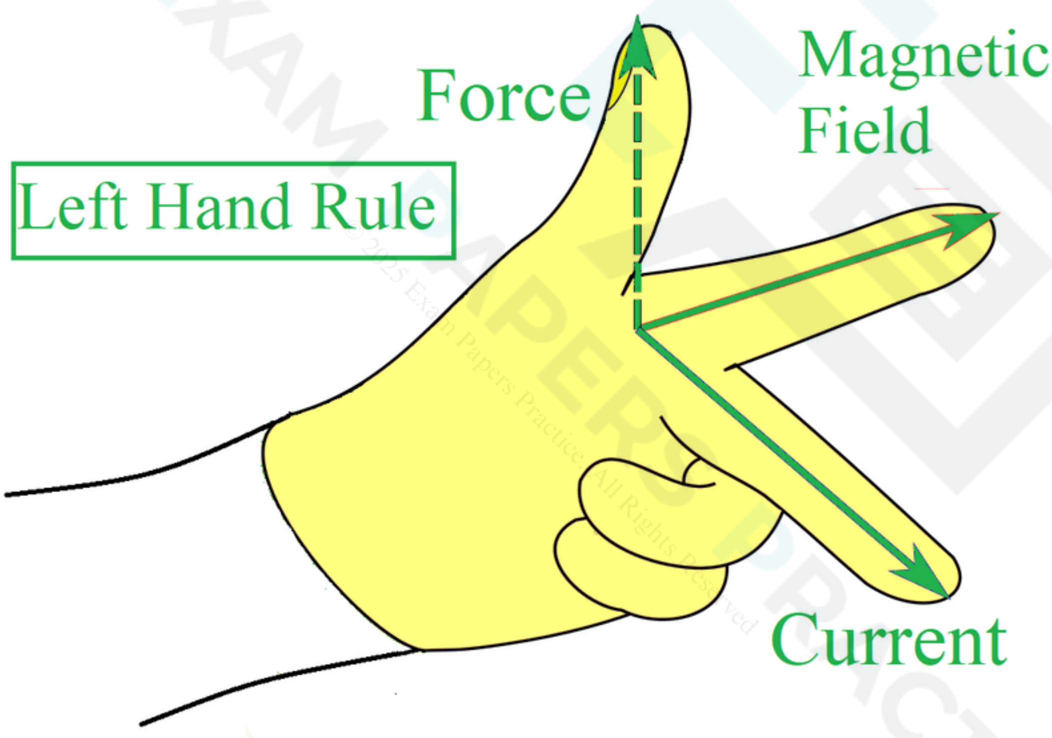
Step 3: Substitute in values and calculate

$F = (80 \times 10^{-3}) \times (0.87) \times (1.4) \times \sin(30) = 0.04872 = 0.049$ N (2 s.f)

Direction of Force on a Current-Carrying Conductor

- When a current-carrying conductor is placed in a magnetic field, the force, B-field and current are all mutually **perpendicular** to each other
  - Their directions can be determined using **Fleming's left-hand rule**
- To use Fleming's left-hand rule, point the thumb, first finger and second finger at right angles to each other
  - The **thumb** points in the direction of **motion** or **force** ***F*** of the conductor
  - The **first** finger points in the direction of the applied **magnetic field** ***B***
  - The **second** finger points in the direction of the flow of conventional **current** ***I*** (from positive to negative)

Fleming's Left-Hand Rule



Fleming's left-hand rule allows us to visualise the 3D arrangement of the force, magnetic field and current

Representing Magnetic Fields in 3D

- When solving problems in three-dimensional space, the current, force or magnetic field could be directed into or out of the page
- When the magnetic field is directed into or out of the page, the following symbols are used:

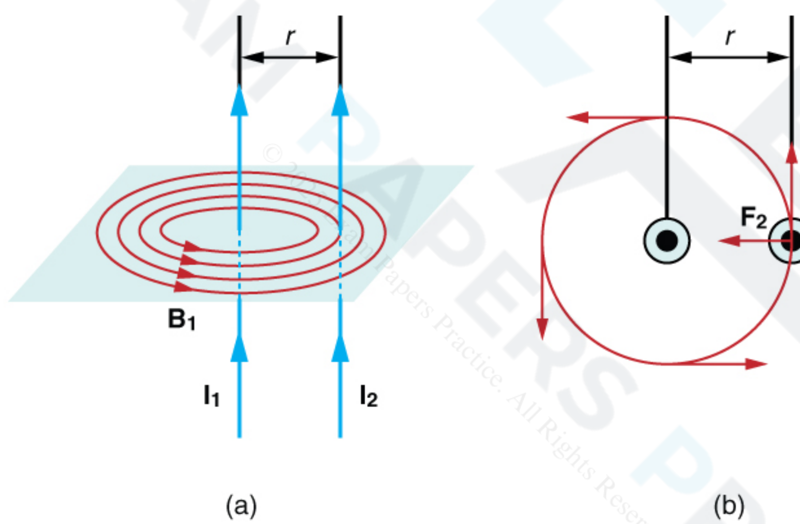
- **Dots** (sometimes with a circle around them) represent the magnetic field directed **out** of the plane of the page
- **Crosses** represent the magnetic field directed **into** the plane of the page

- The way to remember this is by imagining an arrow used in archery or darts:
  - If the arrow is approaching **head-on**, such as out of a page, only the very tip of the arrow can be seen (a dot)
  - When the arrow is **receding away**, such as into a page, only the cross of the feathers at the back can be seen (a cross)

Magnetic Force between Two Parallel Conductors

Magnetic Force Between Two Parallel Conductors

- A current carrying conductor, such as a wire, produces a magnetic field around it
- The direction of the field depends on the direction of the current through the wire
  - This is determined by the **right hand thumb rule**
- Parallel current-carrying conductors will therefore either attract or repel each other
  - If the currents are in the **same** direction in both conductors, the magnetic field lines between the conductors cancel out – the conductors will **attract** each other
  - If the currents are in the **opposite** direction in both conductors, the magnetic field lines between the conductors push each other apart – the conductors will **repel** each other



Both wires will attract if their currents are in the same direction and repel if in opposite directions

- When the conductors **attract**, the direction of the magnetic forces will be **towards** each other
- When the conductors **repel**, the direction of the magnetic forces will be **away** from each other
- The magnitude of each force depends on the amount of current and the length of the wire

Force per Unit Length Between Two Parallel Conductors

- The ratio $\frac{F}{L}$ is the force per unit length between two parallel currents I_1 and I_2 separated by a distance r

- The force is **attractive** if the currents are in the **same direction** and **repulsive** if they are in **opposite directions**
- It is calculated using the equation:

$$\frac{F}{L} = \mu_0 \frac{I_1 I_2}{2\pi r}$$

- Where:
  - F = the force applied between the two parallel wires (N)
  - L = the length of each parallel conductor (m)
  - μ_0 = the constant for the magnetic permeability of free space ($4\pi \times 10^{-7}$ N A^{-2})
  - I_1 = the current through the first conducting wire (A)
  - I_2 = the current through the second conducting wire (A)
  - r = the separation between the two conducting wires (m)

Obtaining the Equation

- The force from wire 2 on wire 1, $\boldsymbol{F_2 = B_2 I_1 L \sin(\theta)}$
- In this situation the magnetic field is perpendicular to the current in the wire, so $\sin(\theta) = 1$
- $F_2 = -F_1$ so the force between them is F
- The force on a unit length of the wires is then given by $\frac{F}{L} = \frac{B_2 I_1 L}{L}$
    - Hence, $\frac{F}{L} = B_2 I_1$
- The magnitude of the magnetic field at a radial distance, r away from the current conducting wire is:

$$B = \frac{\mu_0 I}{2\pi r}$$

    - In this case the magnetic field strength from B_2 at a distance r away from wire 2 is: $B_2 = \frac{\mu_0 I_2}{2\pi r}$
- Substituting for B_2 into the force per unit length equation gives us: $\frac{F}{L} = \left(\frac{\mu_0 I_2}{2\pi r}\right) I_1$

Worked example

Two long, straight, current-carrying conductors, WX and YZ, are held at a close distance, as shown in diagram 1.

diagram 1

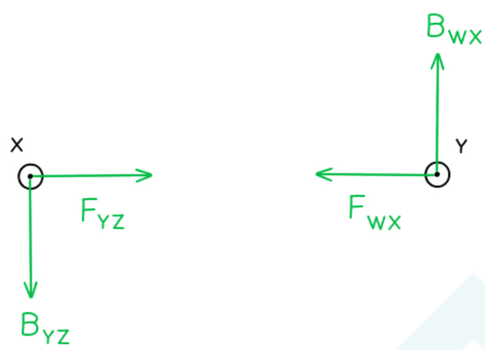
The conductors each carry the same magnitude current in the same direction. A plan view from above the conductors is shown in diagram 2.

diagram 2

On diagram 2, draw arrows, one in each case, to show the direction of:

- The magnetic field at X due to the current in wire YZ (label this arrow B_{YZ})
- The force at X as a result of the magnetic field due to the current in the wire YZ (label this arrow F_{YZ})
- The magnetic field at Y due to the current in wire WX (label this arrow B_{WX})
- The force at Y as a result of the magnetic field due to the current in the wire WX (label this arrow F_{WX})

Answer:



- **Newton's third law**: When two bodies interact, the force on one body is **equal but opposite** in direction to the force on the other body
- Therefore, the forces on the wires act in equal but opposite directions

Magnetic Force on a Charge

Magnetic Force on a Charge

- A moving charge produces its own magnetic field
  - When interacting with an applied magnetic field, it will experience a **force**
- The force *F* on an isolated particle with charge Q moving with speed *v* at an angle θ to a magnetic field with flux density *B* is defined by the equation

$$F = Bqv \sin \theta$$

- Where:
  - F = magnetic force on the particle (N)
  - B = magnetic flux density (T)
  - q = charge of the particle (C)
  - v = speed of the particle (m s^{-1})
- Current is taken as the rate of flow of **positive** charge (i.e. **conventional** current)
  - This means that the direction of the current for a flow of negative charge (e.g. a beam of electrons) is in the opposite direction to its motion
- As with a current-carrying conductor, the **maximum** force on a charged particle occurs when it travels **perpendicular** to the field
  - This is when $\theta = 90°$, so $\sin \theta = 1$
- The equation for the magnetic force becomes:

$$F = Bqv$$

- *F*, *B* and *v* are mutually perpendicular, therefore:
  - If the direction of the particle's motion changes, the magnitude of the force will also change
  - If the particle travels **parallel** to a magnetic field, it will experience **no** magnetic force

- From the diagram above, when a beam of electrons enters a magnetic field which is directed **into** the page:
  - Electrons are negatively charged, so current *I* is directed to the **right** (as motion *v* is directed to the **left**)
  - Using Fleming's left hand rule, the force on an electron will be directed **upwards**

Worked example

An electron moves in a uniform magnetic field of flux density 0.2 T at a velocity of 5.3×10^7 m s^{-1}.

(a) Calculate the force on the electron when it moves perpendicular to the field.
(b) Determine the angle the electron must make with the field for the force in (a) to half.

Answer:

(a)

Step 1: Write out the known quantities

- Velocity of the electron, $v = 5.3 \times 10^7$ m s^{-1}
- Charge of an electron, $q = 1.60 \times 10^{-19}$ C
- Magnetic flux density, $B = 0.2$ T

Step 2: Write down the equation for the magnetic force on an isolated particle

$$F = Bqv \sin \theta$$

- The electron moves perpendicular ($\theta = 90°$) to the field, so $\sin \theta = 1$

$$F = Bqv$$

Step 3: Substitute in values, and calculate the force on the electron

$$F = (0.2) \times (1.60 \times 10^{-19}) \times (5.3 \times 10^7) = 1.7 \times 10^{-12} \text{ N (2 s.f.)}$$

(b)

Step 1: Write an expression for the ratio of the two forces

- When the electron moves perpendicular to the field: $F_\perp = Bqv$
- When the electron moves at angle θ to the field: $F_\theta = Bqv \sin \theta$
- The ratio of these forces is

$$\frac{F_\theta}{F_\perp} = \frac{Bqv \sin \theta}{Bqv} = \sin \theta$$

Step 2: Determine the angle when the ratio of the forces is equal to one-half

- When the force halves, the ratio is

$$\frac{F_\theta}{F_\perp} = \frac{1}{2}$$

- The angle this occurs at is

$$\sin\theta = \frac{1}{2}$$

$$\theta = \sin^{-1}\left(\frac{1}{2}\right) = 30°$$

Direction of Force on a Moving Charge

- The direction of the magnetic force on a charged particle depends on
  - The direction of flow of **current**
  - The direction of the **magnetic field**
- This can be found using **Fleming's left-hand rule**
- The second finger represents the **current** flow or the flow of **positive** charge
  - For a **positive** charge, the current points in the **same** direction as its velocity
  - For a **negative** charge, the current points in the **opposite** direction to its velocity

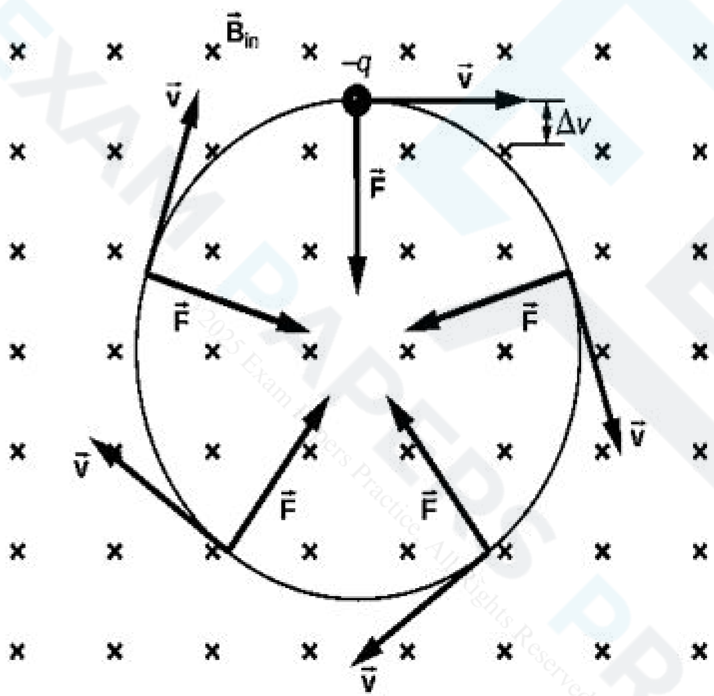
- From the diagram above, when a **positive** charge enters a magnetic field from left to right, using Fleming's left-hand rule:
  - The first finger (**field**) points **into** the page
  - The second finger (**current**) points to the **right**
  - The thumb (**force**) points **upwards**

- When a charged particle moves in a uniform magnetic field, the force acts perpendicular to the field and the particle's velocity
  - As a result, it follows a **circular** path

Charged Particles in Magnetic Fields

Charged Particles in Magnetic Fields

- When a charged particle enters a uniform magnetic field, it travels in a **circular** path
- This is because the direction of the magnetic force *F* will always be
  - perpendicular to the particle's velocity *v*
  - directed towards the centre of the path, resulting in circular motion



In a magnetic field, a charged particle travels in a circular path as the force, velocity and field are all perpendicular

- The magnetic force *F* provides the centripetal force on the particle
- The equation for centripetal force is:

$$F = \frac{mv^2}{r}$$

- Equating this to the magnetic force on a moving charged particle gives the expression:

$$\frac{mv^2}{r} = BQv$$

- Rearranging for the radius *r* gives an expression for the **radius** of the path of a charged particle in a perpendicular magnetic field:

$$r = \frac{mv}{BQ}$$

- Where:
  - r = radius of the path (m)
  - m = mass of the particle (kg)
  - v = linear velocity of the particle (m s^{-1})
  - B = magnetic field strength (T)
  - Q = charge of the particle (C)
- This equation shows that:
  - Faster moving particles with speed *v* move in larger circles (larger *r*): $r \propto v$
  - Particles with greater mass *m* move in larger circles: $r \propto m$
  - Particles with greater charge *q* move in smaller circles: $r \propto \frac{1}{q}$
  - Particles moving in a strong magnetic field *B* move in smaller circles: $r \propto \frac{1}{B}$
- The centripetal acceleration is in the **same direction** as the magnetic (centripetal) force
- This can be found using Newton's second law:

$$F = ma$$

Worked example

An electron travels at right angles to a uniform magnetic field of flux density 6.2 mT. The speed of the electron is 3.0×10^6 m s^{-1}.

Calculate the radius of the circular path of the electron.

Answer:

Step 1: List the known quantities

- Electron charge-to-mass ratio = $\frac{e}{m_e}$ = 1.76×10^{11} C kg^{-1} (from formula sheet)
- Magnetic flux density, B = 6.2 mT = 6.2×10^{-3} T
- Speed of the electron, $v = 3.0 \times 10^6$ m s^{-1}

Step 2: Write an expression for the radius of an electron in a magnetic field

centripetal force = magnetic force

$$\frac{m_e v^2}{r} = Bev$$

$$r = \frac{m_e v}{eB}$$

Step 3: Substitute the known values into the expression

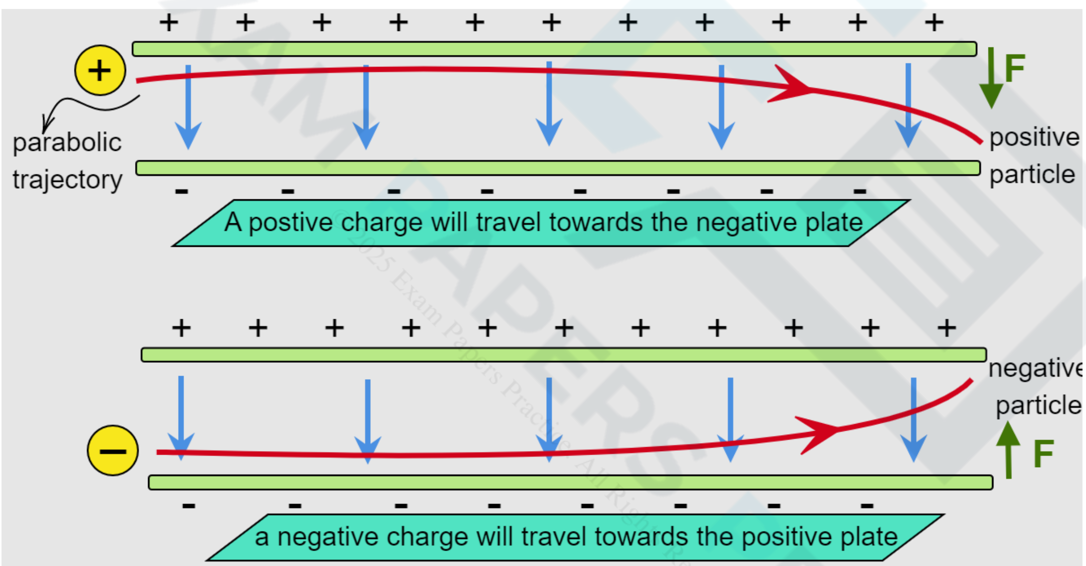
$$\frac{m_e}{e} = \frac{1}{1.76 \times 10^{11}}$$

$$r = \frac{3.0 \times 10^6}{(1.76 \times 10^{11}) \times (6.2 \times 10^{-3})} = 2.7 \times 10^{-3} = 2.7 \text{ mm}$$

Charged Particles in Electric Fields

Charged Particles in Electric Fields

- A **charged** particle in an **electric** field will experience a **force** on it that will cause it to **move**
- If a charged particle remains **stationary** in a uniform electric field, it will move **parallel** to the electric field lines (along or against the field lines depending on its charge)
- If a charged particle is in **motion** through a uniform electric field (e.g. between two charged parallel plates), it will experience a constant electric force and travel in a **parabolic trajectory**



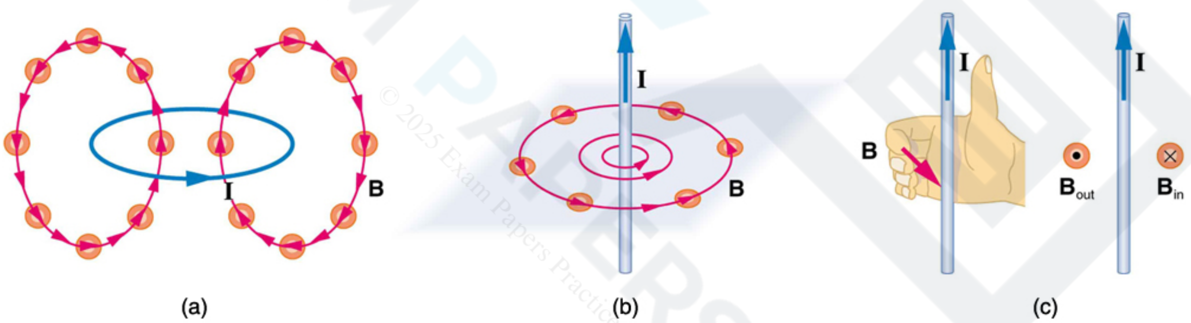
The parabolic path of charged particles in a uniform electric field

- The direction of the parabola will depend on the charge of the particle
  - A **positive** charge will be deflected towards the **negative** plate
  - A **negative** charge will be deflected towards the **positive** plate
- The force on the particle is the same at all points and is always in the same direction
  - **Note:** an uncharged particle, such as a neutron experiences **no** force in an electric field and will therefore travel **straight through** the plates undeflected
- The amount of deflection depends on the following properties of the particles:
  - **Mass** – the **greater** the mass, the **smaller** the deflection and vice versa
  - **Charge** – the **greater** the magnitude of the charge of the particle, the **greater** the deflection and vice versa
  - **Speed** – the **greater** the speed of the particle, the **smaller** the deflection and vice versa

Charged Particles in Electric & Magnetic Fields

Charged Particles in Electric & Magnetic Fields

- A **charged particle** moving in perpendicularly orientated uniform **electric** and **magnetic fields** will experience
  - a force **parallel** to the electric field
  - a force **perpendicular** to the magnetic field
- One particular orientation is:
  - a charged particle moving with speed v to the right of the x-axis
  - an electric field E directed up the y-axis
  - a magnetic field B directed out of the page on the z-axis
- Hence, the three vectors are **perpendicular** to each other



An example of the orientation of an electric field perpendicular to a magnetic field

Motion of a Positively Charged Particle

- When the particle is **positively** charged
  - the electric force acts **upwards**, in the same direction as the electric field
  - the magnetic force acts **downwards**, perpendicular to the magnetic field
- Using Fleming's left hand rule:
  - **Field** (first finger): the magnetic field is directed **out** of the page
  - **Current** (second finger): the positive charge moves to the **right**
  - **Force** (thumb): the magnetic force acts **downwards**
- Hence, the electric force and magnetic force act in **opposite directions** on the positive charge

Motion of a Negatively Charged Particle

- When the particle is **negatively** charged
  - the electric force acts **downwards**, in the opposite direction to the electric field
  - the magnetic force acts **upwards**, perpendicular to the magnetic field
- Using Fleming's left hand rule:
  - **Field** (first finger): the magnetic field is directed **out** of the page
  - **Current** (second finger): the positive charge moves to the **left** (since the negative charge moves to the right, in the opposite direction)
  - **Force** (thumb): the magnetic force acts **upwards**
- Hence, the electric force and magnetic force act in **opposite directions** on the negative charge

Balancing the Electric and Magnetic Fields

- The field strengths of each field can be adjusted until the forces cancel each other out
- If the magnitude of the electric and magnetic forces are equal, the particle will move in a straight line with **constant speed**
- This speed can be determined by equating the two forces:

$$F_E = F_B$$

- Where:
  - The electric force on the particle: $F_E = qE$
  - The magnetic force on the particle: $F_B = Bqv$
- Equating these and rearranging for speed *v* gives:

$$qE = Bqv$$

$$v = \frac{E}{B}$$

- Therefore, the speed *v* is equal to the ratio of the electric and magnetic field strengths

Worked example

An electron passes between two parallel metal plates moving with a constant velocity of 2.1 × 10 m s . The potential difference between the plates is 3100 V. A uniform magnetic field of magnitude 0.054 T acts perpendicular to the electric field and the movement of the electron.

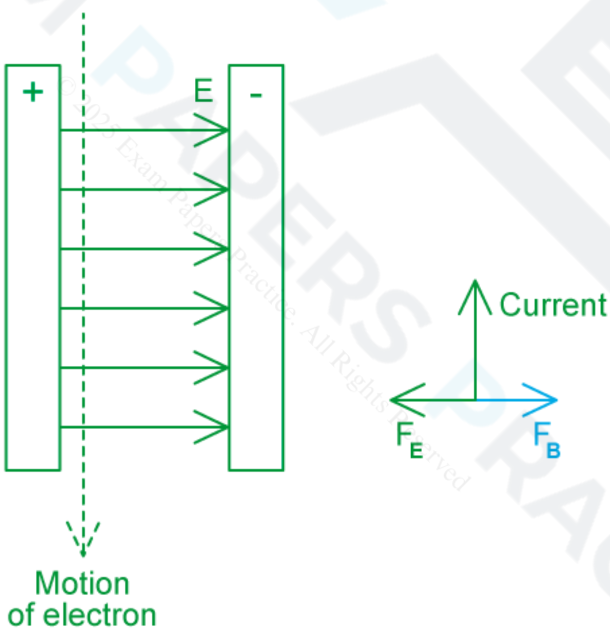
The electric field acts to the right and the electron is moving downwards.

(a) Determine the direction of the magnetic field
(b) Calculate the separation of the plates

Answer:

(a) The direction of the magnetic field:

Step 1: Draw a diagram of the situation



- The electric field goes (from the positive plate to the negative plate), to the right
- The electron is moving vertically downwards
- So, the current is moving upwards in the opposite direction to the electron
- The electric force is acting in the opposite direction to the electric field because the particle is an electron

Step 2: Determine the direction of the magnetic field

- The electron is moving at a constant speed, so the magnetic and electric forces are equal and opposite
  - Hence, the magnetic force acts to the left

(b) Calculate the separation of the plates:

Step 1: Calculate the magnitude of the electric field, *E*

$$v = \frac{E}{B} \quad \Rightarrow \quad E = vB$$

$$E = (2.1 \times 10^7) \times 0.054 = 1.134 \times 10^6 \text{ N C}^{-1}$$

Step 2: Calculate the separation of the plates

- Use the electric field strength equation:

$$E = \frac{V}{d} \quad \Rightarrow \quad d = \frac{V}{E}$$

$$d = \frac{3100}{1.134 \times 10^6}$$

$$d = 2.73 \times 10^{-3} \text{ m}$$

Charge to Mass Ratio of Particles

- The charge-to-mass ratio of a particle is defined as:

  The ratio of the total charge of a particle to its mass

- It can be calculated using the equation:

$$\text{charge-to-mass ratio} = \frac{charge}{mass} = \frac{Q}{m}$$

- The charge-to-mass ratio of an **electron** is $\frac{e}{m_e} = \frac{1.60 \times 10^{-19}}{9.11 \times 10^{-31}} = 1.76 \times 10^{11}$ C kg^{-1}
- The charge-to-mass ratio of a **proton** is $\frac{e}{m_p} = \frac{1.60 \times 10^{-19}}{1.67 \times 10^{-27}} = 9.58 \times 10^{7}$ C kg^{-1}

Determining Charge-to-Mass Ratio

- The charge-to-mass ratio of a charged particle can be determined by investigating its path in a uniform magnetic field
- The method used by J.J. Thomson to determine the charge-to-mass ratio of an electron used:
  - 'Helmholtz coils' to generate a **uniform magnetic field**
  - oppositely charged parallel plates to generate a **uniform electric field**

- When moving in a magnetic field, a charge experiences a force perpendicular to its motion
  - From the diagram above, the magnetic field B is directed **into** the plane of the page
  - From Fleming's left-hand rule, the magnetic force F_B on an electron acts **downwards**
- When moving in a uniform electric field, a charge experiences a force towards either the positive or negative plate
  - The electrons are negative so they experience an **upward** electric force F_E towards the positive plate
- When these forces are equal in magnitude, the beam of electrons is **horizontal** and straight
- The electric field strength between two parallel plates is given by:

$$E = \frac{F}{q} = \frac{V}{d}$$

- The electric force can be adjusted by changing the potential difference V across the plates
- Therefore, the upward electric force F_E is equal to:

$$F_E = \frac{qV}{d}$$

- Where:
  - q = charge of the particle (C)
  - V = potential difference between the plates (V)
  - d = separation between the plates (m)
- The upward electric force is **adjusted** until it is equal to the downward magnetic force, making the electron beam **horizontal**:

$$F_E = F_B$$

- The downward magnetic force F_B on the particle is equal to:

$$F_B = Bqv$$

- Where:
  - v = speed of the particle (m s^{-1})
  - B = magnetic field strength (T)
- Equating the two forces and rearranging for particle speed v:

$$Bqv = \frac{qV}{d}$$

$$v = \frac{V}{Bd}$$

- If the electric field is switched off, the beam will be influenced by the magnetic field only
- The particles will then travel in a circular path with the same speed v

- The radius of the circular path of a charged particle in a magnetic field is given by:

$$r = \frac{mv}{Bq}$$

- Where:
  - r = radius of the path (m)
  - m = mass of the particle (kg)
- Rearranging for the charge-to-mass ratio:

$$\frac{q}{m} = \frac{v}{rB}$$

- To measure these quantities:
  - the radius r of the path and the magnetic field strength B can be measured directly
  - the speed v of the particles can determined using perpendicular electric and magnetic fields
- Combining these two equations gives an expression for the charge-to-mass ratio of a charged particle:

$$\frac{q}{m} = \frac{1}{rB}\left(\frac{V}{Bd}\right) = \frac{V}{rB^2 d}$$

- Therefore, these four quantities (V, d, r, B) are needed to determine a particle's charge-to-mass ratio: