

1. The equation $x^3 + 7x - 2 = 55$ has a solution between 3 and 4.

Use trial and improvement to find this solution.
 Give your answer to 1 decimal place.

x	$x^3 + 7x - 2$	comment
3.5	$(3.5)^3 + 7(3.5) - 2$ $= 65.375$	too big
3.3	$(3.3)^3 + 7(3.3) - 2$ $= 57.037$	too big
3.2	$(3.2)^3 + 7(3.2) - 2$ $= 53.168$	too small
3.25	$(3.25)^3 + 7(3.25) - 2$ $= 55.078125$	too big

..... 3.2 (4)

2. Use trial and improvement to solve $x^3 - x^2 = 85$

Give your answer to 1 decimal place.

x	$x^3 - x^2$	comment
5	$(5)^3 - (5)^2 = 100$	too big
4.8	$(4.8)^3 - (4.8)^2 = 87.552$	too big
4.7	$(4.7)^3 - (4.7)^2 = 81.733$	too small
4.75	$(4.75)^3 - (4.75)^2$ $= 84.609375$	too small

..... 4.8 (4)

3. Use trial and improvement to solve $x^3 + 5x = 70$

Give your answer to 1 decimal place.

x	$x^3 + 5x$	Comment
4	$(4)^3 + 5(4)$ $= 84$	too high
3.8	$(3.8)^3 + 5(3.8)$ $= 73.872$	too high
3.7	$(3.7)^3 + 5(3.7)$ $= 69.153$	too low
3.75	$(3.75)^3 + 5(3.75)$ $= 71.484375$	too high

..... 3.7 (4)

4. An approximate solution to an equation is found using this iterative process:

$$x_{n+1} = \sqrt{(x_n) + 10} \quad \text{and} \quad x_1 = 3$$

a) Work out the values of x_2 and x_3

$$x_2 = \sqrt{(3) + 10} = \sqrt{13} = 3.61 \text{ (2dp)}$$

$$x_3 = \sqrt{(\sqrt{13}) + 10} = 3.69 \text{ (2dp)}$$

$$\dots \sqrt{13} \dots \dots 3.69 \text{ (2dp)} \dots (2)$$

b) Work out the solution to 3 decimal places

$$x_4 = 3.6998068$$

$$x_5 = 3.701325006$$

$$x_6 = 3.70153009$$

$$x_7 = 3.701557792$$

$$x_8 = 3.701561534$$

$$\dots 3.702 \dots (1)$$

5. An approximate solution to an equation is found using this iterative process:

$$x_{n+1} = \frac{(x_n)^3 - 3}{8} \quad \text{and} \quad x_1 = -1$$

a) Work out the values of x_2 and x_3

$$x_2 = \frac{(-1)^3 - 3}{8} = -\frac{1}{2}$$

$$x_3 = \frac{-\frac{25}{64}}$$

$$\dots -\frac{1}{2} \dots \dots \frac{-25}{64} \dots \dots \quad (2)$$

b) Work out the solution to 6 decimal places

$$x_4 = -0.3824505806$$

$$x_5 = -0.3819925565$$

$$x_6 = -0.3819674637$$

$$x_7 = -0.3819660907$$

$$x_8 = -0.3819660156$$

$$-0.381966 \dots \dots \quad (1)$$

6. A sequence is defined by the term-to-term rule:

$$U_{n+1} = U_n^2 - 8U_n + 17$$

a) Given that $U_1 = 4$, find U_2 and U_3

$$U_2 = (4)^2 - 8(4) + 17 = 1$$

$$U_3 = 10$$

$$\dots\dots\dots 1 \dots\dots\dots 10 \dots\dots\dots (2)$$

b) Given instead that $U_1 = 2$, find U_2 , U_3 and U_{100}

$$U_2 = 5$$

$$U_3 = 2$$

$$U_4 = 5$$

$$\dots\dots\dots U_2 = 5 \dots\dots\dots U_3 = 2 \dots\dots\dots U_{100} = 5 \dots\dots\dots (3)$$

7.(a) Show that the equation $x^3 + 4x = 1$ has a solution between $x=0$ and $x=1$

$$x^3 + 4x - 1 = 0$$

$$(0)^3 + 4(0) - 1 = -1$$

$$(1)^3 + 4(1) - 1 = 4$$

change of sign \therefore there is a solution between 0 and 1 (2)

(b) Show that the equation $x^3 + 4x = 1$ can be rearranged

to give $x = \frac{1}{4} - \frac{x^3}{4}$

$$\begin{aligned}
 x^3 + 4x &= 1 \\
 4x &= 1 - x^3 \\
 x &= \frac{1}{4} - \frac{x^3}{4}
 \end{aligned}$$

..... (1)

(c) Starting with $x_0 = 0$, use the iteration formula $x_{n+1} = \frac{1}{4} - \frac{x_n^3}{4}$

twice, to find an estimate to the solution of $x^3 + 4x = 1$

$$x_1 = \frac{1}{4} - \frac{(0)^3}{4} = \frac{1}{4}$$

$$x_2 = 0.24609375$$

$$x_3 = 0.2462740093$$

..... (3)