HL IB Physics

Gravitational Fields

Contents

Newton's Law of Gravitation

Newton's Law of Gravitation

- The gravitational force between two bodies outside a uniform field, e.g. between the Earth and the Sun, is defined by Newton's Law of Gravitation
- Newton's Law of Gravitation states that:

  The gravitational force between two point masses is proportional to the product of the masses and inversely proportional to the square of their separation

- All planets and stars are assumed to be point masses
- In equation form, this can be written as:

$$F = \frac{Gm_1 m_2}{r^2}$$

- Where:
  - F = gravitational force between two masses (N)
  - G = Newton's Gravitational Constant
  - m_1 and m_2 = mass of body 1 and mass of body 2 (kg)
  - r = distance between the centre of the two masses (m)

- Although planets are not point masses, their separation is much larger than their radius
  - Therefore, Newton's law of gravitation applies to planets orbiting the Sun

- The $F \propto \frac{1}{r^2}$ relation is called the inverse square law
- This means that when a mass is twice as far away from another, its force due to gravity reduces by (½) = ¼

Gravitational Field Strength

Gravitational Field Strength

- There is a universal force of attraction between all matter with **mass**
  - This force is known as the 'force due to gravity' or the **weight**
- The Earth's gravitational field is responsible for the weight of all objects on Earth
- A gravitational field is defined as:

  A region of space where a test mass experiences a force due to the gravitational attraction of another mass

- The direction of the gravitational field is always towards the centre of the mass causing the field
  - Gravitational forces are **always** attractive
- Gravity has an infinite range, meaning it affects **all** objects in the universe
  - There is a **greater** gravitational force around objects with a **large mass** (such as planets)
  - There is a **smaller** gravitational force around objects with a **small mass** (almost negligible for atoms)

- The gravitational field strength at a point is defined as:

  The force per unit mass experienced by a test mass at that point

- This can be written in equation form as:

$$g = \frac{F}{m}$$

- Where:
  - g = gravitational field strength (N kg^{-1})
  - F = force due to gravity, or weight (N)
  - m = mass of test mass in the field (kg)
- This equation shows that:
  - On planets with a large value of g, the gravitational force per unit mass is **greater** than on planets with a smaller value of g
- An object's mass remains the **same** at all points in space
  - However, on planets such as Jupiter, the **weight** of an object will be greater than on a less massive planet, such as Earth
  - This means the gravitational force would be so high that humans, for example, would not be able to fully stand up

- Factors that affect the gravitational field strength at the surface of a planet are:
  - The **radius** r (or diameter) of the planet
  - The **mass** M (or density) of the planet
- This can be shown by equating the equation $F = mg$ with Newton's law of gravitation:

$$F = \frac{GMm}{r^2}$$

- Substituting the force *F* with the gravitational force *mg* leads to:

$$mg = \frac{GMm}{r^2}$$

- Cancelling the mass of the test mass *m* leads to the equation:

$$g = \frac{GM}{r^2}$$

- Where:
  - G = Newton's Gravitational Constant
  - *M = mass of the body causing the field (kg)*
  - r = distance from the mass where you are calculating the field strength (m)
- This equation shows that:
  - The gravitational field strength g depends only on the mass of the body M causing the field
  - Hence, objects with any mass m in that field will experience the **same gravitational field strength**
  - The gravitational field strength g is **inversely proportional** to the **square** of the radial distance, r^2

Worked example

The mean density of the Moon is $\frac{3}{5}$ times the mean density of the Earth. The gravitational field strength on the Moon is $\frac{1}{6}$ the gravitational field strength on Earth.

Determine the ratio of the Moon's radius r_M to the Earth's radius r_E.

Answer:

Step 1: Write down the known quantities

- g_M = gravitational field strength on the Moon, ρ_M = mean density of the Moon
- g_E = gravitational field strength on the Earth, ρ_E = mean density of the Earth

$$\rho_M = \frac{3}{5}\rho_E$$

$$g_M = \frac{1}{6}g_E$$

Step 2: Write down the equations for the gravitational field strength, volume and density

Gravitational field strength: $g = \frac{GM}{r^2}$

Volume of a sphere: $V = \frac{4}{3}\pi r^3 \quad \Rightarrow \quad V \propto r^3$

Density: $\rho = \frac{M}{V} \quad \Rightarrow \quad M = \rho V = \frac{4}{3}\pi\rho r^3 \quad \Rightarrow \quad M \propto \rho r^3$

Step 3: Substitute the relationship between M and r into the equation for g

$$g \propto \rho\frac{(r^3)}{r^2} \quad \Rightarrow \quad g \propto \rho r$$

Step 4: Find the ratio of the gravitational field strength

$$g_M \propto \rho_M r_M$$

$$g_E \propto \rho_E r_E$$

$$g_M = \frac{1}{6} g_E \quad \Rightarrow \quad \rho_M r_M = \frac{1}{6} \rho_E r_E$$

Step 5: Substitute the ratio of the densities into the equation

$$\left(\frac{3}{5}\rho_E\right) r_M = \frac{1}{6} \rho_E r_E$$

$$\frac{3}{5} r_M = \frac{1}{6} r_E$$

Step 6: Calculate the ratio of the radii

$$\frac{r_M}{r_E} = \frac{1}{6} \div \frac{3}{5} = \frac{5}{18} = 0.28$$

Gravitational Field Lines

Point Mass Approximation

- For a point outside a uniform sphere, the mass of the sphere may be considered to be a **point mass** at its centre
  - A uniform sphere is one where its mass is **distributed evenly**
- The gravitational field lines around a uniform sphere are therefore **identical to those around a point mass**
- An object can be regarded as a point mass when:

  A body covers a very large distance compared to its size, so, to study its motion, its size or dimensions can be neglected

- An example of this is field lines around planets

- Radial fields are considered **non-uniform** fields
  - So, the gravitational field strength *g* is different depending on how far an object is from the centre of mass of the sphere
- Newton's universal law of gravitation is extended to spherical masses of uniform density by assuming that their mass is concentrated at their centre i.e point masses

Representing Gravitational Fields

- Gravitational fields represent the **action** of gravitational forces between masses, the direction of these forces can be shown using vectors
  - The direction of the **vector** shows the direction of the **gravitational force** that would be exerted on a **mass** if it was placed at that position in the field
  - These vectors are known as **field lines** (or 'lines of force')
- The direction of a gravitational field is represented by gravitational field lines
  - Therefore, gravitational field lines also show the direction of **acceleration** of a mass placed in the field
- Gravitational field lines are always directed toward the centre of mass of a body
  - This is because gravitational forces are **attractive only** (they are never repulsive)
  - Therefore, masses **always** attract each other via the gravitational force
- The gravitational field around a point mass will be **radial** in shape and the field lines will always point towards the centre of mass

- The gravitational field lines around a point mass are **radially inwards**
- The gravitational field lines of a uniform field, where the field strength is the same at all points, is represented by **equally spaced parallel lines**
  - For example, the fields lines on the Earth's surface

- Radial fields are considered **non-uniform fields**
  - The gravitational field strength g is different depending on how far you are from the centre
- Parallel field lines on the Earth's surface are considered a **uniform field**
  - The gravitational field strength g is the same throughout

Gravitational Potential (HL)

Gravitational Potential

- The gravitational potential *V* at a point can, therefore, be defined as:

The work done per unit mass in bringing a test mass from infinity to a defined point

- Gravitational potential is measured in J kg^{-1}
- It is always has a **negative** value because:
  - It is defined as having a value of **zero** at **infinity**
  - Since the gravitational force is **attractive**, work must be done **on** a mass to reach infinity
- On the surface of a mass (such as a planet), gravitational potential has a negative value
  - The value becomes less negative, i.e. it increases, with distance from that mass
- Work has to be done **against** the gravitational pull of the planet to take a unit mass away from the planet
- The gravitational potential at a point depends on:
  - The **mass** of the object
  - The **distance** from the centre of mass of the object to the point

Calculating Gravitational Potential

- The equation for gravitational potential *V* is defined by the mass *M* and distance *r*:

$$V_g = -\frac{GM}{r}$$

- Where:
  - V_g = gravitational potential (J kg^{-1})
  - G = Newton's gravitational constant
  - M = mass of the body producing the gravitational field (kg)
  - r = distance from the centre of the mass to the point mass (m)
- The gravitational potential always is negative near an isolated mass, such as a planet, because:
  - The potential when *r* is at infinity (∞) is defined as zero
  - Work must be done to move a mass away from a planet (V becomes less negative)
- It is also a scalar quantity, unlike the gravitational field strength which is a vector quantity
- Gravitational forces are always **attractive**, this means as *r* decreases, positive work is done by the mass when moving from infinity to that point
  - When a mass is closer to a planet, its gravitational potential becomes smaller (more negative)
  - As a mass moves away from a planet, its gravitational potential becomes larger (less negative) until it reaches 0 at infinity
- This means when the distance *r* becomes very large, the gravitational force tends rapidly towards zero the further away the point is from a planet

Worked example

A planet has a diameter of 7600 km and a mass of 3.5 × 10 kg. A meteor of mass 6000 kg accelerates towards the planet from infinity.

Calculate the gravitational potential of the rock at a distance of 400 km above the planet's surface.

Answer:

- The gravitational potential at a point is

$$V_g = -\frac{GM}{r}$$

- Where *r* is the distance from the centre of the planet to the point i.e. the radius of the planet + the height above the planet's surface

$$r = \frac{7600}{2} + 400 = 4200 \text{ km}$$

- And *M* is the mass of the larger mass, i.e. the planet (not the meteor)

$$V_g = -\frac{(6.67 \times 10^{-11}) \times (3.5 \times 10^{23})}{4200 \times 10^3} = -5.6 \times 10^6 \text{ J kg}^{-1}$$

Gravitational Potential Energy in a Non-Uniform Field (HL)

Gravitational Potential Energy in a Non-Uniform Field

- In a radial field, gravitational potential energy (GPE) describes the energy an object possesses due to its **position** in a gravitational field
- The gravitational potential energy of a system is defined as:

  The work done to assemble the system from infinite separation of the components of the system

- Similarly, the gravitational potential energy of a point mass is defined as:

  The work done in bringing a mass from infinity to a point

Near the Earth's Surface

- The gravitational potential energy near the Earth's surface is equal to

$$E_p = mg\Delta h$$

- The GPE on the surface of the Earth is taken to be zero
  - This means work is done to **lift** the object
- This equation can **only** be used for objects that are **near the Earth's surface**
  - This is because, near Earth's surface, the gravitational field is approximated to be **uniform**
  - Far away from the Earth's surface, the gravitational field is **radial** because the Earth is a **sphere**

Gravitational Potential Energy Equation (HL)

Work Done on a Mass

- When a mass is moved against the force of gravity, work is required
  - This is because gravity is **attractive**, therefore, energy is needed to work against this attractive force
- The work done in moving a mass *m* is given by:

$$\Delta W = m\Delta V_g$$

- Where:
  - ΔW = change in work done (J)
  - m = mass (kg)
  - ΔV_g = change in gravitational potential (J kg^{-1})

Gravitational Potential Energy Equation

- In a radial field, gravitational potential energy (GPE) describes the energy an object possesses due to its **position** in a gravitational field
- The gravitational potential energy of a system is defined as:

  The work done to assemble the system from infinite separation of the components of the system

- Similarly, the gravitational potential energy of a point mass is defined as:

  The work done in bringing a mass from infinity to a point

- The equation for GPE of two point masses *m* and *M* at a distance *r* is:

$$E_p = -\frac{Gm_1 m_2}{r}$$

- Where:
  - G = universal gravitational constant (N m^2 kg^{-2})
  - m_1 = larger mass producing the field (kg)
  - m_2 = mass moving within the field of M (kg)
  - r = distance between the centre of m and M (m)

- Recall that Newton's Law of Gravitation relates the magnitude of the force F between two masses M and m:

$$F = \frac{Gm_1 m_2}{r^2}$$

- Therefore, a **force-distance** graph would be a curve, because F is **inversely proportional** to r^2, or:

$$F \propto \frac{1}{r^2}$$

- The product of **force** and **distance** is equal to work done (or energy transferred)
- Therefore, the **area** under the **force**-**distance** graph for gravitational fields is equal to the **work done**
  - In the case of a mass m moving further away from a mass M, the potential **increases**
  - Since gravity is attractive, this requires **work to be done** on the mass m
  - The area between two points under the **force**-**distance** curve, therefore, gives the change in **gravitational potential energy** of mass m

Change in Gravitational Potential Energy

- Two points at different distances from a mass will have **different** gravitational potentials
  - This is because the gravitational potential **increases** with distance from a mass
- Therefore, there will be a **gravitational potential difference** ΔV between the two points

$$\Delta V = V_f - V_i$$

- Where:
  - V_i = initial gravitational potential (J kg⁻¹)
  - V_f = final gravitational potential (J kg⁻¹)
- The change in work done against a gravitational field is equal to the change in **gravitational potential energy** (GPE)
  - When $V = 0$, then the GPE = 0
- It is usually more useful to find the **change** in the GPE of a system
  - For example, a satellite lifted into space from the Earth's surface
- The change in GPE when a mass moves towards, or away from, another mass is given by:

$$\Delta E_p = -\frac{Gm_1m_2}{r_2} - \left(-\frac{Gm_1m_2}{r_1}\right)$$

$$\Delta E_p = Gm_1m_2\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

- Where:
  - m_1 = mass that is producing the gravitational field (e.g. a planet) (kg)
  - m_2 = mass that is moving in the gravitational field (e.g. a satellite) (kg)
  - r_1 = first distance of m from the centre of M (m)
  - r_2 = second distance of m from the centre of M (m)
- The change in potential ΔV_g is the same, without the mass of the object m_2:

$$\Delta V_g = -\frac{Gm_1}{r_2} - \left(-\frac{Gm_1}{r_1}\right)$$

$$\Delta V_g = Gm_1\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

- Work is done when an object in a planet's gravitational field moves **against** the gravitational field lines i.e. **away** from the planet

Worked example

A spacecraft of mass 300 kg leaves the surface of Mars up to an altitude of 700 km.

Calculate the work done by the spacecraft.

- Radius of Mars = 3400 km
- Mass of Mars, $m_1 = 6.40 \times 10^{23}$ kg

Answer:

- The change in GPE is equal to

$$\Delta E_p = Gm_1 m_2 \left(\frac{1}{r_1} - \frac{1}{r_2} \right)$$

- Where
  - r_1 = radius of Mars = 3400 km
  - r_2 = radius + altitude = 3400 + 700 = 4100 km

$$\Delta E_p = (6.67 \times 10^{-11}) \times (6.40 \times 10^{23}) \times 300 \times \left(\frac{1}{3400 \times 10^3} - \frac{1}{4100 \times 10^3} \right)$$

Work done by satellite: $\Delta E_p = 643.1 \times 10^6 = 640 \text{ MJ}$ (2 s.f.)

Worked example

A satellite of mass 1450 kg moves from an orbit of 980 km above the Earth's surface to a lower orbit of 480 km.

Calculate the change in gravitational potential energy of the satellite.

- Mass of the Earth = 5.97×10^{24} kg
- Radius of the Earth = 6.38×10^{6} m

Answer:

Step 1: Write down the known quantities

- Initial height above Earth's surface, h_1 = 980 km
- Final height above Earth's surface, h_2 = 480 km
- Mass of the satellite, m_1 = 1450 kg
- Mass of the Earth, $m_2 = 5.97 \times 10^{24}$ kg
- Radius of the Earth, $R = 6.38 \times 10^{6}$ m

Step 2: Write down the equation for change in gravitational potential energy

$$\Delta E_p = G m_1 m_2 \left(\frac{1}{r_1} - \frac{1}{r_2} \right)$$

Step 3: Convert distances into standard units and include Earth radius

- Distance from centre of Earth to higher orbit:

$$r_1 = h_1 + R$$

$$r_1 = (980 \times 10^3) + (6.38 \times 10^6) = 7.36 \times 10^6 \text{ m}$$

- Distance from centre of Earth to lower orbit:

$$r_2 = h_2 + R$$

$$r_2 = (480 \times 10^3) + (6.38 \times 10^6) = 6.86 \times 10^6 \text{ m}$$

Step 4: Substitute values into the equation

$$\Delta E_p = (6.67 \times 10^{-11}) \times (5.97 \times 10^{24}) \times 1450 \times \left(\frac{1}{7.36 \times 10^6} - \frac{1}{6.86 \times 10^6} \right)$$

Change in gravitational potential energy: $\Delta E_p = 5.72 \times 10^{9}$ J

Gravitational Potential Gradient (HL)

Gravitational Potential Gradient

- A gravitational field can be defined in terms of the variation of gravitational potential at different points in the field**:**

  The gravitational field at a particular point is equal to the negative gradient of a potential-distance graph at that point

- The potential gradient is defined by the **equipotential lines**
  - These demonstrate the gravitational potential in a gravitational field and are always drawn **perpendicular** to the field lines
- The potential gradient in a gravitational field is defined as:

  The rate of change of gravitational potential with respect to displacement in the direction of the field

- Gravitational field strength, *g* and the gravitational potential, *V* can be graphically represented against the distance from the centre of a planet, *r*

$$g = -\frac{\Delta V_g}{\Delta r}$$

- Where:
  - g = gravitational field strength ($N\ kg^{-1}$)
  - ΔV_g = change in gravitational potential ($J\ kg^{-1}$)
  - Δr = distance from the centre of a point mass (m)
- The graph of V_g against r for a planet is:

- **The key features of this graph are:**
  - The values for V_g are all negative (because the graph is drawn below the horizontal *r* axis)
  - As *r* increases, V_g against *r* follows a $-\frac{1}{r}$ relation
  - The **gradient** of the graph at any particular point is the value of *g* at that point,

$$g = -V_g \times -\frac{1}{r} = \frac{V_g}{r}$$

  - The graph has a shallow increase as *r* increases
- To calculate *g*, draw a tangent to the graph at that point and calculate the gradient of the tangent
- This is a graphical representation of the gravitational potential equation:

$$V_g = -\frac{GM}{r}$$

where G and *M* are constant

Worked example

Determine the change in gravitational potential when travelling from 3 Earth radii (from Earth's centre) to the surface of the Earth.

Take the mass of the Earth to be 5.97×10^{24} kg and the radius of the Earth to be 6.38×10^{6} m.

Answer:

Step 1: List the known quantities

- Mass of the Earth, $M_E = 5.97 \times 10^{24}$ kg
- Radius of the Earth, $r_E = 6.38 \times 10^{6}$ m
- Initial distance, $r_1 = 3r_E = 3 \times (6.38 \times 10^{6})$ m $= 1.914 \times 10^{7}$ m
- Final distance, $r_2 = r_E = 6.38 \times 10^{6}$ m
- Gravitational constant, $G = 6.67 \times 10^{-11}$ m^3 kg^{-1} s^{-2}

Step 2: Write down the equation for potential difference

$$\Delta V_g = -GM_E\left(\frac{1}{r_2} - \frac{1}{r_1}\right)$$

Step 3: Substitute the values into the equation

$$\Delta V_g = -(6.67 \times 10^{-11}) \times (5.97 \times 10^{24}) \times \left(\frac{1}{6.38 \times 10^{6}} - \frac{1}{1.914 \times 10^{7}}\right)$$

$$\Delta V_g = -4.16 \times 10^{7} \text{ J kg}^{-1}$$

Gravitational Equipotential Surfaces (HL)

Gravitational Equipotential Surfaces

- Equipotential lines (when working in 2D) and surfaces (when working in 3D) join together points that have the same gravitational potential
- These are always:
  - **Perpendicular** to the gravitational field lines in both radial and uniform fields
  - Represented by **dotted** lines (unlike field lines, which are solid lines with arrows)
- In a radial field (e.g. a planet), the equipotential lines:
  - Are concentric circles around the planet
  - Become further apart further away from the planet
  - Remember: **radial** field is made up of lines which follow the **radius** of a circle
- In a uniform field (e.g. near the Earth's surface), the equipotential lines are:
  - Horizontal straight lines
  - Parallel
  - Equally spaced
  - Remember: **uniform** field is made up of lines which are a **uniform** distance apart
- Potential gradient is defined by the **equipotential lines**
- **No work is done** when moving along an equipotential line or surface, only **between** equipotential lines or surfaces
  - This means that an object travelling along an equipotential doesn't lose or gain energy and $\Delta V = 0$

Kepler's Laws of Planetary Motion

Kepler's Laws of Planetary Motion

Kepler's First Law

- Kepler's First Law describes the **shape** of planetary orbits
- It states:
- An ellipse is just a 'squashed' circle
  - Some planets, like Pluto, have highly elliptical orbits around the Sun
  - Other planets, like Earth, have near circular orbits around the Sun

Kepler's Second Law

- Kepler's Second Law describes the **motion** of all planets around the Sun
- It states:
- The consequence of Kepler's Second Law is that planets move **faster** nearer the Sun and **slower** further away from it

Kepler's Third Law

- Kepler's Third Law states

  For planets or satellites in a circular orbit about the same central body, the square of the time period is proportional to the cube of the radius of the orbit

- This law describes the relationship between the **time** of an orbit and its **radius**

$$T^2 \propto r^3$$

- Where:
  - T = orbital time period (s)
  - r = mean orbital radius (m)

Time Period & Orbital Radius Relation

- Since a planet or a satellite is travelling in circular motion when in order, its orbital time period T to travel the circumference of the orbit $2\pi r$, the linear speed v is:

$$v = \frac{2\pi r}{T}$$

- This is a result of the well-known equation, speed = distance / time and first introduced in the circular motion topic
- Substituting the value of the linear speed v from equating the gravitational and centripetal force into the above equation gives:

$$v^2 = \left(\frac{2\pi r}{T}\right)^2 = \frac{GM}{r}$$

- Squaring out the brackets and rearranging for T^2 gives the equation relating the time period T and orbital radius r:

$$T^2 = \frac{4\pi^2 r^3}{GM}$$

- Where:
  - T = time period of the orbit (s)
  - r = orbital radius (m)
  - G = Gravitational Constant
  - M = mass of the object being orbited (kg)
- The relationship between T and r can be shown using a logarithmic plot

$$T^2 \propto r^3 \quad \Rightarrow \quad 2\log T \propto 3\log r$$

Worked example

Planets A and B orbit the same star.

Planet A is located an average distance *r* from the star. Planet B is located an average distance 6*r* from the star

What is $\frac{\text{orbital period of planet } A}{\text{orbital period of planet } B}$?

A. $\frac{1}{\sqrt[3]{6}}$ **B.** $\frac{1}{\sqrt{6}}$ **C.** $\frac{1}{\sqrt[3]{6^2}}$ **D.** $\frac{1}{\sqrt{6^3}}$

Answer: **D**

- Kepler's third law states $T^2 \propto r^3$
- The orbital period of planet A: $T_A \propto \sqrt{r^3}$
- The orbital period of planet B: $T_B \propto \sqrt{(6r)^3}$
- Therefore the ratio is equal to:

$$\frac{T_A}{T_B} = \frac{\sqrt{r^3}}{\sqrt{(6r)^3}} = \frac{1}{\sqrt{6^3}}$$

Escape Speed (HL)

Escape Speed

- To escape a gravitational field, a mass must travel at, or above, the minimum **escape speed**
  - This is dependent on the mass and radius of the object creating the gravitational field, such as a planet, a moon or a black hole
- Escape speed is defined as:

  The minimum speed that will allow an object to escape a gravitational field with no further energy input

- It is the same for all masses in the same gravitational field
  - For example, the escape speed of a rocket is the same as a tennis ball on Earth
- The escape speed of an object is the speed at which all its kinetic energy has been transferred to gravitational potential energy
- This is calculated by equating the equations:

$$\frac{1}{2}mv_{esc}^2 = \frac{GMm}{r}$$

- Where:
  - m = mass of the object in the gravitational field (kg)
  - v_{esc} = escape velocity of the object (m s^{-1})
  - G = Newton's Gravitational Constant
  - M = mass of the object to be escaped from (i.e. a planet) (kg)
  - r = distance from the centre of mass M (m)
- Since mass m is the same on both sides of the equation, it can cancel on both sides of the equation:

$$\frac{1}{2}v_{esc}^2 = \frac{GM}{r}$$

- Multiplying both sides by 2 and taking the square root gives the equation for escape velocity v_{esc}:

$$v_{esc} = \sqrt{\frac{2GM}{r}}$$

- Rockets launched from the Earth's surface do **not** need to achieve escape velocity to reach their orbit around the Earth
- This is because:
  - They are continuously given energy through fuel and thrust to help them move
  - Less energy is needed to achieve orbit than to escape from Earth's gravitational field
- The escape velocity is **not** the velocity needed to escape the planet but to escape the planet's **gravitational field** altogether
  - This could be quite a large distance away from the planet

Worked example

Calculate the escape speed at the surface of the Moon.

- Density of the Moon = 3340 kg m^{-3}
- Mass of the Moon = 7.35×10^{22} kg

Answer:

Step 1: List the known quantities

- Gravitational constant, $G = 6.67 \times 10^{-11}$ N m^2 kg^{-2}
- Density of the Moon, ρ = 3340 kg m^{-3}
- Mass of the Moon, $M = 7.35 \times 10^{22}$ kg

Step 2: Rearrange the density equation for radius *r*

Density: $\rho = \frac{M}{V}$ and volume of a sphere: $V = \frac{4}{3}\pi r^3$

$$\rho = \frac{M}{\frac{4}{3}\pi r^3} = \frac{3M}{4\pi r^3}$$

$$r = \sqrt[3]{\frac{3M}{4\pi\rho}}$$

Step 3: Calculate the radius by substituting in the values

$$r = \sqrt[3]{\frac{3 \times (7.35 \times 10^{22})}{4\pi \times 3340}} = 1.7384 \times 10^6 \text{ m}$$

Step 4: Substitute r into the escape speed equation

$$v_{esc} = \sqrt{\frac{2GM}{r}} = \sqrt{\frac{2 \times (6.67 \times 10^{-11}) \times (7.35 \times 10^{22})}{1.7384 \times 10^6}}$$

Escape speed of the Moon: v_{esc} = 2.37 km s^{-1}

Orbital Motion, Speed & Energy (HL)

Orbital Motion, Speed & Energy

- Since most planets and satellites have near-circular orbits, the gravitational force F_G between two bodies (e.g. planet & star, planet & satellite) provides the centripetal force needed to stay in an orbit
  - Both the gravitational force and centripetal force are **perpendicular** to the direction of travel of the planet
- Consider a satellite with mass *m* orbiting Earth with mass *M* at a distance *r* from the centre travelling with linear speed *v*

$$F_G = F_{circ}$$

- Equating the gravitational force to the centripetal force for a planet or satellite in orbit gives:

$$\frac{GMm}{r^2} = \frac{mv^2}{r}$$

- The mass of the satellite *m* will cancel out on both sides to give:

$$v^2 = \frac{GM}{r} \Rightarrow v_{orbital} = \sqrt{\frac{GM}{r}}$$

- Where:
  - $v_{orbital}$ = orbital speed of the smaller mass (m s^{-1})
  - *G* = Newton's Gravitational Constant
  - *M* = mass of the larger mass being orbited (kg)
  - *r* = orbital radius (m)
- This means that all satellites, **whatever their mass**, will travel at the same speed *v* in a particular orbit radius *r*
  - Since the direction of a planet orbiting in circular motion is constantly changing, the **centripetal acceleration** acts towards the planet

Energy of an Orbiting Satellite

- An orbiting satellite follows a circular path around a planet
- Just like an object moving in circular motion, it has both kinetic energy (E_k) **and** gravitational potential energy (E_p) and its **total** energy is always **constant**
- An orbiting satellite's total energy is calculated by:

Total energy = Kinetic energy + Gravitational potential energy

$$\omega = \sqrt{\frac{GM_2}{R_1(R_1 + R_2)^2}}$$

(b) Orbital period:

- The relation between angular speed and orbital period is

$$\omega = \frac{2\pi}{T} \quad \Rightarrow \quad T = \frac{2\pi}{\omega}$$

- Using the expression for angular velocity from part (a)

$$T = 2\pi \div \sqrt{\frac{GM_2}{R_1(R_1 + R_2)^2}} = 2\pi\sqrt{\frac{R_1(R_1 + R_2)^2}{GM_2}}$$

Worked example

Two identical satellites, X and Y, orbit a planet at radii *R* and 3*R* respectively.

Which one of the following statements is **incorrect**?

A. Satellite X has more kinetic energy and less potential energy than satellite Y

B. Satellite X has a shorter orbital period and travels faster than satellite Y

C. Satellite Y has less kinetic energy and more potential energy than satellite X

D. Satellite Y has a longer orbital period and travels faster than satellite X

Answer: D

- Satellite Y is at a larger orbital radius, therefore it will have a **longer** orbital period, since $T^2 \propto R^3$
- Being at a larger orbital radius means the gravitational force will be weaker for Y than for X
- So, satellite Y will travel much **slower** than X as centripetal force: $F \propto v^2$
- Travelling at a slower speed means satellite Y will have **less** kinetic energy, as $E_K \propto v^2$, and, therefore, **more** potential energy than X

	Satellite X	Satellite Y
orbital radius	smaller	larger
orbital period	shorter	longer
orbital speed	faster	slower
kinetic energy	greater	lower
potential energy	lower	greater

- Therefore, all statements are correct except in **D** where it says 'Satellite Y travels faster than satellite X'

Effects of Drag on Orbital Motion (HL)

Effects of Drag on Orbital Motion

- Satellites in low orbits (<600 km) may be slightly affected by viscous drag, or air resistance
- The effects of drag on the motion of the satellite are usually very small, but over time, it can have a significant effect on the height and speed of the satellite's orbit

- The density of the air in the very upper layers of the atmosphere is very low, but not zero
- As a result, satellites travelling through these thin layers of air will experience a small dissipation of kinetic energy into thermal energy
  - This heating is due to the friction between the air particles and the surface of the satellite

- As some of the kinetic energy is dissipated into the surroundings, the satellite's total energy is reduced
  - When a satellite loses energy, its **orbital radius decreases**
  - However, as the satellite's orbit becomes lower, some of its **potential** energy is transferred to **kinetic** energy
- Overall, its speed increases and the effects of air resistance become even greater in its lower orbit resulting in greater dissipation of kinetic energy into thermal energy
- If the overall **decrease** in potential energy is **larger** than the overall **increase** in kinetic energy, the **total energy will decrease**

$$\Delta E_{total} < 0 \text{ if } \Delta E_p > \Delta E_k$$