HL IB Physics

Gas Laws

Contents

Gas Pressure

Calculating Gas Pressure

- **Pressure** is defined as

  The force applied per unit area

- For example, when a drawing pin is pushed downwards:
  - It is pushed into the surface, rather than up towards the finger
  - This is because the sharp point is more **concentrated** (a small area) creating a **larger** pressure

- When an object is immersed in a liquid, the liquid will exert pressure, squeezing the object
  - The pressure exerted on objects in fluids creates **forces** against surfaces
  - These forces act at **90 degrees** (at right angles) to the surface

- The equation for pressure is:

$$P = \frac{F}{A}$$

- Where:
  - P = pressure (Pa)
  - F = force (N)
  - A = cross-sectional area (m^2)
- Pressure is measured in **Pascals (Pa)**
- This equation is only relevant when gas molecules exert a force **perpendicular** to the surface

- It is possible for someone to experience this force by closing their mouth and forcing air into their cheeks
- The strain on the cheeks is due to the force of the gas particles pushing at right angles to the cheeks

- This equation means:
  - If a force is spread over a **large** area it will result in a **small** pressure
  - If it is spread over a **small** area it will result in a **large** pressure

Amount of Substance

Amount of Substance

- The **mole** is one of the seven SI base units
  - It is used to measure the **amount of substance**, not a mass
- One mole is defined as follows:

  The amount of substance that contains as many elementary entities as the number of atoms in 12 g of carbon–12

- This amount of substance is exactly $6.02214076 \times 10^{23}$ elementary entities (i.e. particles, atoms, molecules)
  - At IB level, this number can be rounded to 6.02×10^{23}
- One mole of gas contains a number of particles (atoms or molecules) equal to the **Avogadro Constant**
- For example, 1 mole of sodium (Na) contains 6.02×10^{23} **atoms** of sodium
- The number of atoms can be determined if the number of moles is known by multiplying by N_A.
  - For example: 2.0 mol of argon contains: $2.0 \times N_A = 2.0 \times 6.02 \times 10^{23} = \mathbf{1.20 \times 10^{24}}$ atoms
- The number of moles, n of a substance can be calculated using the equation

$$n = \frac{N}{N_A}$$

- Where:
  - N = number of particles (molecules or atoms, depending on the substance)
  - n = number of moles of gas (mol)
  - $N_A = 6.02 \times 10^{23}$ mol^{-1} (Avogadro constant)

Molar Mass

- One mole of any element is equal to the **relative atomic mass** of that element in grams
  - E.g. Helium has a relative atomic mass of 4 - this means 1 mole of helium has a mass of 4 g

- If the substance is a compound, add up the relative atomic masses, for example, water (H_2O) is made up of
  - 2 hydrogen atoms (each with atomic mass of 1) and 1 oxygen atom (atomic mass of 16)
  - So, 1 mole of water would have a mass of (2 × 1) + 16 = 18 g
- The **molar mass,** m of a substance is defined as the mass m of the substance divided by the amount (in moles) of that substance
- The molar mass is calculated as follows:

$$m_r = \frac{m}{n}$$

- Where:
  - m_r = molar mass in g mol^{-1}
  - m = mass in grams (g)
  - n = number of moles (mol)

Worked example

120 moles of nitrogen gas are in a container.

Calculate the number of nitrogen gas molecules in the container.

Answer:

Step 1: List the known quantities

- Number of moles, n = 120

Step 2: Rearrange the number of moles equation for the number of molecules, *N*

$$N = nN_A$$

Step 3: Substitute in the values

$$N = (6.02 \times 10^{23}) \times 120 = 7.2 \times 10^{25} \text{ molecules}$$

Worked example

A container is filled with 2.6×10^{20} molecules of argon gas which has a mass number of 40.

Calculate the total mass of argon in the container.

Answer:

Step 1: List the known quantities

- Number of argon gas molecules, $N = 2.6 \times 10^{20}$
- Molar mass of argon, m_r = 40 g mol^{-1} (same as the atomic mass)

Step 2: Calculate the number of moles of argon in the container

$$n = \frac{N}{N_A} = \frac{2.6 \times 10^{20}}{6.02 \times 10^{23}} = 4.3 \times 10^{-4} \text{ moles}$$

Step 3: Calculate the total mass of the argon in the container

$$m = m_r \times n = 40 \times (4.3 \times 10^{-4}) = 0.0172\ g$$

Gas Laws

Gas Laws

- An **ideal gas** is one which obeys the relation:

$$pV \propto T$$

- Where:
  - p = pressure of the gas (Pa)
  - V = volume of the gas (m^3)
  - T = thermodynamic temperature (K)
- Turning this into an equation gives:

$$\frac{PV}{T} = \text{constant}$$

- This is derived from the **empirical** gas laws for constant pressure, constant temperature and constant volume

Empirical Gas Laws

- The ideal gas laws are the experimental relationships between pressure *P*, volume *V* and the temperature *T* of an ideal gas
  - **Boyle's Law** (constant temperature)
  - **Charles's Law** (constant pressure)
  - **Gay-Lussac's Law** (constant volume)
- The mass and the number of molecules of the gas are assumed to be **constant** for all experiments

Boyle's Law

- If the temperature *T* of an ideal gas is constant, then **Boyle's Law** is given by:

$$P \propto \frac{1}{V}$$

- This means the pressure is **inversely proportional** to the volume of a gas
- The relationship between the pressure and volume for a fixed mass of gas at constant temperature can also be written as:

$$P_1V_1 = P_2V_2$$

- Where:
  - P_1 = initial pressure (Pa)
  - P_2 = final pressure (Pa)
  - V_1 = initial volume (m^3)
  - V_2 = final volume (m^3)

Charles's Law

- If the pressure P of an ideal gas is constant, then **Charles's law** is given by:

$$V \propto T$$

- This means the volume is **proportional** to the temperature of a gas
- The relationship between the volume and thermodynamic temperature for a fixed mass of gas at constant pressure can also be written as:

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

- Where:
  - V_1 = initial volume (m^3)
  - V_2 = final volume (m^3)
  - T_1 = initial temperature (K)
  - T_2 = final temperature (K)
- The variation of volume and temperature at a constant pressure is shown below:

Gay-Lussac's (Pressure) Law

- If the volume V of an ideal gas is constant, the **Gay-Lussac's** or **Pressure law** is given by:

$$P \propto T$$

- This means the pressure is **proportional** to the temperature
- The relationship between the pressure and thermodynamic temperature for a fixed mass of gas at constant volume can also be written as:

$$\frac{P_1}{T_1} = \frac{P_2}{T_2}$$

- Where:
  - P_1 = initial pressure (Pa)
  - P_2 = final pressure (Pa)
  - T_1 = initial temperature (K)
  - T_2 = final temperature (K)
- The variation of pressure and temperature at a constant volume is shown below:
- Changes in the pressure, temperature or volume of an ideal gas are normally represented on a **pressure-volume** graph

Worked example

An ideal gas is in a container of volume 4.5×10^{-3} m^3. The gas is at a temperature of 30 °C and a pressure of 6.2×10^5 Pa.

Calculate the pressure of the ideal gas in the same container when it is heated to 40 °C.

Answer:

Step 1: List the known quantities

- Volume, $V = 4.5 \times 10^{-3}$ m^3
- Inital temperature, $T_1 = 30$ °C
- Pressure, $P = 6.2 \times 10^5$ Pa
- Final temperature, $T_2 = 40$ °C

Step 2: State the ideal gas equation

$$pV \propto T$$

$$pV = kT$$

- Where k = the constant of proportionality

Step 3: Rearrange for the constant of proportionality

$$k = \frac{pV}{T}$$

Step 4: Convert temperature T into Kelvin

$$\theta\ °C + 273.15 = T\ K$$

$$30\ °C + 273.15 = 303.15\ K$$

Step 5: Substitute in known value into the constant of the proportionality equation

$$k = \frac{6.2 \times 10^5 \times 4.5 \times 10^{-3}}{303.15} = 9.203...$$

Step 6: Rearrange the ideal gas relation equation for pressure

$$p = \frac{kT}{V}$$

Step 7: Substitute in new values

$$k = 9.203$$

$$T = 40\ °C = 40 + 273.15 = 313.15\ K$$

$$p = \frac{(9.203...) \times 313.15}{4.5 \times 10^{-3}} = 640.45... \times 10^{3}\ \text{Pa} = \mathbf{640\ kPa}$$

Ideal Gas Equation

Ideal Gas Equation

- The two ideal gas equations, derived from the empirical gas laws, are:

$$PV = nRT$$

$$PV = Nk_BT$$

- The variables will be outlined below
- The empirical gas laws can be combined to give a single constant, known as the **ideal gas constant**, ***R***

	Boyle's Law	Charles' Law	Pressure Law
relationship	$PV = constant$	$V \propto T$	$P \propto T$
constants	T, n	P, n	V, n

- An ideal gas is defined as:

A gas which obeys the ideal gas equation at all pressures, volumes and temperatures

- Combining the gas laws leads to the **ideal gas equation**:

$$PV = nRT$$

- Where:
  - P = pressure (Pa)
  - V = volume (m^3)
  - n = number of moles (mol)
  - R = 8.31 J K^{-1} mol^{-1} (ideal gas constant)
  - T = temperature (K)

Constants

- The **ideal gas constant *R*** is the macroscopic equivalent of the Boltzmann constant k_B
  - The ideal gas constant is associated with macroscopic quantities such as volume and temperature
  - The Boltzmann constant is associated with the thermal energy of **microscopic** particles
- The **Boltzmann constant** is defined as the ratio of the ideal gas constant R and Avogadro's constant N_A:

$$k_B = \frac{R}{N_A}$$

- Recall from the Amount of Substance revision note that $N_A = \frac{N}{n}$
- This gives **another form** of the **ideal gas equation**:

$$PV = Nk_BT$$

- Where:
  - N = number of molecules
  - $k_B = 1.38 \times 10^{-23}$ J K^{-1} (Boltzmann constant)

Worked example

A gas has a temperature of –55°C and a pressure of 0.5 MPa. It occupies a volume of 0.02 m^3.

Calculate the number of gas particles.

Answer:

Step 1: Write down the known quantities

- Temperature, T = –55°C = 218 K
- Pressure, p = 0.5 MPa = 0.5×10^6 Pa
- Volume, V = 0.02 m^3

Step 2: Write down the equation of state of ideal gases

$$PV = nRT$$

Step 3: Rearrange the above equation to calculate the number of moles *n*

$$n = \frac{PV}{RT}$$

Step 4: Substitute numbers into the equation

- From the data booklet, R = 8.31 J K^{-1} mol^{-1}

$$n = \frac{(0.5 \times 10^6) \times 0.02}{8.31 \times 218} = 5.5 \text{ moles}$$

Step 5: Calculate the number of particles *N*

$$n = \frac{N}{N_A} \Rightarrow N = nN_A$$

$$N = 5.5 \times (6.02 \times 10^{23}) = 3.3 \times 10^{24}$$

Kinetic Theory of Gases

Kinetic Theory of Gases

- Ideal gases are described in terms of the **kinetic theory**
  - This is a modelled system used to approximate the behaviour of real gases
- A gas consists of atoms or molecules moving around **randomly** at **high speeds**
- The kinetic theory of gases models the thermodynamic behaviour of gases by linking:
  - The **microscopic properties** of particles e.g. mass and speed
  - The **macroscopic properties** of particles e.g. pressure and volume
- Kinetic theory and ideal gases are based on a set of the following **assumptions**:
  1. A gas consists of **many** identical molecules in a container. They all have the same mass
  2. The volume of the molecules is negligible **compared to** the volume of the container. This means they can be considered **point** particles
  3. The molecules are in continuous **random** motion at **high speeds**
  4. The molecules obey Newton's laws of motion
  5. The molecules collide elastically with each other and the walls of the container
  6. There are no intermolecular forces between the molecules **except during** collisions
  7. The time of a collision between molecules is negligible compared to the time between collisions
  8. External forces (e.g. gravity) are ignored
  9. The number of molecules of gas in a container is **very large.** Therefore the **average** behaviour (eg. speed) is usually considered
  10. Each particle exerts a force on the wall of the container with which it collides. This means the average of the forces produced by all gas particles results in a **uniform gas pressure**

Real & Ideal Gases

- **Real gases** will not always obey the **ideal gas** equation
- This is because the assumptions of the kinetic theory of gases may not always be valid
  - For example, the assumption that there are no intermolecular forces between gas molecules is not particularly realistic
- However, under certain conditions, they can be **considered** ideal gases
- An ideal gas is a good approximation of a real gas when:
  - The gas pressure is **low**
  - The gas density is **low**
  - Temperature is sufficiently **higher** than the boiling point of the substance
- At very **high pressures**, **densities** and **low temperatures**, real gases do not obey the kinetic theory model
- This is because at high pressures and densities:
  - Molecules are closer together
  - Therefore, there will be attractive forces between the molecules
  - The volume of the molecules cannot be considered negligible due to the high density
- At low temperatures:
  - A gas can change into a liquid, so the substance will no longer behave like a gas
- No gas is completely ideal, but air at normal room temperature and pressure, based on experiments, behaves very similar to an ideal gas

Derivation of the Kinetic Theory of Gases Equation

Derivation of the Kinetic Theory of Gases Equation

- Gas pressure arises due to the collisions of the gas particles with the walls of the container that holds it
  - When a gas particle collides with a wall in the container, it exerts a **force** on the wall
  - The random motion of a large number of molecules exerting a force on the walls creates an overall **pressure**
  - This is because pressure = force per unit area

Derivation

- Picture a single molecule in a cube-shaped box with sides of equal length l
- The molecule has a mass *m* and moves with speed *c*, parallel to one side of the box
- It collides at regular intervals with the ends of the box, exerting a force and contributing to the pressure of the gas
- By calculating the pressure this one molecule exerts on one end of the box, the total pressure produced by all the molecules can be deduced

1. Find the change in momentum as a single molecule hits a wall perpendicularly

- One assumption of the kinetic theory is that molecules **rebound elastically**
  - This means there is no kinetic energy lost in the collision, so initial and final velocities are equal in magnitude
- When a gas particle of mass *m*, travels at an **average speed** *v*, and hits a wall of the container, it undergoes a **change in momentum** due to the force exerted by the wall on the particle

$$\text{initial momentum} = mv$$

$$\text{final momentum} = -mv$$

- Therefore, the change in momentum Δp is:

Δp = final momentum – initial momentum

$$\Delta p = -mv - (mv) = -2mv$$

- Force is equal to the rate of change of momentum (Newton's Second Law), so the force exerted by the wall on a molecule is:

$$F = \frac{\Delta p}{\Delta t} = -\frac{2mv}{\Delta t}$$

- According to Newton's Third Law, the particle exerts an **equal** and **opposite** force on the wall
- Therefore force exerted **on the wall** by the particle is:

$$F = \frac{2mv}{\Delta t}$$

2. Calculate the number of collisions per second by the molecule on a wall

- The time between collisions from one wall to the other, and then back again, over a distance of $2l$ with speed *v* is:

$$\text{time between collisions} = \frac{distance}{speed} = \frac{2l}{v}$$

3. Find the change in momentum per second

- The force the molecule exerts on one wall, *F*, from step 1 now becomes the following when substituting the time between collisions for Δ*t*:

$$F = \frac{2mv}{\Delta t} = \frac{2mv}{\frac{2l}{v}} = \frac{mv^2}{l}$$

4. Calculate the total pressure from *N* molecules

- The area of one wall is l^2
- The pressure, *P* can be written as:

$$P = \frac{F}{A} = \frac{\frac{mv^2}{l}}{l^2} = \frac{mv^2}{l^3}$$

- This is the pressure **exerted from one molecule**
- To account for the large number of *N* molecules, the pressure can now be written as:

$$P = \frac{Nmv^2}{l^3}$$

5. Consider the effect of the molecule moving in 3D space

- The pressure equation written above still assumes all the molecules are travelling in the same *x* direction and colliding with the same pair of opposite faces of the cube
  - To reflect this, it can be rewritten as:

$$P = \frac{Nm{v_x}^2}{l^3}$$

- Where v_x is the *x* component of the average velocity of all the particles
- In reality, all molecules will be moving in three dimensions equally
- Splitting the velocity into its components v_x, v_y and v_z to denote the amount in the x, y and z directions, v^2 (the square of the average speed) can be defined using pythagoras' theorem in 3D:

$$v^2 = {v_x}^2 + {v_y}^2 + {v_z}^2$$

- Since there is nothing special about any particular direction, it can be determined that the square of the average speed in each direction is equal:

$$v_x^2 = v_y^2 = v_z^2$$

$$v^2 = 3v_x^2$$

- Therefore, by combining the two equations, *v* can be defined as:

$$v_x^2 = \frac{1}{3}v^2$$

6. Re-write the pressure equation

- The box is a cube with a side length l
- Therefore, volume of the cube $V = l^3$

$$P = \frac{Nmv^2}{3V}$$

- This can also be written using the density *ρ* of the gas:

$$\rho = \frac{mass}{volume} = \frac{Nm}{V}$$

  - *m* is the mass of a single particle so *Nm* is the total mass of all the particles
- Rearranging the pressure equation for p and substituting the density *ρ*:

$$P = \frac{1}{3}\rho v^2$$

- Where:
  - *P* = pressure of the gas (Pa)
  - *ρ* = density of the gas (kg m^{-3})
  - *v* = mean square speed (m s^{-1})
- This is known as the **Kinetic theory of gases equation**

Worked example

An ideal gas has a density of 4.5 kg m^{-3} at a pressure of 9.3×10^5 Pa and a temperature of 504 K.

Determine the mean square speed of the gas molecules at 504 K.

Answer:

Step 1: List the known quantities

- Density of the gas, ρ = 4.5 kg m^{-3}
- Gas pressure, $P = 9.3 \times 10^5$ Pa
- Temperature, T = 504 K

Step 2: Rearrange the pressure equation for the mean square speed

$$P = \frac{1}{3}\rho v^2$$

$$v = \sqrt{\frac{3P}{\rho}}$$

Step 3: Substitute in the values

$$v = \sqrt{3 \times \frac{9.3 \times 10^5}{4.5}} = 787.4 = 790 \text{ m s}^{-1}$$

Average Kinetic Energy of a Molecule

Average Kinetic Energy of a Molecule

- We think of an ideal gas as molecules that collide elastically in random motion
- When the molecules collide, momentum and energy are conserved
  - We assume that, when the molecules are **not in contact**, no forces act between them
- This means they have no **potential energy**
  - Therefore, the internal energy U of an ideal, monatomic gas is equal to its **average kinetic energy** if the molecules are far enough apart
- The average kinetic energy, E_k for **one** molecule is equal to:

$$E_k = \frac{1}{2}mv^2 = \frac{3}{2}k_B T$$

- Where:
  - E_k = average kinetic energy of one molecule (J)
  - v = mean square speed of one molecule (m s^{-1})
  - m = mass of one molecule (kg)
  - k_B = Boltzmann constant
  - T = temperature of the gas (K)
- A consequence of this equation is that a **greater gas temperature** means a **greater average kinetic energy** of the particles
- Since the **total** internal energy is the **total** kinetic energy, the internal energy of a gas U is defined as:

$$U = \frac{3}{2}Nk_B T$$

- Where:
  - U = internal energy (J)
  - N = number of molecules

- The relation to the amount of substance is:

$$U = \frac{3}{2}nRT$$

- Where:
  - n = number of moles
  - R = ideal gas constant
- When heat is transferred to a fixed volume of gas, the internal energy **increases** and hence, so does the temperature, since the equations show that

$$U \propto T$$

- This is only relevant for **monatomic** gases. Examples include:
  - Helium
  - Neon
  - Argon
- A monatomic (one atom) molecule only has translational energy, whilst a diatomic (two-atom) molecule has both **translational** and **rotational energy**

Worked example

600 J of thermal energy is transferred to 3 g of helium gas kept at a constant volume in a cylinder.

Helium has a mass number of 4.

Calculate the temperature of the gas.

Answer:

Step 1: List the known quantities

- Thermal (internal) energy, U = 600 J
- Molar mass of helium, m_r = 4 g mol^{-1} (from its mass number)
- Mass of helium gas, m = 3 g

Step 2: State the relevant equation

$$U = \frac{3}{2}nRT$$

Step 3: Calculate the number of moles of the gas

$$m_r = \frac{m}{n} \Rightarrow n = \frac{m}{m_r}$$

$$n = \frac{3}{4} = 0.75 \text{ moles}$$

Step 4: Rearrange the internal energy for the temperature, *T*

$$T = \frac{2U}{3nR}$$

Step 5: Substitute in the values

$$T = \frac{2 \times 600}{3 \times 0.75 \times 8.31} = 64 \text{ K}$$