HL IB Physics

Fusion & Stars

Contents

Fusion Reactions in Stars

Fusion Reactions in Stars

- In the centre of a stable star, hydrogen atoms undergo **nuclear fusion** to form helium
- The equation for this reaction is:

$$^{2}_{1}H + ^{3}_{1}H \longrightarrow ^{4}_{2}He + ^{1}_{0}n$$

Deuterium and tritium are both isotopes of hydrogen. They can be formed through other fusion reactions in the star

- Fusion is defined as:

The joining of two small nuclei to produce a larger nucleus

- Low-mass nuclei (such as hydrogen and helium) can undergo fusion and release energy
- A **huge** amount of energy is released in the reaction
- This provides a **radiation pressure** that prevents the star from collapsing under its gravity
- When two protons fuse, a deuterium nucleus is produced
  - In the centre of stars, the deuterium combines with a tritium nucleus to form a helium nucleus
  - The total mass of the helium nucleus is less than the total mass of the individual nucleons
  - Hence, the reaction **releases energy**, which provides fuel for the star to continue burning

Fusion & The Strong Nuclear Force

- For two nuclei to fuse, both nuclei must have **high kinetic energy**
- This is because
  - Nuclei must overcome the repulsive coulomb forces between protons
  - The strong nuclear force, which binds nucleons together, has a very short range
- Therefore, nuclei must get very close together for the strong nuclear force to take effect
- This means an **extremely hot** and **dense** environment is required to achieve fusion

Energy Released in Fusion Reactions

Energy Released in Fusion Reactions

- When two small nuclei undergo a fusion reaction, the single larger nucleus produced as a result will have a **higher binding energy per nucleon** than the original two nuclei
- As a result of the mass defect between the parent nuclei and the daughter nucleus, **energy is released**

- When two protons fuse, the element deuterium is produced
- In the centre of stars, the deuterium combines with a tritium nucleus to form a helium nucleus, plus the release of energy, which provides fuel for the star to continue burning

Worked example

In the Sun, fusion occurs via a process known as the proton-proton cycle.

It is predicted that 80% of the total power output of the Sun is produced through the following cycle:

$$\left.\begin{array}{r} {}^{1}_{1}H + {}^{1}_{1}H \rightarrow {}^{2}_{1}H + {}^{0}_{1}e^{+} + \nu_e \\ {}^{1}_{1}H + {}^{2}_{1}H \rightarrow {}^{2}_{1}He + \gamma \\ {}^{3}_{2}He + {}^{3}_{2}He \rightarrow {}^{4}_{2}He + {}^{1}_{1}H + {}^{1}_{1}H \end{array}\right\} 4{}^{1}_{1}H \rightarrow {}^{4}_{2}He + 2{}^{0}_{1}e^{+} + 2\nu_e \text{ (overall reaction)}$$

nucleus	rest mass / u
hydrogen-1	1.007825
helium-4	4.002603

The neutrinos produced in the first step carry away 2% of the energy released by the process.

Determine the mass of hydrogen-1 that must be fused each second to produce this output.

Luminosity of the Sun = 3.85×10^{26} W.

Answer:

Step 1: Determine the energy released per overall fusion reaction

- In the overall reaction, 4 hydrogen-1 nuclei fuse into a helium-4 nucleus, so the mass defect is

mass defect: $\Delta m = 4(1.007825\text{u}) - 4.002603\text{u} = 0.028697\text{u}$

- Where atomic mass unit, $\text{u} = 1.66 \times 10^{-27}$ kg
- Using mass-energy equivalence, the energy released by one reaction is

$\Delta E = \Delta mc^2 = 0.028697 \times (1.66 \times 10^{-27}) \times (3 \times 10^8)^2$

energy released: $\Delta E = 4.287 \times 10^{-12}$ J

Step 2: Determine the energy released minus the energy that is carried away by neutrinos

- Per reaction, neutrinos carry away 2% of 4.287×10^{-12} J, so 98% of the energy contributes to the luminosity of the Sun

energy released: $\Delta E = 0.98 \times (4.287 \times 10^{-12}) = 4.201 \times 10^{-12}$ J

Step 3: Determine the number of fusion reactions that happen each second

- This process accounts for 80% of the luminosity of the Sun,
- So, the total power output of the reaction = $0.8 \times (3.85 \times 10^{26})$ W
- The number of fusion reactions each second is:

$$\text{number of reactions} = \frac{power\ output}{energy\ released\ per\ reaction} = \frac{0.8 \times (3.85 \times 10^{26})}{4.201 \times 10^{-12}}$$

number of reactions = 7.332×10^{37} s^{-1}

Step 4: Determine the mass of hydrogen that fuses each second

- Every reaction fuses 4 hydrogen-1 nuclei, so the mass per reaction is $4 \times 1.007825u$
- The mass of hydrogen-1 that fuses each second in this process is

= $4u \times$ number of reactions

mass of hydrogen-1 = $4 \times 1.007825 \times (1.66 \times 10^{-27}) \times (7.332 \times 10^{37})$

mass of hydrogen-1 = **4.91×10^{11} kg s^{-1}**

Star Formation

Conditions for Fusion

- For nuclear fusion to occur, both nuclei must have sufficiently **high kinetic energy** to overcome the electrostatic repulsion between protons
- The conditions required to achieve this are:
  - **Very high temperature** (on the scale of 100 million Kelvin)
  - **Very high pressure and density**
- Four hydrogen nuclei (protons) are fused into one helium nucleus, producing two gamma-ray photons, two neutrinos and two positrons
  - Massive amounts of energy are released
  - The momentum of the gamma-ray photons results in an outward acting pressure called radiation pressure

Equilibrium in Stars

- Once the core temperature of a star reaches millions of degrees kelvin and the fusion of hydrogen nuclei to helium nuclei begins
  - The protostar's gravitational field continues to attract more gas and dust, increasing the temperature and pressure of the core
  - With more frequent collisions, the kinetic energy of the particles increases, increasing the probability that fusion will occur
  - Eventually, when the core becomes **hot** enough and fusion reactions can occur, they will begin to produce an **outward radiation pressure** which balances the inward pull of gravity
- The star reaches a **stable state** where the inward and outward **forces** are in **equilibrium**
  - As the temperature of the star increases and its volume decreases due to gravitational collapse, the gas pressure increases
  - The gas pressure and the radiation pressure act **outwards** to balance the gravitational force (weight, $F = mg$) acting **inwards**

- If the **temperature** of a star **increases**, the **outward pressure** will also **increase**
  - If outward pressure > gravitational force, the star will **expand**
- If the **temperature drops** the **outward pressure** will also **decrease**
  - If outward pressure < gravitational force, the star will **contract**
- As long as these two forces are **balanced**, the star will remain **stable**

Life Cycle of a Star

The Life Cycle of Stars

- The life cycle of a star follows predictable stages
- The exact route a star's development takes depends on its initial mass

Initial Stages for All Masses

- The first four stages in the life cycle of stars are the same for stars of all masses
- After these stages, the life-cycle branches depending on whether the star is:
  - **Low mass**: stars with a **core mass** of **less** than about 1.4 times the mass of the Sun (< 1.4 M_{Sun})
  - **High mass**: stars with a **core mass** of **more** than about 1.4 times the mass of the Sun (> 1.4 M_{Sun})

1. Nebula

- All stars form from a giant cloud of **hydrogen gas** and **dust** called a **nebula**
  - **Gravitational attraction** between individual atoms forms denser clumps of matter
  - This inward movement of matter is called gravitational collapse

2. Protostar

- The gravitational collapse causes the gas to heat up and glow, forming a **protostar**
  - Work done on the particles of gas and dust by collisions between the particles causes an increase in their kinetic energy, resulting in an increase in **temperature**
  - Protostars can be detected by telescopes that can observe **infrared radiation**
- Eventually the temperature will reach millions of **degrees Kelvin** and the fusion of hydrogen nuclei to helium nuclei begins
  - The protostar's gravitational field continues to attract more gas and dust, increasing the temperature and pressure of the core
  - With more frequent collisions, the kinetic energy of the particles increases, increasing the probability that fusion will occur

3. Main Sequence Star

- The star reaches a **stable state** where the inward and outward **forces** are in **equilibrium**
- As the temperature of the star increases and its volume decreases due to gravitational collapse, the **gas pressure** increases

- A star will spend most of its life cycle on the **main sequence**
  - 90% of stars are on the main sequence
  - Main sequence stars can vary in mass from ~10% of the mass of the Sun to 200 times the mass of the Sun
  - The Sun has been on the main sequence for 4.6 billion years and will remain there for an estimated 6.5 billion years

Next Stages for Low Mass Stars

- The fate of a star beyond the main sequence depends on its **mass**
  - A star is classed as a low-mass star if it has a **total** mass **less** than 4 times the mass of the Sun
  - A low-mass star will become a **red giant** before turning into a **white dwarf**

4. Red Giant

- Hydrogen supplies in the core begins to run out
  - Most of the hydrogen nuclei in the core of the star have been fused into helium
  - Nuclear fusion **slows**
  - Energy released by hydrogen fusion decreases, but it continues in the **shell** around the core
- The star initially shrinks which causes the core to become hotter
- When the temperature is high enough, helium fusion begins
- This releases massive amounts of energy which causes the outer layers to swell and cool to form a **red giant**

5. Planetary Nebula

- The helium supply in the core begins to run out
- The core contracts, but it does not get hot enough for further fusion reactions
- The **outer layers** of the star are **released**

6. White Dwarf

- The solid **core collapses** under gravity
- The remnant left behind is a very hot, dense core called a **white dwarf**

Next Stages for Massive Stars

- A star is classed as a high-mass star if it has a **total** mass **greater** than 4 times the mass of the Sun
- A high-mass star will become a **red supergiant** before exploding as a **supernova**
- The remnant of the core will either be a **neutron star** or **black hole**

4. Red Super Giant

- The star follows the same process as the formation of a red giant
- The **shell-burning** and **core-burning** cycle in massive stars goes beyond that of low-mass stars, fusing elements up to **iron**

5. Supernova

- The iron core collapses
- The outer shell is blown out in an explosive **supernova**

6. Neutron Star (or Black Hole)

- After the supernova explosion, the collapsed **neutron core** can remain intact having formed a **neutron star**
- If the remnant core has a mass greater than 3 times the solar mass, the pressure becomes so great that it collapses and produces a **black hole**

Worked example

Stars less massive than our Sun will leave the main sequence and become red giants.

Describe and explain the next stages of evolution for such stars.

Answer:

Step 1: Plan your answer

- Make a list of the remaining stages in the evolution of a low-mass star adding any important points or keywords

Red giant	Planetary nebula	White dwarf
▪ Fuel runs out ▪ Forces no longer balanced ▪ Expands and cools ▪ Fusion continues in shell	▪ Carbon-oxygen core not hot enough for further fusion ▪ Outer layers released	▪ Hot, dense remnant of the core

Step 2: Use the plan to keep the answer concise and logically sequenced

Low-mass stars leave the main sequence and become red giants when the hydrogen in the core runs out. Reduced energy released by fusion leads to radiation pressure decreasing

Radiation pressure and gas pressure no longer balance the gravitational pressure and the core collapses. Fusion no longer takes place inside the core

The outer layers expand and cool to form a red giant. Temperatures generated by the collapsing core are high enough for fusion to occur in the shell around the core.

Contraction of the core produces temperatures great enough for the fusion of helium into carbon and oxygen. The carbon-oxygen core is not hot enough for further fusion, so the core collapses

The outer layers are ejected forming a planetary nebula.

The remnant core remains intact leaving a hot, dense, solid core called a white dwarf.

Worked example

Describe the evolution of a star much more massive than our Sun from its formation to its eventual death.

Answer:

Step 1: Plan your answer

- List the stages that a massive star goes through, this will help you form your answer in a logical sequence of events

Nebula	Protostar	Main sequence	Red supergiant	Supernova	Neutron star/black hole
▪ gravitational collapse	▪ heats up and glows	▪ H to He generates energy ▪ stable, forces balanced	▪ expands and cools ▪ fusion up to iron	▪ iron core collapses ▪ shockwave explosion	▪ super dense remnants

Step 2: Use the plan to keep the answer concise and logically sequenced

A star more massive than our Sun will form from clouds of gas and dust called a nebula. The gravitational collapse of matter increases the temperature of the cloud causing it to glow - this is a protostar.

Nuclear fusion of hydrogen nuclei to helium nuclei generates massive amounts of energy. The outward radiation and gas pressure balance the inward gravitational pressure allowing the star to become stable as it enters the main sequence stage.

When the hydrogen runs out, the outer layers of the star expand and cool to form a red supergiant. The core becomes hot enough for helium fusion. Once helium fusion ends, successive cycles of expansion and collapse occur as heavier elements are fused in the core, up to iron.

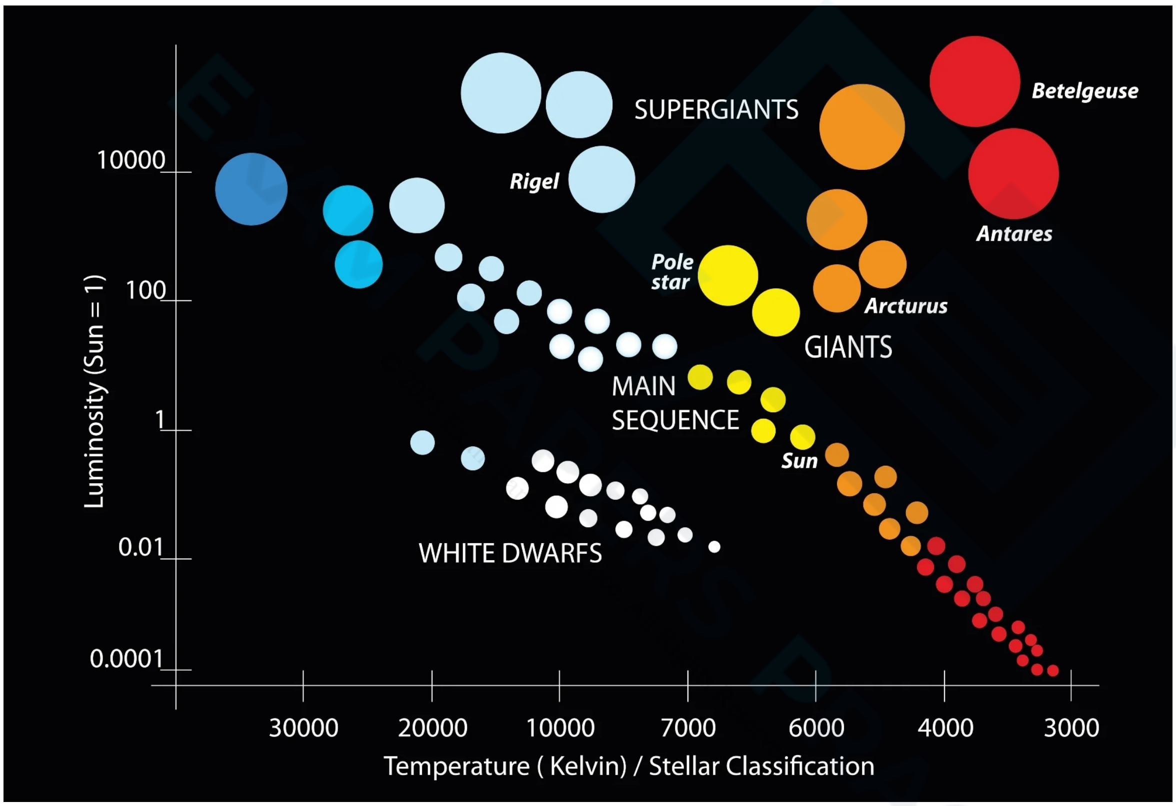
Eventually, once iron has formed in the core and fusion reactions can no longer continue, the outward layers of the star collapse and the star undergoes a shockwave explosion known as a supernova.

The remnant of the core collapses further and forms either a neutron star or a black hole.

The Hertzsprung–Russell (HR) Diagram

The Hertzsprung–Russell (HR) Diagram

- Danish astronomer Ejnar Hertzsprung, and American astronomer Henry Noris Russell, independently plotted the luminosity of different stars against their **temperature**
  - **Luminosity**, relative to the Sun, on the y-axis, goes from dim (at the bottom) to bright (at the top)
  - **Temperature**, in degrees Kelvin, on the x-axis, goes from **hot** (on the left) **to cool** (on the right)



The Hertzsprung-Russell Diagram depicts the luminosity of stars against their temperature

- Hertzsprung and Russel found that the stars clustered in **distinct areas**
- Most stars are clustered in a band called the **main sequence**
  - For main sequence stars, luminosity **increases** with surface temperature
- A smaller number of stars clustered above the main sequence in two areas, **red giants,** and **red supergiants**
  - These stars show an increase in luminosity at **cooler** temperatures
  - The only explanation for this is that these stars are much **larger** than main sequence stars
- Below and to the left of the main sequence are the **white dwarf stars**
  - These stars are **hot**, but not very luminous
  - Therefore, they must be much **smaller** than main sequence stars
- The Hertzsprung-Russell Diagram only shows stars that are in **stable phases**
  - Transitory phases happen quickly in relation to the lifetime of a star
  - Black holes cannot be seen since they emit no light

Emission & Absorption Spectra in Stars

Emission & Absorption Spectra in Stars

Types of Spectra

- There are three types of light spectra:
  - Continuous emission spectra
  - Emission line spectra
  - Absorption line spectra

Continuous Spectra

- In a **continuous spectrum**, photons emitted from the core of a star contain all the wavelengths and frequencies of the electromagnetic spectrum
- Continuous spectra are produced from **hot, dense sources**, such as the cores of stars

Emission Spectra

- When an electron transitions from a higher energy level to a lower energy level, this results in the **emission** of a photon
- Each transition corresponds to a different wavelength of light and this corresponds to a line in the spectrum
- The resulting emission spectrum contains a set of discrete wavelengths represented by **coloured lines** on a **black** background
- Emission line spectra are produced by **hot, low-pressure gases**

Absorption Spectra

- An atom can be raised to an excited state by the absorption of a photon
- Absorption spectra are observed when white light passes through a **cool, low-pressure gas**
- Some wavelengths appear to be missing in an absorption spectrum which correspond to the lines in the emission spectra of the same element

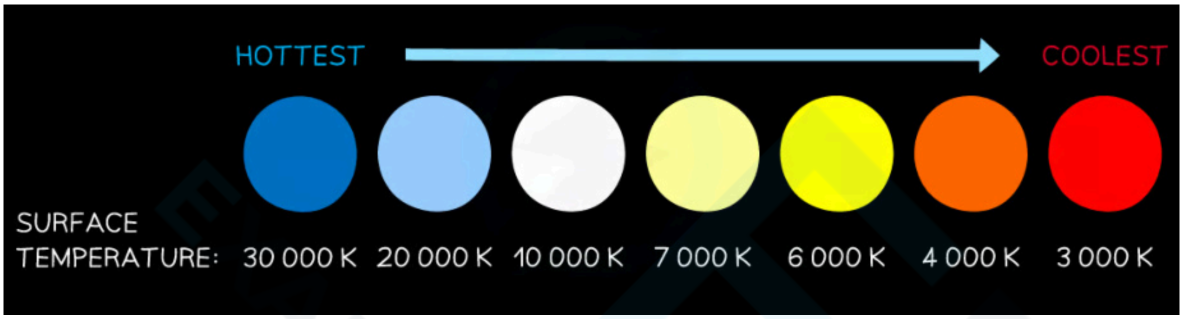
Chemical Composition of Stars

- Stellar spectral lines are caused by the interactions between **photons** and the **atoms** present in **gaseous layers** of stars
- Photons produced by fusion reactions in a star's core move towards the layers of gas in the outer atmosphere of the star
  - The photons produced in the core form a **continuous spectrum**
  - Photons are absorbed by the gas atoms, which excite and re-emit other photons of various frequencies in random directions
- The light from a star can be analysed using **spectroscopy**
  - The atmospheres of stars are not hot enough to produce an emission line spectrum
  - Therefore, stars are found to emit an **absorption line spectrum**
- An absorption line spectrum is the equivalent of an emission line spectrum but it is made of **dark lines** on top of a **continuous spectrum**
  - The dark lines represent the frequencies or wavelengths that are absorbed by a medium, such as a gas, when light passes through it
  - The absorption spectral lines represent the **energy** that has been **absorbed by electrons** in the outer atmosphere of the star
- Each gas produces a **unique pattern** of spectral lines due to the specific transition between the element's energy levels
  - The presence of spectral lines in a star's absorption spectrum act as **fingerprints**
  - They can be used to determine the presence of a **certain element** within the star
- The **chemical composition** of a star can be investigated even when extremely distant
  - If the element is present in the star, its characteristic pattern of spectral lines will appear as dark lines in the absorption line spectrum of the star
- The Sun is predominantly made up of **hydrogen** and **helium** gas
  - The chemical composition of the Sun can be verified using the **emission line spectra** of the two gases compared with the **absorption line spectrum** of the Sun
- For example, the hydrogen emission line spectrum includes lines at:
  - 2 nm, 486.3 nm and 656.5 nm
- While helium spectrum includes lines at:
  - 7 nm and 587.7 nm

- The same wavelengths can be seen as dark lines on top of the Sun's continuous spectrum

Surface Temperature of Stars

- The spectra of stars consist of a wide distribution of wavelengths
- Each wavelength of radiation has a different intensity
- The peak wavelength refers to the wavelength with the highest intensity



The colour of a star correlates to its temperature

- Stars are the closest approximation to black-body radiators that exist
- Therefore, the colour of a star i.e. its peak emission wavelength, can be attributed to its temperature according to Wien's law, where:
  - A shorter peak wavelength corresponds to a higher temperature at the peak intensity, so **hotter** stars tend to be **white or blue**
  - A longer peak wavelength corresponds to a lower temperature at the peak intensity, so **cooler** stars tend to be **red or yellow**

Worked example

Explain why:

a) Hot, dense sources produce continuous spectra
b) Hot, low pressure gases produce emission spectra
c) Hot, dense sources observed through cool, low pressure gases produce absorption spectra

Answer:

(a) Hot, dense sources, such as the cores of stars, produce continuous spectra because:

- In a hot, dense material, the atoms or molecules are so close together that they interact with one another
- This leads to a spread of energy states that are not clearly defined
- Therefore, photons of all frequencies are emitted leading to an uninterrupted band of colour

(b) Hot, low pressure gases produce emission line spectra, because:

- Hot gases produce emission line spectra when photons are emitted due to the transition of electrons
  between discrete energy levels in atoms of the gas
- The line spectrum has certain, fixed frequencies related to the differences in energy between the various energy levels of the atoms of the gas
- In a low pressure gas, the atoms or molecules are not close together
- This means the energy levels of the gas atoms or molecules are clearly quantised and well-defined
- Therefore, only photons which correspond to the differences in energy between the energy levels of a bound electron are seen

(c) Hot, dense sources observed through cold gases produce absorption spectra because:

- Atoms of different elements in the cold gas absorb energy emitted from the hot source but only at particular energy values
- These particular energy values correspond to the differences in energy between the energy levels of a bound electron
- This means that particular frequencies of light are absorbed, creating black lines in the continuous emission spectrum

Stellar Parallax

Astronomical Unit Conversions

- Astronomical distances are very large and as a result, are usually measured using:
  - Astronomical Units (AU)
  - Light-years (ly)
  - Parsecs (pc)

Astronomical Unit (AU)

- The astronomical unit (AU) is defined as

  The mean distance from the centre of the Earth to the centre of the Sun

- As the Earth's orbit around the Sun is **elliptical** it will be slightly closer to the Sun in January (1.471×10^{11} m) than it is in July (1.521×10^{11} m)
- Calculating the mean of these two values gives:

$$\frac{(1.471 \times 10^{11}) + (1.521 \times 10^{11})}{2} = 1.496 \times 10^{11} \text{ m}$$

- Therefore, 1 astronomical unit = 1.496×10^{11} m $\approx 1.5 \times 10^{11}$ m
- The astronomical unit is useful for studying distances on the scale of the **solar system**

Light-year (ly)

- A light-year is defined as:

  The distance travelled by light in one year

- This can be calculated using:

  Distance = speed × time

- Where:
  - The speed of light is 3×10^8 m s^{-1}
  - 1 year = 60 × 60 × 24 × 365 = 3.15×10^7 s
- Hence, the distance travelled by light in one year = $(3 \times 10^8) \times (3.15 \times 10^7) = 9.46 \times 10^{15}$ m
  - Therefore, 1 light-year ≈ 9.5×10^{15} m

Parsec (pc)

- Angles smaller than 1 degree can be measured in arcminutes or arcseconds
  - 1 degree = 60 arcminutes
  - 1 arcminute = 60 arcseconds
  - Therefore, 1 degree = 60 x 60 = 3600 arcseconds
  - 1 arcsecond = 1/3600 degree
- The parsec is defined as

  A unit of distance that gives a parallax angle of 1 second of an arc (of a degree), using the radius of the Earth's orbit (1 AU) as the baseline of a right-angled triangle

- Given that 1 AU = 1.496×10^{11} m, trigonometry can be used to express 1 parsec in metres:

$$\tan\theta = \frac{opp}{adj} = \frac{1\ AU}{1\ pc}$$

$$\tan\left(\frac{1}{3600}\right) = \frac{1\ AU}{1\ pc}$$

$$1\ \text{pc} = \frac{1\ AU}{\tan\left(\frac{1}{3600}\right)} = \frac{1.496 \times 10^{11}}{\tan\left(\frac{1}{3600}\right)} = 3.09 \times 10^{16}\ \text{m}$$

- Therefore, 1 parsec ≈ 3.1×10^{16} m

- The parsec (1 pc = 3.1×10^{16} m) and the light-year (1 ly = 9.5×10^{15} m) are much **greater** in size than the astronomical unit (1 AU = 1.496×10^{11} m)
- This makes them useful when studying **interstellar distances**
  - For example, on the scale of distances between the Earth and stars, or neighbouring galaxies

Worked example

The closest star to Earth is a triple-star system called Alpha Centauri, which is approximately 4.35 light-years from Earth.

Calculate the distance between the Earth and Alpha Centauri in:

(a) Astronomical units
(b) Parsecs

An astronomical unit is 1.496×10^{11} m.

Answer:

(a)

Step 1: List the known quantities

- 1 light-year ≈ 9.5×10^{15} m (from data booklet)
- 1 AU = 1.496×10^{11} m
- Distance to Alpha Centauri = 4.35 ly

Step 2: Convert 4.35 light-years into metres

- 4.35 ly = $4.35 \times (9.5 \times 10^{15}) = 4.13 \times 10^{16}$ m

Step 3: Convert from metres into AU

- 4.13×10^{16} m = $\dfrac{4.13 \times 10^{16}}{1.496 \times 10^{11}}$ = 2.8×10^{5} AU (to 2 s.f)

(b)

Step 1: List the known quantities

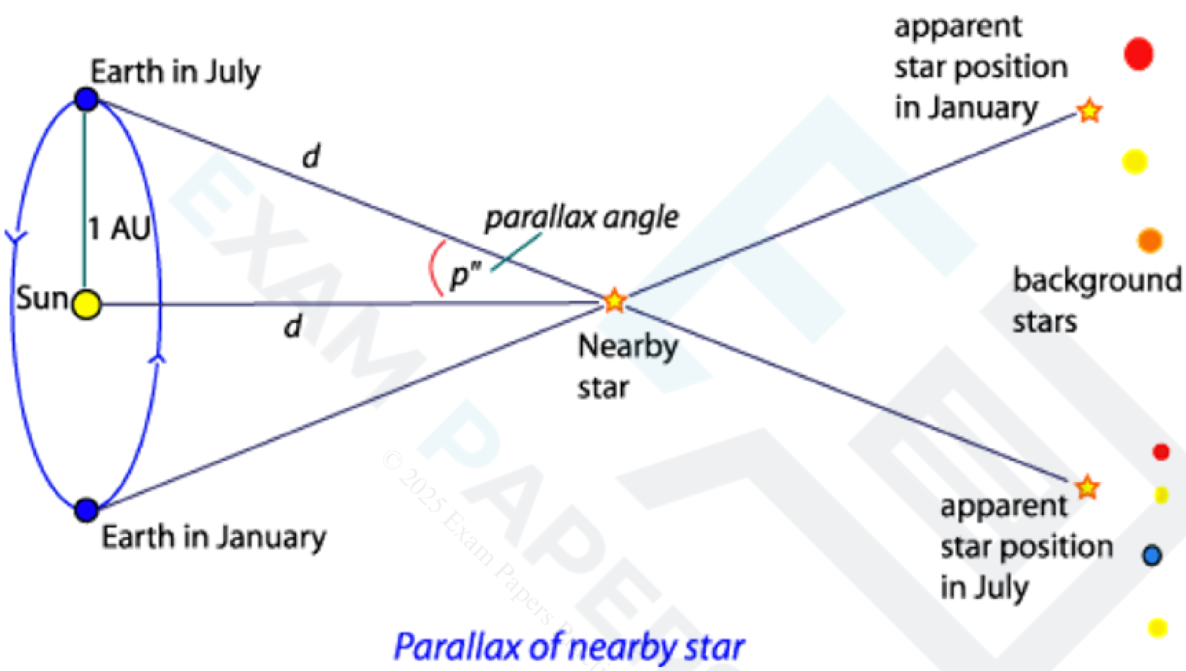
- 1 parsec ≈ 3.1×10^{16} m (from data booklet)
- 4.35 ly = 4.13×10^{16} m (from part a)

Step 2: Convert from metres into parsecs

- 4.13×10^{16} m = $\dfrac{4.13 \times 10^{16}}{3.1 \times 10^{16}}$ = 1.3 pc (to 2 s.f)

Parallax Calculations

- The **principle of parallax** is based on how the position of an object appears to change depending on where it is observed from
- When **observing** the volume of liquid in a measuring cylinder the parallax principle will result in the observer obtaining different values based on where they viewed the bottom of the meniscus from



Parallax error describes the false readings that can be made by taking measurements from different angles

- **Stellar parallax** can be used to measure the distance to **nearby stars**
- Stellar Parallax is defined as:

  The apparent shifting in position of a nearby star against a background of distant stars when viewed from different positions of the Earth, during the Earth's orbit about the Sun

- It involves observing how the position of a nearby star changes over a period of time against a fixed background of distant stars
  - To an **observer,** the position of distant stars does not change with time
- If a **nearby star** is viewed from the Earth at 6 months intervals (e.g. once in January and once again in July), the Earth will be at a **different position** in its orbit around the Sun
  - The nearby star will appear in a different position compared to the backdrop of distant stars
  - The distant stats will appear to not have moved

- This **apparent movement** of the nearby star is called the stellar parallax

- Applying trigonometry to the parallax equation:

$$\tan p = \frac{1\ AU}{d}$$

- Where:
  - 1 AU = radius of the Earth's orbit around the sun
  - *p* = parallax angle from Earth to the nearby star
  - *d* = distance to the nearby star
- For small angles, expressed in radians, $\tan p \approx p$, therefore:

$$p = \frac{1\ AU}{d}$$

- If the distance to the nearby star is to be measured in parsec, then it can be shown that the relationship between the distance to a star from Earth and the angle of stellar parallax is given by

$$p = \frac{1}{d}$$

- Where:
  - *p* = parallax (")
  - *d* = the distance to the nearby star (pc)
- This equation is accurate for distances of up to 100 pc
- For distances larger than 100 pc, the angles involved are so small they are too difficult to measure accurately

Worked example

The nearest star to Earth, Proxima Centauri, has a parallax of 0.768 seconds of arc.

Calculate the distance of Proxima Centauri from Earth

(a) in parsecs

(b) in light-years

Answer:

(a)

Step 1: List the known quantities

- Parallax, p = 0.768"

Step 2: State the parallax equation

$$p = \frac{1}{d}$$

Step 3: Rearrange and calculate the distance d

$$d = \frac{1}{p} = \frac{1}{0.768} = 1.30\ pc$$

(b)

Step 1: State the conversion between parsecs and metres

- From the data booklet:

$$1 \text{ parsec} \approx 3.1 \times 10^{16} \text{ m}$$

Step 2: Convert 1.30 pc to m

$$1.30 \text{ pc} = 1.30 \times (3.1 \times 10^{16}) = 4.03 \times 10^{16} \text{ m}$$

Step 3: State the conversion between light-years and metres

- From the data booklet

$$1 \text{ light-year} \approx 9.5 \times 10^{15} \text{ m}$$

Step 4: Convert 4.03×10^{16} m into light-years

$$\frac{4.03 \times 10^{16}}{9.5 \times 10^{15}} = 4.2 \text{ ly (to 2 s.f)}$$

Determination of Stellar Radii

Determination of Stellar Radii

- The radius of a star can be estimated by combining Wien's displacement law and the Stefan–Boltzmann law
- The procedure for this is as follows:
  - Using Wien's displacement law to find the surface temperature of the star
  - Using the inverse square law of flux equation to find the luminosity of the star (if given the radiant flux and stellar distance)
  - Then, using the Stefan-Boltzmann law, the stellar radius can be obtained

Worked example

Betelguese is our nearest red giant star. It has a luminosity of 4.49×10^{31} W and emits radiation with a peak wavelength of 850 nm.

Calculate the ratio of the radius of Betelguese r_B to the radius of the Sun r_S.

Radius of the Sun, $r_S = 6.95 \times 10^8$ m.

Answer:

Step 1: List the known quantities

- Luminosity of Betelgeuse, $L = 4.49 \times 10^{31}$ W
- Peak wavelength of Betelgeuse, $\lambda_{max} = 850$ nm $= 850 \times 10^{-9}$ m
- Radius of the Sun, $r_S = 6.95 \times 10^8$ m

Step 2: Write down Wien's displacement law

$$\lambda_{max} T = 2.9 \times 10^{-3} \text{ m K}$$

Step 3: Rearrange Wien's displacement law to find the surface temperature of Betelguese

$$T = \frac{2.9 \times 10^{-3}}{\lambda_{max}} = \frac{2.9 \times 10^{-3}}{850 \times 10^{-9}} = 3410 \text{ K}$$

Step 4: Write down the Stefan-Boltzmann law

$$L = 4\pi r^2 \sigma T^4$$

Step 5: Rearrange for r and calculate the stellar radius of Betelguese

$$r_B = \sqrt{\frac{L}{4\pi\sigma T^4}} = \sqrt{\frac{(4.49 \times 10^{31})}{4\pi \times (5.67 \times 10^{-8}) \times (3410)^4}} = 6.83 \times 10^{11} \text{ m}$$

Step 6: Calculate the ratio r_B / r_s

$$\frac{r_B}{r_s} = \frac{6.83 \times 10^{11}}{6.95 \times 10^8} = \mathbf{983}$$

- Therefore, the radius of Betelguese is about 1000 times larger than the Sun's radius