HL IB Physics

Doppler Effect

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The Doppler Effect

- When a source of sound, such as the whistle of a train or the siren of an ambulance, moves **away** from an observer:
  - It appears to **decrease** in frequency, i.e. it sounds **lower** in pitch
  - The **source** of the sound however, remains at a **constant** frequency
- This frequency change due to the relative motion between a source of sound or light and an observer is known as the **Doppler effect** (or **Doppler shift**)
- When the observer and the source of sound (e.g. ambulance siren) are both **stationary**:
  - The waves appear to remain at the **same** frequency for both the observer and the source
- When the observer and the source of sound (e.g. ambulance siren) are **moving** relative to each other
  - The waves appear to have a **different** frequency for both the observer and the source
- When the source starts to move **towards** the observer, the wavelength of the waves is **shortened**
  - The sound, therefore, appears at a **higher** frequency to the observer
- Notice how the waves are **closer** together between the source and the observer compared to point **P** and the source
- This also works if the source is moving **away** from the observer
  - If the observer was at point **P** instead, they would hear the sound at a lower frequency due to the wavelength of the waves **broadening**
- The frequency is **increased** when the source is moving **towards** the observer
- The frequency is **decreased** when the source is moving **away** from the observer
- The same phenomena occurs for electromagnetic waves, such as light
- Waves moving **away** from the observer are **red-shifted**
  - Their wavelengths shift to the red end of the electromagnetic spectrum
  - This is equivalent to sound waves appearing at a **lower** frequency to the observer
- Waves moving **towards** the observer are **blue-shifted**
  - Their wavelengths shift to the blue end of the electromagnetic spectrum
  - This is equivalent to sound waves appearing at a **higher** frequency to the observer
- This is because red light has a **longer wavelength** than blue light

Representing The Doppler Effect

- Wavefront diagrams help visualise the Doppler effect for moving wave sources and stationary observers
- $\Delta\lambda$ is the **change in** wavelength
  - The bigger the change, the bigger the doppler shift
- A moving object will cause the **wavelength**, λ, (and frequency) of the waves to change:
  - The **wavelength** of the waves **in front** of the source **decreases** ($\lambda - \Delta\lambda$) and the **frequency increases**
  - The wavelength **behind** the source **increases** ($\lambda + \Delta\lambda$) and the **frequency decreases**
- The Doppler shift is observed by **all** waves including **sound** and **light**

The Doppler Effect of Light

The Doppler Effect of Light

- The Doppler shift for a light-emitting non-relativistic source can be described using the equation:

$$\frac{\Delta f}{f} = \frac{\Delta \lambda}{\lambda} \approx \frac{\Delta v}{c}$$

- Where:
  - Δf = change in frequency (Hz)
  - f = reference (original) frequency (Hz)
  - $\Delta\lambda$ = change in wavelength (m)
  - λ = reference (original) wavelength (m)
  - Δv = relative velocity of the source and observer (m s^{-1})
  - c = the speed of light (m s^{-1})
- The sign ≈ means 'approximately equals to'
- This equation only works if $v << c$
- The change in wavelength $\Delta\lambda$ is equal to:

$$\Delta \lambda = \lambda_0 - \lambda$$

- Where:
  - λ_0 = **observed** wavelength of the source (m)
- Since the fractions have the **same** units on the numerator (top number) and denominator (bottom number), the Doppler shift has **no units**
- The **relative speed** between the source and observer along the line joining them is given by:

$$\Delta v = v_s - v_o$$

- Where:
  - v_s = velocity of the **source** of the light (m s^{-1})
  - v_o = velocity of the **observer** (m s^{-1})

- Usually, we calculate the speed of the source of electromagnetic waves **relative** to an observer which we assume to be **stationary**
  - Therefore $v_o = 0$, hence $\Delta v = v_s = v$
  - Where v is the velocity at which the source of the electromagnetic waves is moving from the observer
- Hence, the Doppler shift equation can be written in terms of wavelength:

$$\frac{\Delta \lambda}{\lambda} = \frac{\lambda_0 - \lambda}{\lambda} \approx \frac{v}{c}$$

- It can also be written in terms of frequency:

$$\frac{\Delta f}{f} = \frac{f_0 - f}{f} \approx \frac{v}{c}$$

Spectral Lines

- Doppler shift can easily be seen in atomic spectral lines from planets and stars
- Each line represents an element making up the composition of the galaxy
- The lines are identical to those measured in the lab and the light measured from the distant galaxy
- Since the lines all move to the left (the red end of the spectrum) this means the galaxy is travelling **away** from Earth

Worked example

A stationary source of light is found to have a spectral line of wavelength 438 nm. The same line from a distant star that is moving away from us has a wavelength of 608 nm.

Calculate the speed at which the star is travelling away from Earth.

Answer:

Step 1: List the known quantities

- Unshifted wavelength, λ = 438 nm
- Shifted wavelength, λ_0 = 608 nm
- Change in wavelength, $\Delta\lambda$ = (608 – 438) nm = 170 nm
- Speed of light, c = 3.0×10^8 m s^{-1}

Step 2: Write down the Doppler equation and rearrange for velocity v

$$\frac{\Delta\lambda}{\lambda} = \frac{v}{c}$$

$$v = \frac{c\Delta\lambda}{\lambda}$$

Step 3: Substitute values to calculate v

$$v = \frac{(3.0 \times 10^8) \times 170}{438} = 1.16 \times 10^8 \text{ m s}^{-1}$$

Worked example

The stars in a distant galaxy can be seen to orbit about a galactic centre. The galaxy can be observed 'edge-on' from the Earth.

Light emitted from a star on the left-hand side of the galaxy is measured to have a wavelength of 656.44 nm. The same spectral line from a star on the right-hand side is measured to have a wavelength of 656.12 nm.

The wavelength of the same spectral line measured on Earth is 656.28 nm.

(a) State and explain which side of the galaxy is moving towards the Earth.
(b) Calculate the rotational speed of the galaxy.

Answer:

(a)

- The light from the right-hand side (656.12 nm) is observed to be at a shorter wavelength than the reference line (656.28 nm)
- Therefore, the right-hand side has been blue-shifted and must be moving towards the Earth

(b)

Step 1: List the known quantities

- Observed wavelength on LHS, λ_{LHS} = 656.44 nm
- Observed wavelength on RHS, λ_{RHS} = 656.12 nm
- Reference wavelength, λ = 656.28 nm
- Speed of light, $c = 3.0 \times 10^8$ m s^{-1}

Step 2: Calculate the average change in wavelength

$$\Delta\lambda = \frac{\lambda_{LHS} - \lambda_{RHS}}{2} = \frac{656.44 - 656.12}{2}$$

$\Delta\lambda$ = 0.32 nm

Step 3: Write down the Doppler equation and rearrange for velocity v

$$\frac{\Delta\lambda}{\lambda} = \frac{v}{c}$$

$$v = \frac{c\Delta\lambda}{\lambda}$$

Step 4: Substitute values into the velocity equation

$$v = \frac{(3 \times 10^8) \times 0.32}{656.28}$$

Rotational speed: $v = 1.46 \times 10^5 \text{ m s}^{-1} = 146 \text{ km s}^{-1}$

Galactic Redshift

Galactic Redshift

- In space, the Doppler effect of **light** can be observed when spectra of distant stars and galaxies are observed, this is known as:
  - **Redshift** if the object is moving **away** from the Earth
    - The wavelength is increasing but the frequency is decreasing
  - **Blueshift** if the object is moving **towards** the Earth
    - The wavelength is decreasing but the frequency is increasing
- Redshift is defined as:

  The fractional increase in wavelength (or decrease in frequency) due to the source and observer receding from each other

- Redshift can be observed by comparing the **light spectrum** produced from a close object, such as our Sun, with that of a distant galaxy
  - The light from the distant galaxy is shifted towards the **red** end of the spectrum (compared to the Sun's spectra)
  - This provides evidence that the universe is **expanding**

Positive and Negative Velocities

- If the speed of the galaxy relative to Earth is **positive** then the galaxy is moving **towards** the Earth
  - This is the case when the observed frequency f_0 is **greater** than the reference frequency f
- If the speed of the galaxy relative to Earth is **negative** then the galaxy is moving **away** from the Earth
  - This is the case when the observed frequency f_0 is **less** than the reference frequency f

An Expanding Universe

- After the discovery of Doppler redshift, astronomers began to realize that almost all the galaxies in the universe are receding
- This led to the idea that the space between the Earth and the galaxies must be **expanding**
- This expansion stretches out the light waves as they travel through space, shifting them towards the red end of the spectrum

- The expansion of the universe can be compared to dots on an inflating balloon
  - As the balloon is inflated, the dots all move away **from each other**
  - In the same way, as the rubber stretches when the balloon is inflated, space itself is **stretching out between galaxies**
  - Just like the dots, the galaxies move away from each other, however, **they themselves** do not move
- Another observation from looking at the light spectra produced by distant galaxies is that the **greater** the **distance** to the galaxy, the **greater** the **redshift**
  - This means that the greater the degree of redshift, the **faster** the galaxy is moving away from Earth

Equations for the Doppler Effect of Sound (HL)

The Doppler Effect of Sound

- When a source of sound waves moves relative to a stationary observer, the observed frequency can be calculated using the equation below:

- The wave velocity for sound waves is 340 ms^{-1}
- The ± depends on whether the source is moving towards or away from the observer
  - If the source is moving **towards** the observer, the denominator is $v - u_s$
  - If the source is moving **away** from the observer, the denominator is $v + u_s$

- The ± depends on whether the observer is moving towards or away from the source
  - If the observer is moving **towards** the source, the numerator is $v + u_o$
  - If the observer is moving **away** from the source, the numerator is $v - u_o$

- The ± depends on whether the source is moving **towards or away** from the observer
  - If the source is moving **towards**, the term in the brackets is $1 - \frac{u_S}{v}$
  - If the source is moving **away**, the term in the brackets is $1 + \frac{u_S}{v}$

Worked example

A bank robbery has occurred and the alarm is sounding at a frequency of 3 kHz. The robber jumps into a car which accelerates and reaches a constant speed.

As he drives away at a constant speed, he hears the frequency of the alarm decrease to 2.85 kHz.

Determine the speed at which the robber must be driving away from the bank.

Speed of sound = 340 m s^{-1}

Answer:

Step 1: List the known quantities

- Source frequency, f = 3 kHz
- Observed frequency, f' = 2.85 kHz
- Speed of sound, v = 340 m s^{-1}

Step 2: Write down the Doppler shift equation

- The observer is moving away from a stationary source of sound, so the equation to use is

$$f' = f\left(\frac{v - u_o}{v}\right)$$

Step 3: Rearrange to find the desired quantity

$$\frac{f'}{f} = \left(\frac{v - u_o}{v}\right) \Rightarrow \frac{vf'}{f} = v - u_o$$

$$\frac{vf'}{f} + u_o = v \Rightarrow u_o = v - \frac{vf'}{f}$$

$$u_o = v\left(1 - \frac{f'}{f}\right)$$

Step 4: Substitute the values into the equation

$$u_o = 340 \times \left(1 - \frac{2.85}{3}\right) = 17 \text{ m s}^{-1}$$

- The robber must be driving away at a constant speed of **17 m s^{-1}** based on the change in frequency heard