

1. The weight of a piece of wire is directly proportional to its length.

A piece of wire is 25 cm long and has a weight of 6 grams.  
Another piece of the same wire is 30 cm long.

Calculate the weight of the 30 cm piece of wire.

$$\begin{aligned}
 W &= kl \\
 6 &= k(25) \\
 k &= \frac{6}{25}
 \end{aligned}
 \qquad
 \begin{aligned}
 w &= \frac{6}{25}(l) \\
 &= \frac{6}{25}(30) \\
 &= \frac{180}{25} \\
 &= 7.2 \text{ grams}
 \end{aligned}$$

(Total 2 marks)

2. A ball falls vertically after being dropped.  
The ball falls a distance  $d$  metres in a time of  $t$  seconds.  
 $d$  is directly proportional to the square of  $t$ .

The ball falls 20 metres in a time of 2 seconds.

- (a) Find a formula for  $d$  in terms of  $t$ .

$$\begin{aligned}
 d &= kt^2 \\
 20 &= k(2)^2 \\
 20 &= 4k \\
 k &= 5
 \end{aligned}$$

$$d = 5t^2$$

(3)

- (b) Calculate the distance the ball falls in 3 seconds.

$$d = 5(3)^2$$

$$45 \text{ m}$$

(1)

- (c) Calculate the time the ball takes to fall 605 m.

$$\begin{aligned}
 605 &= 5t^2 \\
 121 &= t^2
 \end{aligned}$$

$$11 \text{ seconds}$$

(3)  
(Total 7 marks)

3. The time,  $T$  seconds, it takes a water heater to boil some water is directly proportional to the mass of water,  $m$  kg, in the water heater.

When  $m = 250$ ,  $T = 600$

- (a) Find  $T$  when  $m = 400$

$$T = km$$

$$600 = k(250)$$

$$k = \frac{600}{250} = \frac{12}{5} = 2.4$$

$$T = 2.4(m)$$

$$= 2.4(400)$$

$$= 960$$

$$T = \underline{\quad 960 \quad}$$

(3)

The time,  $T$  seconds, it takes a water heater to boil a constant mass of water is inversely proportional to the power,  $P$  watts, of the water heater.

When  $P = 1400$ ,  $T = 360$

- (b) Find the value of  $T$  when  $P = 900$

$$T = \frac{k}{P}$$

$$360 = \frac{k}{1400}$$

$$k = 504000$$

$$T = \frac{504000}{900}$$

$$= 560$$

$$T = \underline{\quad 560 \quad}$$

(3)

(Total 6 marks)

4.  $D$  is proportional to  $S^2$ .

$D = 900$  when  $S = 20$

Calculate the value of  $D$  when  $S = 25$

$$D = kS^2$$

$$900 = k(20)^2$$

$$\frac{900}{400} = k$$

$$k = 2.25$$

$$D = 2.25S^2$$

$$D = 2.25(25)^2$$

$$= 1406.25$$

$$D = \underline{\quad 1406.25 \quad}$$

(Total 4 marks)

5. In a spring, the tension ( $T$  newtons) is directly proportional to its extension ( $x$  cm).

When the tension is 150 newtons, the extension is 6 cm.

(a) Find a formula for  $T$  in terms of  $x$ .

$$T = kx$$

$$150 = k(6)$$

$$k = 25$$

$$T = 25x$$

$$T = \underline{25x} \dots\dots\dots (3)$$

(b) Calculate the tension, in newtons, when the extension is 15 cm.

$$T = 25x$$

$$= 25(15)$$

$$= 375$$

$$\underline{375} \dots\dots\dots \text{newtons} (1)$$

(c) Calculate the extension, in cm, when the tension is 600 newtons.

$$T = 25x$$

$$600 = 25x$$

$$\frac{600}{25} = x$$

$$x = 24$$

$$\underline{24} \dots\dots\dots \text{cm} (1)$$

(Total 5 marks)

6.  $d$  is directly proportional to the square of  $t$ .

$d = 80$  when  $t = 4$

(a) Express  $d$  in terms of  $t$ .

$$d = k t^2$$

$$80 = k (4)^2$$

$$80 = 16k$$

$$k = 5$$

$$d = 5t^2$$

$$\dots d = 5t^2 \dots$$

(3)

(b) Work out the value of  $d$  when  $t = 7$

$$d = 5t^2$$

$$= 5(7)^2$$

$$= 245$$

$$d = \dots 245 \dots$$

(1)

(c) Work out the positive value of  $t$  when  $d = 45$

$$d = 5t^2$$

$$45 = 5t^2$$

$$9 = t^2$$

$$t = 3$$

$$t = \dots 3 \dots$$

(2)

(Total 6 marks)

7. The distance,  $D$ , travelled by a particle is directly proportional to the square of the time,  $t$ , taken.

When  $t = 40$ ,  $D = 30$

- (a) Find a formula for  $D$  in terms of  $t$ .

$$D = kt^2$$

$$30 = k(40)^2$$

$$30 = k(1600)$$

$$k = \frac{3}{160}$$

$$D = \frac{3}{160}t^2 \dots\dots\dots$$

- (b) Calculate the value of  $D$  when  $t = 64$

$$D = \frac{3}{160}t^2$$

$$= \frac{3}{160}(64)^2$$

$$= 76.8$$

$$\underline{\underline{76.8}}$$

(1)

- (c) Calculate the value of  $t$  when  $D = 12$   
Give your answer correct to 3 significant figures.

$$D = \frac{3}{160}t^2$$

$$12 = \frac{3}{160}t^2$$

$$640 = t^2$$

$$t = 25.3 \quad (3 \text{ s.f.})$$

$$\underline{\underline{25.3}}$$

(2)

(Total 6 marks)

8.  $M$  is directly proportional to  $L^3$ .

When  $L = 2$ ,  $M = 160$

Find the value of  $M$  when  $L = 3$

$$M = kL^3$$

$$160 = k(2)^3$$

$$160 = 8k$$

$$k = 20$$

$$\begin{aligned} m &= 20L^3 \\ &= 20(3)^3 \\ &= 540 \end{aligned}$$

540

(Total 4 marks)

9.  $p$  is inversely proportional to  $m$ .  
 $p = 48$  when  $m = 9$

Calculate the value of  $p$  when  $m = 12$

$$p = \frac{k}{m}$$

$$48 = \frac{k}{9}$$

$$k = 432$$

$$p = \frac{432}{m}$$

$$= \frac{432}{12}$$

36

(Total 2 marks)

10.  $r$  is inversely proportional to  $t$ .  
 $r = 12$  when  $t = 0.2$

Calculate the value of  $r$  when  $t = 4$ .

$$r = \frac{k}{t}$$

$$12 = \frac{k}{0.2}$$

$$k = 2.4$$

$$r = \frac{2.4}{t}$$

$$r = \frac{2.4}{4} = 0.6$$

..... 0.6 .....

(Total 3 marks)

11.  $f$  is inversely proportional to  $d$ .

When  $d = 50$ ,  $f = 256$

Find the value of  $f$  when  $d = 80$

$$f = \frac{k}{d}$$

$$256 = \frac{k}{50}$$

$$k = 12800$$

$$f = \frac{12800}{d}$$

$$f = \frac{12800}{80}$$

$f =$  ..... 160 .....

(Total 3 marks)

12.  $y$  is inversely proportional to  $x^2$ .

Given that  $y = 2.5$  when  $x = 24$ ,

(i) find an expression for  $y$  in terms of  $x$

$$y = \frac{k}{x^2}$$

$$2.5 = \frac{k}{(24)^2}$$

$$k = 1440$$

$$y = \frac{1440}{x^2}$$

(ii) find the value of  $y$  when  $x = 20$

$$y = \frac{1440}{x^2}$$

$$= \frac{1440}{(20)^2}$$

$$y = 3.6$$

(iii) find a value of  $x$  when  $y = 1.6$

$$y = \frac{1440}{x^2}$$

$$1.6 = \frac{1440}{x^2}$$

$$x^2 = \frac{1440}{1.6} = 900$$

$$x = 30$$

(Total 6 marks)

13.  $P$  is inversely proportional to  $d^2$ .

$P = 10\,000$  when  $d = 0.4$

Find the value of  $P$  when  $d = 0.8$

$$P = \frac{k}{d^2}$$

$$10000 = \frac{k}{(0.4)^2}$$

$$k = 1600$$

$$P = \frac{1600}{d^2}$$

$$= \frac{1600}{0.8^2}$$

$$P = 2500$$

(Total 3 marks)



14. The shutter speed,  $S$ , of a camera varies inversely as the square of the aperture setting,  $f$ .

When  $f = 8$ ,  $S = 125$

(a) Find a formula for  $S$  in terms of  $f$ .

$$S = \frac{k}{f^2}$$
$$125 = \frac{k}{8^2}$$
$$k = 8000$$

$$S = \frac{8000}{f^2} \quad (3)$$

(b) Hence, or otherwise, calculate the value of  $S$  when  $f = 4$

$$S = \frac{8000}{4^2}$$

$$S = 500$$

$$S = \underline{500} \quad (1)$$

(Total 4 marks)

15.  $q$  is inversely proportional to the square of  $t$ .

When  $t = 4$ ,  $q = 8.5$

(a) Find a formula for  $q$  in terms of  $t$ .

$$q = \frac{k}{t^2}$$
$$8.5 = \frac{k}{(4)^2}$$
$$k = 136$$

$$q = \frac{136}{t^2} \dots\dots\dots (3)$$

(b) Calculate the value of  $q$  when  $t = 5$

$$q = \frac{136}{5^2}$$
$$= 5.44$$

$$\dots\dots\dots 5.44 \dots\dots\dots (1)$$

(Total 4 marks)

16.  $P$  is inversely proportional to  $V$ .

When  $V = 8$ ,  $P = 5$

(a) Find a formula for  $P$  in terms of  $V$ .

$$P = \frac{k}{V}$$

$$5 = \frac{k}{8}$$

$$k = 40$$

$$P = \frac{40}{V} \dots\dots\dots (3)$$

(b) Calculate the value of  $P$  when  $V = 2$

$$P = \frac{40}{2}$$

$$\dots\dots\dots 20 \dots\dots\dots (1)$$

(Total 4 marks)

17. The force,  $F$ , between two magnets is inversely proportional to the square of the distance,  $x$ , between them.

When  $x = 3$ ,  $F = 4$ .

(a) Calculate  $F$  when  $x = 2$ .

$$F = \frac{k}{x^2}$$

$$4 = \frac{k}{3^2}$$

$$k = 36$$

$$F = \frac{36}{x^2}$$

$$= \frac{36}{2^2}$$

$$\dots\dots\dots 9 \dots\dots\dots (4)$$

(b) Calculate  $x$  when  $F = 64$ .

$$F = \frac{36}{x^2}$$

$$x = \sqrt{\frac{36}{64}}$$

$$64 = \frac{36}{x^2}$$

$$x^2 = \frac{36}{64}$$

$$\dots\dots\dots x = \frac{3}{4} \dots\dots\dots (2)$$

(Total 6 marks)