## Statistical distributions

Name: $\qquad$
Class: $\qquad$
Date: $\qquad$

Time:
Total marks available:
Total marks achieved: $\qquad$


A Level Mathematics: Statistics
Subject: Mathematics
Topic 4 : Statistical distributions

Type: Topic Questions

To be used by all students preparing for Edexcel A Level Mathematics - Students of other Boards may also find this useful

Q1.
Magali is studying the mean total cloud cover, in oktas, for Leuchars in 1987 using data from the large data set. The daily mean total cloud cover for all 184 days from the large data set is summarised in the table below.

| Daily mean total cloud cover (oktas) | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Frequency (number of days) | 0 | 1 | 4 | 7 | 10 | 30 | 52 | 52 | 28 |

One of the 184 days is selected at random.
(a) Find the probability that it has a daily mean total cloud cover of 6 or greater.

Magali is investigating whether the daily mean total cloud cover can be modelled using a binomial distribution.

She uses the random variable $X$ to denote the daily mean total cloud cover and believes that $X \sim$ B(8, 0.76)

Using Magali's model,
(b) (i) find $\mathrm{P}(X \geq 6)$
(ii) find, to 1 decimal place, the expected number of days in a sample of 184 days with a daily mean total cloud cover of 7

(c) Explain whether or not your answers to part (b) support the use of Magali's model.

There were 28 days that had a daily mean total cloud cover of 8
For these 28 days the daily mean total cloud cover for the following day is shown in the table below.

| Daily mean total cloud cover (oktas) | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Frequency (number of days) | 0 | 0 | 1 | 1 | 2 | 1 | 5 | 9 | 9 |

(d) Find the proportion of these days when the daily mean total cloud cover was 6 or greater.
(e) Comment on Magali's model in light of your answer to part (d).

Q2.
(a) State one disadvantage of using quota sampling compared with simple random sampling.

In a university 8\% of students are members of the university dance club.
A random sample of 36 students is taken from the university.
The random variable $X$ represents the number of these students who are members of the dance club.
(b) Using a suitable model for $X$, find
(i) $\mathrm{P}(X=4)$
(ii) $\mathrm{P}(X \geq 7)$

Only $40 \%$ of the university dance club members can dance the tango.
(c) Find the probability that a student is a member of the university dance club and can dance the tango.

A random sample of 50 students is taken from the university.
(d) Find the probability that fewer than 3 of these students are members of the university dance club and can dance the tango.
(Total for question = $\mathbf{7}$ marks)

Q3.
A health centre claims that the time a doctor spends with a patient can be modelled by a normal distribution with a mean of 10 minutes and a standard deviation of 4 minutes.
(a) Using this model, find the probability that the time spent with a randomly selected patient is
more than 15 minutes.

Some patients complain that the mean time the doctor spends with a patient is more than 10 minutes.

The receptionist takes a random sample of 20 patients and finds that the mean time the doctor spends with a patient is 11.5 minutes.
(b) Stating your hypotheses clearly and using a 5\% significance level, test whether or not there is evidence to support the patients' complaint.

The health centre also claims that the time a dentist spends with a patient during a routine appointment,
$T$ minutes, can be modelled by the normal distribution where $T \sim N\left(5,3.5^{2}\right)$
(c) Using this model,
(i) find the probability that a routine appointment with the dentist takes less than 2 minutes
(ii) find $\mathrm{P}(T<2 \mid T>0)$
(iii) hence explain why this normal distribution may not be a good model for $T$.

The dentist believes that she cannot complete a routine appointment in less than 2 minutes. She suggests that the health centre should use a refined model only including values of $T>2$
(d) Find the median time for a routine appointment using this new model, giving your answer correct to one decimal place.

A machine puts liquid into bottles of perfume. The amount of liquid put into each bottle,

D ml, follows a normal distribution with mean 25 ml
Given that 15\% of bottles contain less than 24.63 ml
(a) find, to 2 decimal places, the value of $k$ such that $\mathrm{P}(24.63<D<k)=0.45$

A random sample of 200 bottles is taken.
(b) Using a normal approximation, find the probability that fewer than half of these bottles contain between 24.63 ml and kml

The machine is adjusted so that the standard deviation of the liquid put in the bottles is now 0.16 ml

Following the adjustments, Hannah believes that the mean amount of liquid put in each bottle is less than 25 ml

She takes a random sample of 20 bottles and finds the mean amount of liquid to be 24.94 ml
(c) Test Hannah's belief at the $5 \%$ level of significance.

You should state your hypotheses clearly.

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\text { (Total for question = } 13 \text { marks) }
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Q5.
George throws a ball at a target 15 times.
Each time George throws the ball, the probability of the ball hitting the target is 0.48
The random variable $X$ represents the number of times George hits the target in 15 throws.
(a) Find
(i) $\mathrm{P}(X=3)$
(ii) $\mathrm{P}(X \geq 5)$

George now throws the ball at the target 250 times.
(b) Use a normal approximation to calculate the probability that he will hit the target more than 110 times.
(Total for question = 6 marks)

