

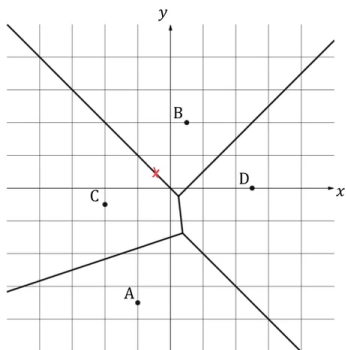
Voronoi Diagrams

Mark Schemes

Question 1

Points $A(-2, -7)$, $B(1, 4)$, $C(-4, -1)$ and $D(5, 0)$ on the Voronoi diagram below represent the locations of four cinemas in Berlin, Germany.

Horizontal scale: 1 unit represents 1 km. Vertical scale: 1 unit represents 1 km.



Amy wants to go to a cinema and her house is located at $(-1, 1)$.

- (a) (i) Determine which cinemas Amy's house is closest to.
 (ii) Calculate the distance from Amy's house to these two cinemas.

[4]

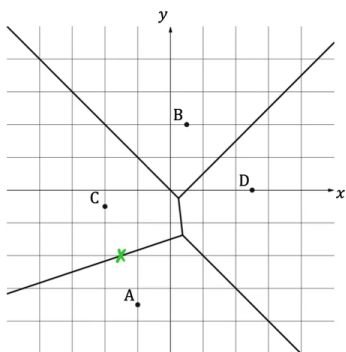
Kayla's apartment is an equal distance from cinemas A and C.

- (b) Find the shortest possible distance Kayla's apartment could be from cinemas A and C.

[2]

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- (a) (i) Determine which cinemas Amy's house is closest to.
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Kayla's apartment is an equal distance from cinemas A and C.

- (b) Find the shortest possible distance Kayla's apartment could be from cinemas A and C.

[2]

a) i) Amy's house lies on the perpendicular bisector of sites B and C.

∴ Amy's house is closest to sites B and C.

ii) The distance is the same to either site.

Distance between two points formula

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} \quad (\text{in formula booklet})$$

$$H(-1, 1) \quad B(1, 4)$$

Sub H and B into formula.

$$d = \sqrt{(-1 - 1)^2 + (1 - 4)^2}$$

$$d = \sqrt{13} = 3.6055\dots$$

$d = 3.61 \text{ km (3sf)}$

b) Kayla's apartment is at the midpoint of sites A and C.

Midpoint formula

$$\text{Midpoint} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \quad (\text{in formula booklet})$$

$$A(-2, -7) \quad C(-4, -1)$$

Sub A and C into formula.

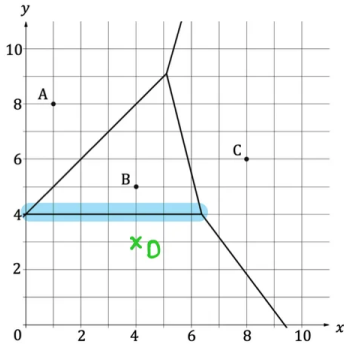
$$\text{Midpoint} = \left(\frac{-2 + (-4)}{2}, \frac{-7 + (-1)}{2} \right)$$

$$\text{Midpoint} = (-3, -4)$$

∴ Kayla's apartment is at $(-3, -4)$.

Question 2

Sites A, B and C on the Voronoi diagram below represent the location of solar panels.
 Horizontal scale: 1 unit represents 10 km. Vertical scale: 1 unit represents 10 km.



(a) A fourth site, D, is missing from the diagram. Write down the coordinates for site D.

[1]

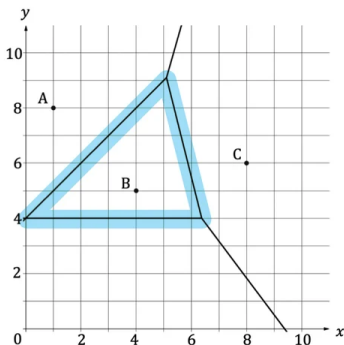
(b) The perpendicular bisectors surrounding B intersect at points (0, 4), (6.375, 4) and (5.1, 9.1).
 Calculate the area of cell B.

[4]

(c) A point E is located at (5, 10). Find the distance from E to the nearest solar panel.

[2]

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 Calculate the area of cell B.

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[2]

The daily average number of watts produced by each solar panel is given in the table below.

Site	A	B	C	D
Watts per day	276	293	312	322

(d) Estimate the watts produced at F(9, 1).

[1]

a) Site D will be an equal distance from the edge below site B.

$$D(4, 3)$$

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Site	A	B	C	D
Watts per day	276	293	312	322

(d) Estimate the watts produced at F(9, 1).

[1]

b) Area of a triangle formula
 $A = \frac{1}{2}bh$ (in formula booklet)
 b is the base, h is the perpendicular height
 $b = 6.375$ $h = 9.1 - 4 = 5.1$

Sub b and h into formula.

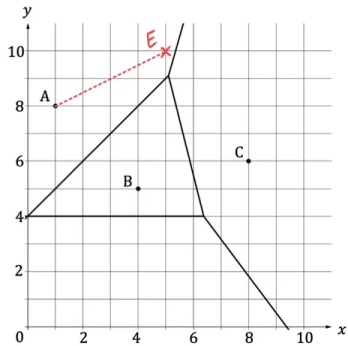
$$A = \frac{1}{2} (6.375)(5.1)$$

$$A = 16.25625$$

$$A = 16.3 \text{ units}^2 \text{ (3sf)}$$

$$A = 1630 \text{ km}^2 \text{ (3sf)}$$

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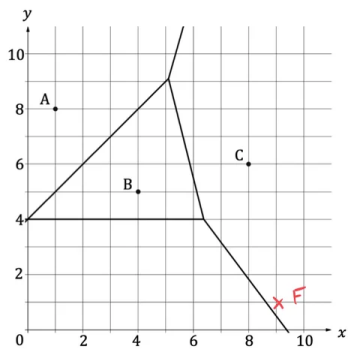
(b) The perpendicular bisectors surrounding B intersect at points (0, 4), (6.375, 4) and (5.1, 9.1). Calculate the area of cell B.

[4]

(c) A point E is located at (5, 10). Find the distance from E to the nearest solar panel.

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[1]

c) Closest site to E(5,10) is site A(1,8).
Distance between two points formula
 $d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$ (in formula booklet)
E(5,10) A(1,8).
Sub E and A into formula.
 $d = \sqrt{(5-1)^2 + (10-8)^2}$
 $d = \sqrt{20}$
 $d = 4.4721...$
 $d = 4.47$ (3sf)
 $d = 44.7$ km (3sf)

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(d) Estimate the watts produced at F(9, 1).

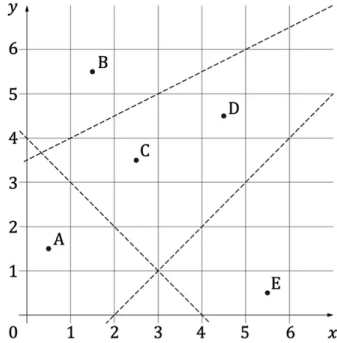
[1]

d) Nearest neighbour interpolation
Nearest site to F(9,1) is site C(8,6).
 $\therefore 312$ watts produced per day

Question 3

Points A(0.5, 1.5), B(1.5, 5.5), C(2.5, 3.5), D(4.5, 4.5) and E(5.5, 0.5) represent mechanics in a city. The mechanics are shown below on an incomplete Voronoi diagram.

Horizontal scale: 1 unit represents 1 km. Vertical scale: 1 unit represents 1 km.



(a) Calculate the gradient of the line segment CD.

[2]

(b) Find the equation of the line which would complete the Voronoi cell containing site C. Give your answer in the form $ax + by + d = 0$ where $a, b, d \in \mathbb{Z}$.

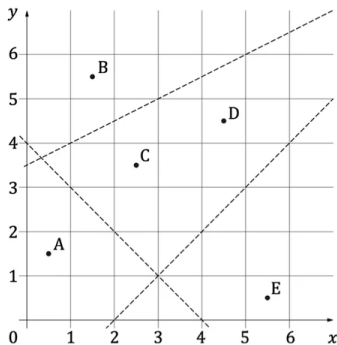
[3]

(c) In the context of the question, explain the significance of the Voronoi cell containing site C.

[1]

Points A(0.5, 1.5), B(1.5, 5.5), C(2.5, 3.5), D(4.5, 4.5) and E(5.5, 0.5) represent mechanics in a city. The mechanics are shown below on an incomplete Voronoi diagram.

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(a) Calculate the gradient of the line segment CD.

$$m_{CD} = \frac{1}{2}$$

[2]

(b) Find the equation of the line which would complete the Voronoi cell containing site C. Give your answer in the form $ax + by + d = 0$ where $a, b, d \in \mathbb{Z}$.

[3]

(c) In the context of the question, explain the significance of the Voronoi cell containing site C.

[1]

a) Gradient of a line formula

$$m = \frac{y_2 - y_1}{x_2 - x_1} \quad (\text{in formula booklet})$$

$$C(2.5, 3.5) \quad D(4.5, 4.5)$$

Sub C and D into formula.

$$m_{CD} = \frac{4.5 - 3.5}{4.5 - 2.5}$$

$$m_{CD} = \frac{1}{2}$$

b) The perpendicular bisector of CD is needed.

Perpendicular gradient formula

$$m_2 = -\frac{1}{m_1} \quad (\text{in formula booklet})$$

$$m_{CD} = m_1 = \frac{1}{2} \quad \therefore m_{\perp CD} = m_2 = -2$$

Midpoint formula

$$MP = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \quad (\text{in formula booklet})$$

$$C(2.5, 3.5) \quad D(4.5, 4.5)$$

Sub C and D into formula.

$$MP = \left(\frac{2.5 + 4.5}{2}, \frac{3.5 + 4.5}{2} \right)$$

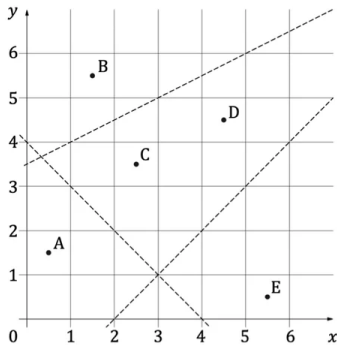
$$MP = (3.5, 4)$$

Sub MP and m into $y - y_1 = m(x - x_1)$.

$$y - 4 = -2(x - 3.5) \quad \left. \begin{array}{l} \text{expand and rearrange} \\ \text{into form} \end{array} \right\}$$

$$2x + y - 11 = 0$$

Points A(0.5, 1.5), B(1.5, 5.5), C(2.5, 3.5), D(4.5, 4.5) and E(5.5, 0.5) represent mechanics in a city. The mechanics are shown below on an incomplete Voronoi diagram.
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(b) Find the equation of the line which would complete the Voronoi cell containing site C. Give your answer in the form $ax + by + d = 0$ where $a, b, d \in \mathbb{Z}$.

[3]

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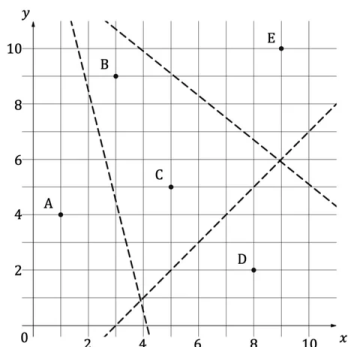
[1]

c) That Voronoi cell contains all the points that are closer to C than they are to A, B, D and E. Therefore the closest mechanic for the houses that lie in cell C is site C.

Question 4

Rangers use aerial imagery to help locate wolfpacks in Yellowstone National Park. This week the plane is not available so they must use last week's image which shows the last known locations of five wolfpacks at points A(1, 4), B(3, 9), C(5, 5), D(8, 2) and E(9, 10) as illustrated on the following coordinate axes.

Horizontal scale: 1 unit represents 10 km. Vertical scale: 1 unit represents 10 km.



Wolfpacks stick to very rigid territories keeping their distance from other packs to avoid confrontation. Using the aerial image, rangers draw three straight lines to form an incomplete Voronoi diagram.

(a) Calculate the gradient of the line segment BC.

[2]

(b) Find the equation of the line which would complete the Voronoi cell containing site C. Give your answer in the form $ax + by + d = 0$ where $a, b, d \in \mathbb{Z}$.

[3]

a) Gradient of a line formula

$$m = \frac{y_2 - y_1}{x_2 - x_1} \quad (\text{in formula booklet})$$

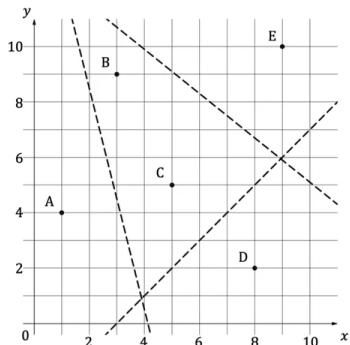
$$B(3, 9) \quad C(5, 5)$$

Sub B and C into formula.

$$m = \frac{5 - 9}{5 - 3}$$

$$m = -2$$

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(a) Calculate the gradient of the line segment BC.

$$m = -2$$

[2]

(b) Find the equation of the line which would complete the Voronoi cell containing site C. Give your answer in the form $ax + by + d = 0$ where $a, b, d \in \mathbb{Z}$.

[3]

b) The perpendicular bisector of BC is needed.

Perpendicular gradient formula

$$m_2 = -\frac{1}{m_1} \quad (\text{in formula booklet})$$

$$m_{BC} = m_1 = -2 \quad \therefore m_{\perp BC} = m_2 = \frac{1}{2}$$

Midpoint formula

$$MP = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \quad (\text{in formula booklet})$$

$$B(3, 9) \quad C(5, 5)$$

Sub B and C into formula.

$$MP = \left(\frac{3+5}{2}, \frac{9+5}{2} \right)$$

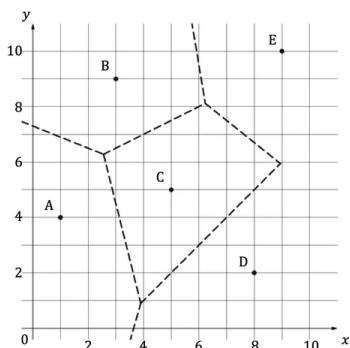
$$MP = (4, 7)$$

Sub MP and m into $y - y_1 = m(x - x_1)$.

$$y - 7 = \frac{1}{2}(x - 4) \quad \left. \begin{array}{l} \text{expand and rearrange} \\ \text{into form} \end{array} \right\}$$

$$x - 2y + 10 = 0$$

There is one straight line missing from the Voronoi diagram below.



(c) Find the equation of the missing line. Give your answer in the form $ax + by + d = 0$ where $a, b, d \in \mathbb{Z}$.

[3]

A family of rabbits are sighted at point F(10, 5).

(d) Write down the wolfpack territory the family of rabbits are in.

[1]

c) The perpendicular bisector of DE is needed.

$$D(8, 2) \quad E(9, 10)$$

$$m_{DE} = \frac{10 - 2}{9 - 8} = 8$$

$$\therefore m_{\perp DE} = -\frac{1}{8}$$

$$MP = \left(\frac{8+9}{2}, \frac{2+10}{2} \right)$$

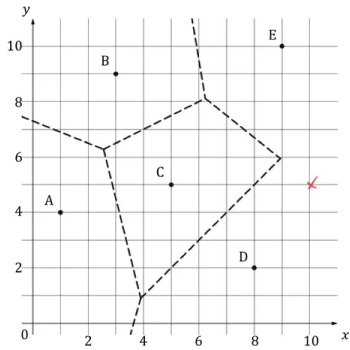
$$MP = (8.5, 6)$$

Sub MP and $m_{\perp DE}$ into $y - y_1 = m(x - x_1)$.

$$y - 6 = -\frac{1}{8}(x - 8.5) \quad \left. \begin{array}{l} \text{expand and rearrange} \\ \text{into form} \end{array} \right\}$$

$$2x + 16y - 113 = 0$$

There is one straight line missing from the Voronoi diagram below.



(c) Find the equation of the missing line. Give your answer in the form $ax + by + d = 0$ where $a, b, d \in \mathbb{Z}$.

[3]

A family of rabbits are sighted at point $F(10, 5)$.

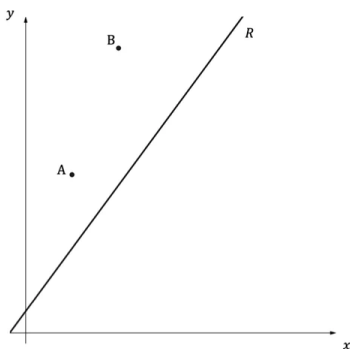
(d) Write down the wolfpack territory the family of rabbits are in.

[1]

d) The rabbits are in wolfpack O's territory.

Question 5

Two schools are represented by points $A(3, 15)$ and $B(6, 27)$ on the graph below. A road, represented by the line R with equation $y - 2x - 2 = 0$, passes near the schools. An architect is asked to determine the location of a new bus stop on the road such that it is the same distance from the two schools.



(a) Find the equation of the perpendicular bisector of $[AB]$. Give your equation in the form $ax + by + d = 0$ where $a, b, d \in \mathbb{Z}$.

[5]

(b) Determine the coordinates of the point on R where the bus stop should be located.

[2]

a) Perpendicular bisector of AB .

$A(3, 15) \quad B(6, 27)$

Find midpoint, MP .

$$MP = \left(\frac{3+6}{2}, \frac{15+27}{2} \right)$$

$$MP = (4.5, 21)$$

Find perpendicular gradient, $m_{\perp AB}$.

$$m_{AB} = \frac{27-15}{6-3} = 4 \quad \therefore m_{\perp AB} = -\frac{1}{4}$$

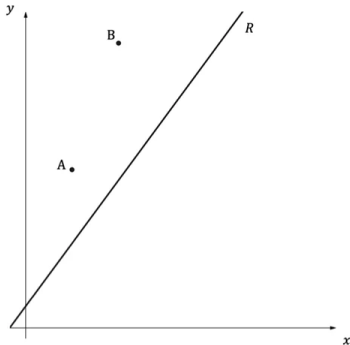
Sub MP and $m_{\perp AB}$ into $y - y_1 = m(x - x_1)$.

$$y - 21 = -\frac{1}{4}(x - 4.5)$$

} expand and rearrange into form

$$2x + 8y - 177 = 0$$

Two schools are represented by points A(3, 15) and B(6, 27) on the graph below. A road, represented by the line R with equation $y - 2x - 2 = 0$, passes near the schools. An architect is asked to determine the location of a new bus stop on the road such that it is the same distance from the two schools.



(a) Find the equation of the perpendicular bisector of [AB]. Give your equation in the form $ax + by + d = 0$ where $a, b, d \in \mathbb{Z}$.

$$2x + 8y - 177 = 0$$

[5]

(b) Determine the coordinates of the point on R where the bus stop should be located.

[2]

b) Simultaneous equations

$$2x + 8y - 177 = 0, \quad y = 2x + 2$$

Sub ① and ② into your GDC.

$$x = \frac{161}{18} \quad y = \frac{179}{9}$$

$$x = 8.9444... \quad y = 19.8888...$$

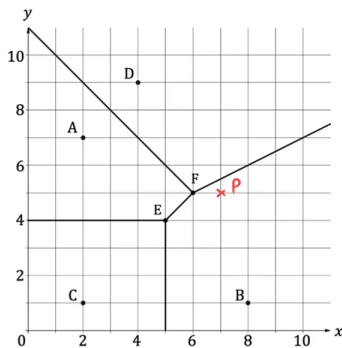
$$x = 8.94 \text{ (3sf)} \quad y = 19.9 \text{ (3sf)}$$

The bus stop should be located at $(\frac{161}{18}, \frac{179}{9})$ or (8.94, 19.9).

Question 6

The sites A, B, C and D in the Voronoi diagram below represent the locations of active volcanos in an Indonesian region and points E and F are intersections.

Horizontal scale: 1 unit represent 10 km. Vertical scale: 1 unit represent 10 km.



There is a population centre at P(7, 5).

(a) Calculate the distance from the population centre to the closest volcano.

[2]

(b) Determine the optimal position for a central shopping centre in the region, such that it is as far away from a volcano as possible.

[3]

(c) A geologist says a safe distance for a shopping centre from a volcano is 45 km. Determine whether the position of the shopping centre will be safe.

[3]

a) The population centre (P) is closest to volcano B.

Distance between two points formula

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} \quad (\text{in formula booklet})$$

$$P(7, 5) \quad B(8, 1)$$

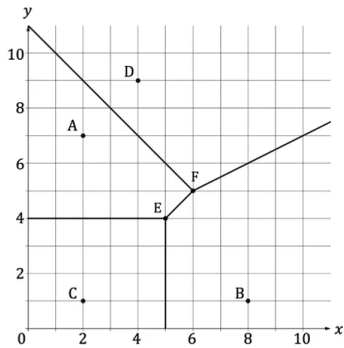
Sub P and B into formula.

$$d = \sqrt{(7-8)^2 + (5-1)^2}$$

$$d = \sqrt{17} = 4.1231...$$

$$d = 41.2 \text{ km}$$

The sites A, B, C and D in the Voronoi diagram below represent the locations of active volcanos in an Indonesian region and points E and F are intersections.
Horizontal scale: 1 unit represent 10 km. Vertical scale: 1 unit represent 10 km.



There is a population centre at P(7, 5).

- (a) Calculate the distance from the population centre to the closest volcano. [2]
- (b) Determine the optimal position for a central shopping centre in the region, such that it is as far away from a volcano as possible. [3]
- (c) A geologist says a safe distance for a shopping centre from a volcano is 45 km. Determine whether the position of the shopping centre will be safe. [3]

b) The optimal position will be the coordinates of the largest empty circle.
This will be either E(5, 4) or F(6, 5).

E is equidistant from A, B and C.

$$A(2, 7) \quad E(5, 4)$$

$$d_{AE} = \sqrt{(2-5)^2 + (7-4)^2}$$

$$d_{AE} = \sqrt{18} \text{ units} \quad (42.4 \text{ km})$$

F is equidistant from A, B and D.

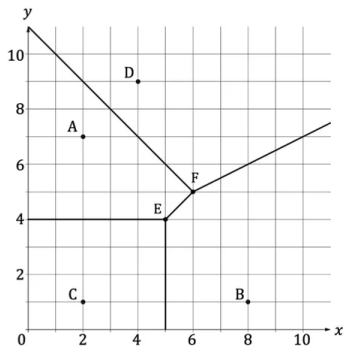
$$A(2, 7) \quad F(6, 5)$$

$$d_{AF} = \sqrt{(2-6)^2 + (7-5)^2}$$

$$d_{AF} = \sqrt{20} \text{ units} \quad (44.7 \text{ km})$$

$\therefore F(6, 5)$ is the optimal position for the shopping centre.

The sites A, B, C and D in the Voronoi diagram below represent the locations of active volcanos in an Indonesian region and points E and F are intersections.
Horizontal scale: 1 unit represent 10 km. Vertical scale: 1 unit represent 10 km.



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- (a) Calculate the distance from the population centre to the closest volcano. [2]
- (b) Determine the optimal position for a central shopping centre in the region, such that it is as far away from a volcano as possible. [3]
- (c) A geologist says a safe distance for a shopping centre from a volcano is 45 km. Determine whether the position of the shopping centre will be safe. [3]

c) The distance is 44.7 km, therefore the position is not safe.