EXAM PAPERS PRACTICE

GCSE OCR Math J560

Vectors

Answers

"We will help you to achieve A Star "

EXAM PAPERS PRACTICE

Answer 1

P is the point on AB such that $AP : PB = 3 : 1$

(b) Find $\overrightarrow{OP}$ in terms of **a** and **b**.
Give your answer in its simplest form.

$$\overrightarrow{OP} = \overrightarrow{OA} + \overrightarrow{AP}$$

$$= \overrightarrow{OA} + \frac{3}{4} \times \overrightarrow{AB}$$

$$= a + \frac{3}{4}(b - a)$$

$$= a + \frac{3}{4}b - \frac{3}{4}a$$

$$= \underline{\frac{1}{4}a + \frac{3}{4}b} \qquad = \underline{\frac{1}{4}(a + 3b)}$$

Answer 2

$\overrightarrow{NQ} = \frac{2}{5}\overrightarrow{SQ}$

(b) Express $\overrightarrow{NR}$ in terms of **a** and **b**.

$$\overrightarrow{NR} = \overrightarrow{NQ} + \overrightarrow{QR}$$

$$= \frac{2}{5}(a-b) + b$$

$$= \frac{2}{5}a - \frac{2}{5}b + b$$

$$= \frac{2}{5}a + \frac{3}{5}b$$

Answer 3

(b) Work out $\mathbf{a} + 2\mathbf{b}$ as a column vector.

$$a + 2b = \begin{pmatrix} 1 \\ 2 \end{pmatrix} + 2\begin{pmatrix} 1 \\ -3 \end{pmatrix}$$

$$= \begin{pmatrix} 1 + 2 \times 1 \\ 2 + 2 \times (-3) \end{pmatrix}$$

$$= \begin{pmatrix} 3 \\ -4 \end{pmatrix}$$

Answer 4

VECTORS

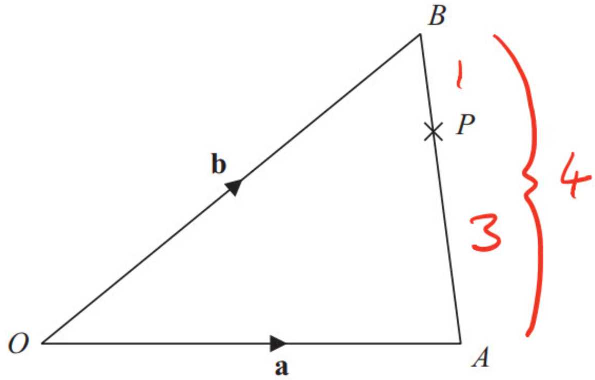


Diagram **NOT** accurately drawn

$\overrightarrow{AP} = \frac{3}{4} \times \overrightarrow{AB}$

OAB is a triangle.

$\overrightarrow{OA} = \mathbf{a}$
$\overrightarrow{OB} = \mathbf{b}$

(a) Find $\overrightarrow{AB}$ in terms of **a** and **b**.

$\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB}$

$= -a + b$

$b - a$

Answer 5

VECTORS

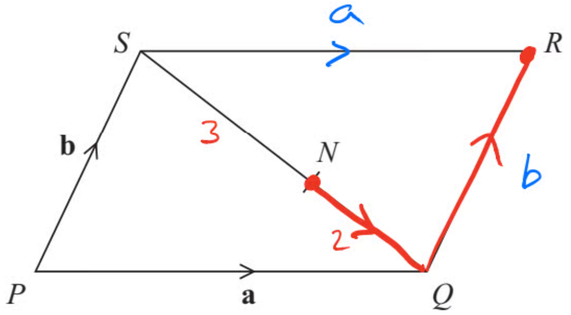


Diagram **NOT** accurately drawn

PQRS is a parallelogram.
N is the point on *SQ* such that *SN* : *NQ* = 3 : 2

TOTAL = 5

$\overrightarrow{NQ} = \frac{2}{5}\overrightarrow{SQ}$

$\overrightarrow{PQ}$ = **a**
$\overrightarrow{PS}$ = **b**

(a) Write down, in terms of **a** and **b**, an expression for $\overrightarrow{SQ}$.

$\overrightarrow{SQ} = \overrightarrow{SP} + \overrightarrow{PQ} = -b + a$

$\overrightarrow{SQ}$ = $a - b$

Answer 6

X is the point on AB such that $AX : XB = 1 : 2$

and $\overrightarrow{BY} = 5\mathbf{a} - \mathbf{b}$

*(b) Prove that $\overrightarrow{OX} = \frac{2}{5}\overrightarrow{OY}$

$$\boxed{\overrightarrow{AX} = \frac{1}{3} \times \overrightarrow{AB}}$$

$$\begin{aligned}\overrightarrow{OX} &= \overrightarrow{OA} + \overrightarrow{AX}\\ &= 3a + \tfrac{1}{3}(-3a + 6b)\\ &= 3a - a + 2b\\ &= 2a + 2b\\ &= \underline{\underline{2(a+b)}}\end{aligned}$$

$$\begin{aligned}\overrightarrow{OY} &= \overrightarrow{OB} + \overrightarrow{BY}\\ &= 6b + 5a - b\\ &= 5a + 5b\\ &= \underline{\underline{5(a+b)}}\end{aligned}$$

So $\underline{\underline{\overrightarrow{OX} = \frac{2}{5}\overrightarrow{OY}}}$

Answer 7

VECTORS

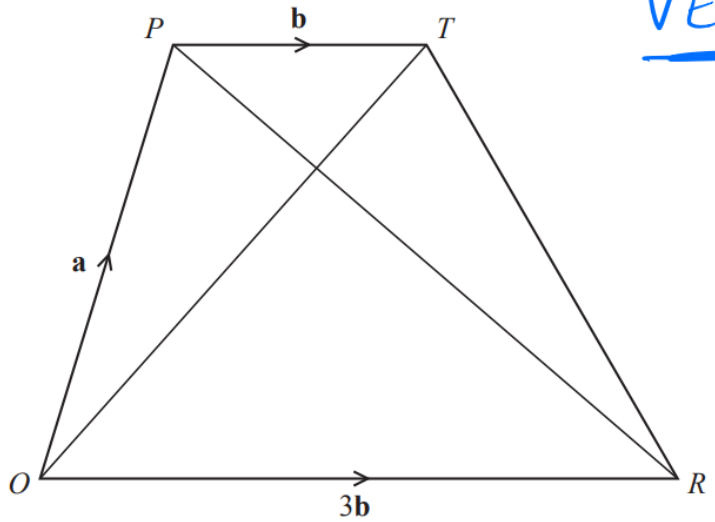


Diagram **NOT** accurately drawn

OPTR is a trapezium.

$\overrightarrow{OP} = \mathbf{a}$
$\overrightarrow{PT} = \mathbf{b}$
$\overrightarrow{OR} = 3\mathbf{b}$

(a) (i) Find $\overrightarrow{OT}$ in terms of **a** and **b**

$\overrightarrow{OT} = \overrightarrow{OP} + \overrightarrow{PT}$

$\overrightarrow{OT} = a + b$

(ii) Find $\overrightarrow{PR}$ in terms of **a** and **b**
Give your answer in its simplest form.

$\overrightarrow{PR} = \overrightarrow{PO} + \overrightarrow{OR}$

$\overrightarrow{PR} = -a + 3b$

Answer 8

*(c) What does your answer to part (b) tell you about the position of point S?

$\overrightarrow{OS} = \frac{3}{4}(a+b)$

SINCE $\overrightarrow{OT} = a+b$

$\overrightarrow{OS} = \frac{3}{4} \times \overrightarrow{OT}$

S LIES ON OT.

EXAM PAPERS PRACTICE

Answer 9

VECTORS

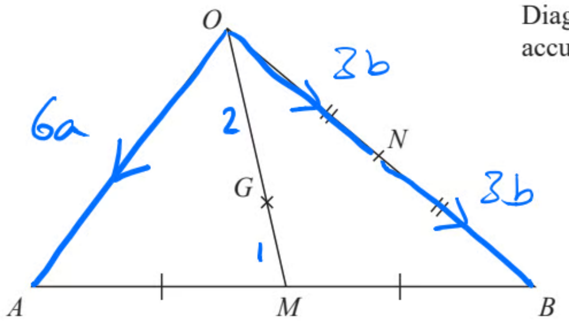


Diagram **NOT** accurately drawn

$\overrightarrow{OA} = 6\mathbf{a}$ and $\overrightarrow{OB} = 6\mathbf{b}$
M is the midpoint of AB.

$\overrightarrow{AM} = \frac{1}{2} \times \overrightarrow{AB}$

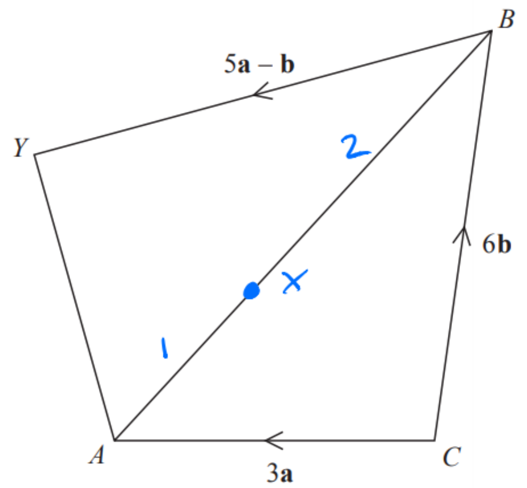
(a) Write $\overrightarrow{OM}$ in terms of **a** and **b**.
Give your answer in its simplest form.

$\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB}$
$= -6a + 6b$
$= 6b - 6a$

$\overrightarrow{OM} = \overrightarrow{OA} + \overrightarrow{AM}$
$= \overrightarrow{OA} + \frac{1}{2}\overrightarrow{AB}$
$= 6a + \frac{1}{2}(6b - 6a)$
$= 6a + 3b - 3a$
$\overrightarrow{OM} = \underline{3a + 3b}$

Answer 10

VECTORS



CAYB is a quadrilateral.

$\overrightarrow{CA} = 3\mathbf{a}$
$\overrightarrow{CB} = 6\mathbf{b}$
$\overrightarrow{BY} = 5\mathbf{a} - \mathbf{b}$

X is the point on *AB* such that $AX : XB = 1 : 2$

Prove that $\overrightarrow{CX} = \frac{2}{5}\overrightarrow{CY}$

$$\overrightarrow{AX} = \frac{1}{3} \times \overrightarrow{AB}$$

$$\overrightarrow{AX} = \frac{1}{3}(\overrightarrow{AC} + \overrightarrow{CB})$$

$$= \frac{1}{3}(-3a + 6b)$$

$$\overrightarrow{CX} = \overrightarrow{CA} + \overrightarrow{AX}$$

$$\overrightarrow{CX} = 3a + \frac{1}{3}(-3a + 6b)$$

$$= 3a - a + 2b$$

$$= 2a + 2b$$

$$\overrightarrow{CX} = 2(a+b)$$

$$\overrightarrow{CY} = \overrightarrow{CB} + \overrightarrow{BY}$$

$$= 6b + 5a - b$$

$$= 5a + 5b$$

$$\overrightarrow{CY} = 5(a+b)$$

$$\frac{2}{5}\overrightarrow{CY} = \frac{2}{\cancel{5}} \times \cancel{5}(a+b)$$

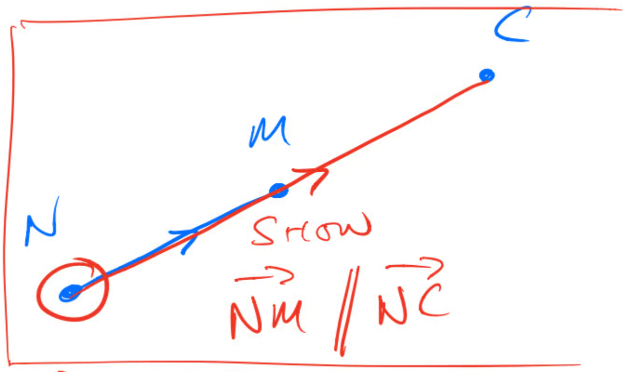
$$\frac{2}{5}\overrightarrow{CY} = 2(a+b)$$

$$\frac{2}{5}\overrightarrow{CY} = \overrightarrow{CX}$$

Answer 11

B is the midpoint of *AC*.
M is the midpoint of *PB*.

*(b) Show that *NMC* is a straight line.



$$\overrightarrow{NM} = \overrightarrow{NP} + \overrightarrow{PM}$$
$$= \overrightarrow{NP} + \tfrac{1}{2}\overrightarrow{PB}$$
$$= b + \tfrac{1}{2}(a - 3b)$$
$$= \tfrac{1}{2}a + b - \tfrac{3}{2}b$$
$$= \underline{\tfrac{1}{2}a - \tfrac{1}{2}b}$$

$$\overrightarrow{NC} = \overrightarrow{NA} + \overrightarrow{AC}$$
$$= -2b + 2a$$
$$= \underline{2a - 2b}$$

SINCE $\overrightarrow{NC} = 4\overrightarrow{NM}$, $\overrightarrow{NC}$ IS PARALLEL TO $\overrightarrow{NM}$ AND HENCE, NMC IS A STRAIGHT LINE.

Answer 12

*(b) Prove that *OND* is a straight line.

→ SHOW THAT $\overrightarrow{OD}$ IS PARALLEL TO $\overrightarrow{ON}$

$$\overrightarrow{OD} = \overrightarrow{OA} + \overrightarrow{AD}$$

$$= \underline{\underline{a + 2b}}$$

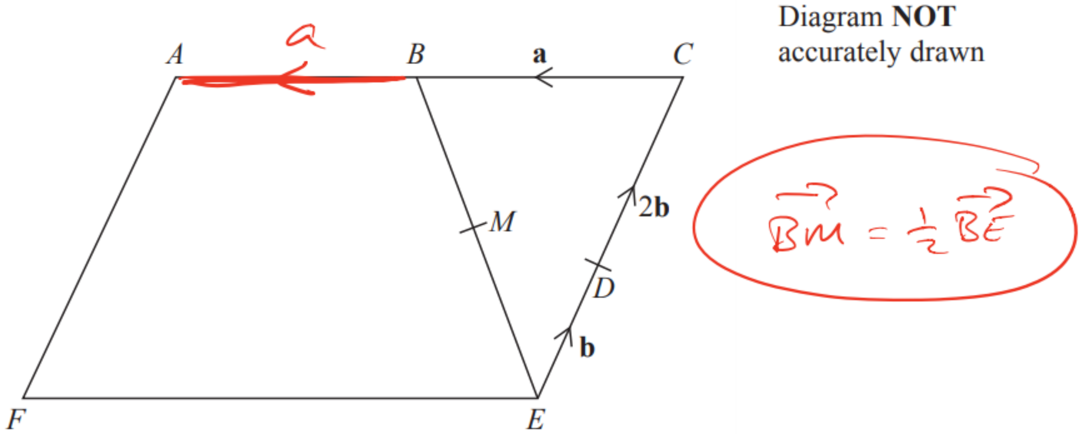
$$\overrightarrow{ON} = \frac{2}{3}b + \frac{1}{3}a$$

$$= \frac{1}{3}(2b + a)$$

$\overrightarrow{ON} = \frac{1}{3} \times \overrightarrow{OD}$ SO $\overrightarrow{ON} \parallel \overrightarrow{OD}$ SO OND IS A STRAIGHT LINE

Answer 13

VECTORS



NEED TO SHOW THAT $\overrightarrow{AD} = k \times \overrightarrow{AM}$

ACEF is a parallelogram.
B is the midpoint of *AC*.
M is the midpoint of *BE*.

$\overrightarrow{CB} = \mathbf{a}$ $\quad \overrightarrow{BA} = a$

$\overrightarrow{ED} = \mathbf{b}$

$\overrightarrow{DC} = 2\mathbf{b}$

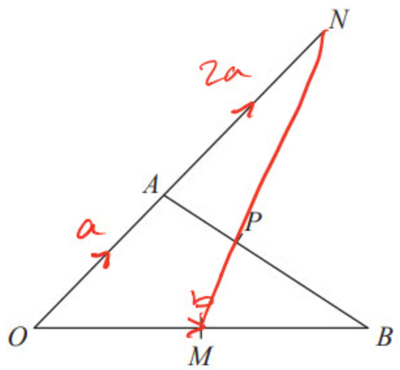
Show that *AMD* is a straight line.

$$\begin{aligned}
\overrightarrow{AM} &= \overrightarrow{AB} + \overrightarrow{BM} \\
&= -a + \tfrac{1}{2}\overrightarrow{BE} \\
&= -a + \tfrac{1}{2}(\overrightarrow{BC} + \overrightarrow{CE}) \\
&= -a + \tfrac{1}{2}(-a - 3b) \\
&= -a - \tfrac{1}{2}a - \tfrac{3}{2}b \\
&= -\tfrac{3}{2}a - \tfrac{3}{2}b \\
&= -\tfrac{3}{2}(a + b)
\end{aligned}$$

$$\begin{aligned}
\overrightarrow{AD} &= \overrightarrow{AC} + \overrightarrow{CD} \\
&= -2a - 2b \\
&= -2(a + b)
\end{aligned}$$

SO $\overrightarrow{AD}$ IS A MULTIPLE OF $\overrightarrow{AM}$ AND SO ADM IS A STRAIGHT LINE.

Answer 14



OAN, OMB and APB are straight lines.
$AN = 2OA$.
M is the midpoint of OB.

$\overrightarrow{OA} = \mathbf{a}$ $\overrightarrow{OB} = \mathbf{b}$

$\overrightarrow{AP} = k\overrightarrow{AB}$ where k is a scalar quantity. "NUMBER"

Given that MPN is a straight line, find the value of k.

$\overrightarrow{MN}$ IS A MULTIPLE OF $\overrightarrow{MP}$.

$$\overrightarrow{MN} = \overrightarrow{MO} + \overrightarrow{ON}$$
$$= -\tfrac{1}{2}b + 3a$$
$$= 3a - \tfrac{1}{2}b$$

$$\overrightarrow{MP} = \overrightarrow{MO} + \overrightarrow{OA} + \overrightarrow{AP}$$
$$= \overrightarrow{MO} + \overrightarrow{OA} + k\overrightarrow{AB}$$
$$= -\tfrac{1}{2}b + a + kb - ka$$
$$= (1-k)a + (k - \tfrac{1}{2})b$$

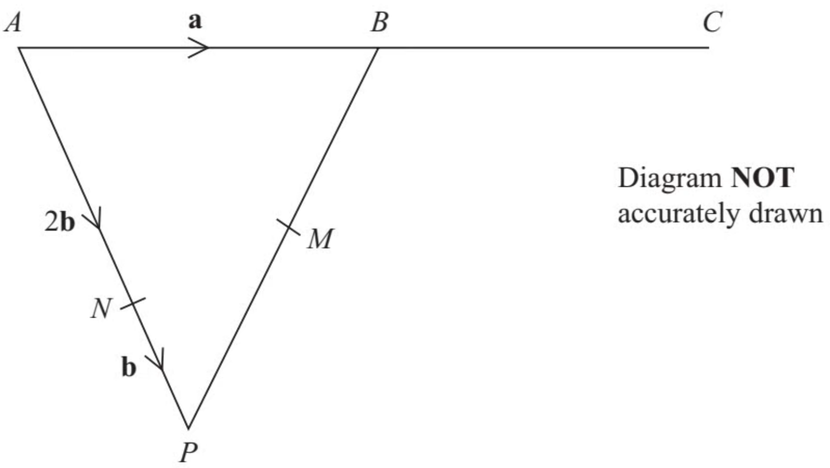
$$\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB}$$
$$= -a + b$$
$$= b - a$$

$$\frac{1-k}{3} = \frac{k - \frac{1}{2}}{-\frac{1}{2}}$$

$$\frac{1-k}{3} = 1 - 2k$$

$$1 - k = 3 - 6k$$
$$5k = 2$$
$$k = \frac{2}{5}$$

Answer 15



APB is a triangle.
N is a point on *AP*.

$\overrightarrow{AB} = \mathbf{a}$ $\overrightarrow{AN} = 2\mathbf{b}$ $\overrightarrow{NP} = \mathbf{b}$

(a) Find the vector $\overrightarrow{PB}$, in terms of **a** and **b**.

$\overrightarrow{PB} = \overrightarrow{PA} + \overrightarrow{AB}$

$= -b - 2b + a = \underline{a - 3b}$