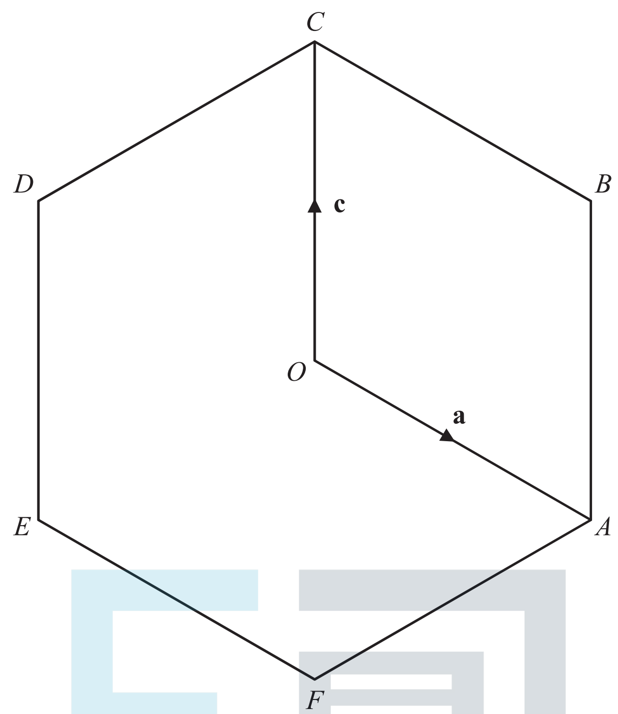
EXAM PAPERS PRACTICE

Vectors

Model Answer



O is the origin.
$ABCDEF$ is a regular hexagon and O is the midpoint of AD.

$\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OC} = \mathbf{c}$.

Find, in terms of **a** and **c**, in their simplest form

(a) $\overrightarrow{BE}$, [2]

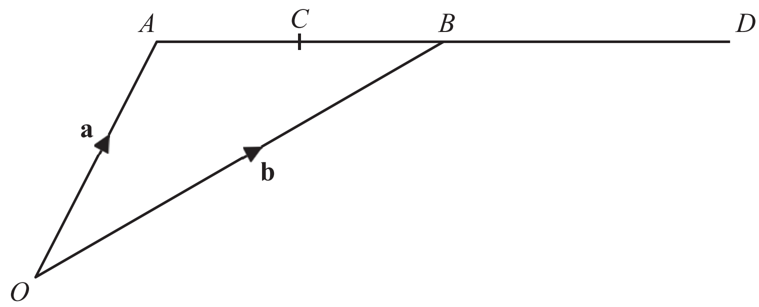
The vector $\overrightarrow{BE}$ in terms of $\mathbf{a}$ and $\mathbf{c}$ is $\mathbf{a} - \mathbf{c}$.

(b) $\overrightarrow{DB}$, [2]

In terms of $\mathbf{a}$ and $\mathbf{c}$, the vector $\overrightarrow{DB}$ is $\mathbf{a} - \mathbf{c}$.

(c) the position vector of E. [2]

The position vector of point E in terms of $\mathbf{a}$ and $\mathbf{c}$ is $\mathbf{a} - 2\mathbf{c}$.



A and B have position vectors **a** and **b** relative to the origin O.
C is the midpoint of AB and B is the midpoint of AD.

Find, in terms of **a** and **b**, in their simplest form

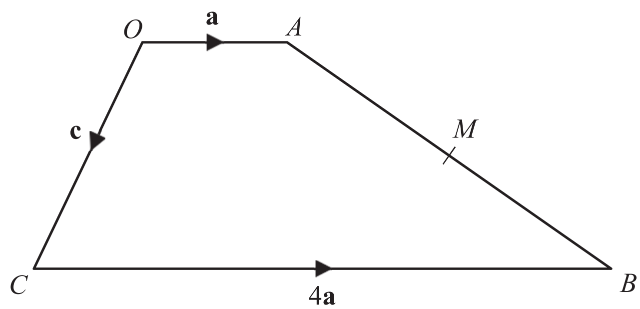
(a) the position vector of C, [2]

The position vector of point C, the midpoint of A and B, is given by $\frac{1}{2}(\mathbf{a}+\mathbf{b})$.

(b) the vector $\overrightarrow{CD}$. [2]

The vector $\overrightarrow{CD}$ can be expressed in terms of **a** and **b** as follows:

$\overrightarrow{CD} = \frac{1}{2}(\mathbf{a}+\mathbf{b}) - \mathbf{b} = \frac{1}{2}\mathbf{a} - \frac{1}{2}\mathbf{b}$.



O is the origin, $\overrightarrow{OA} = \mathbf{a}$, $\overrightarrow{OC} = \mathbf{c}$ and $\overrightarrow{CB} = 4\mathbf{a}$.
M is the midpoint of AB.

(a) Find, in terms of **a** and **c**, in their simplest form

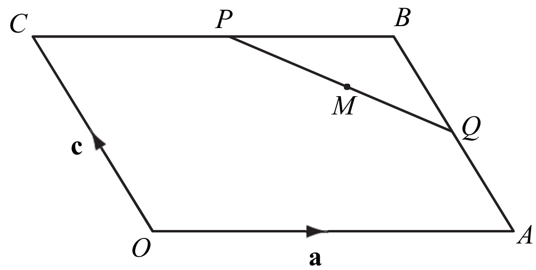
(i) the vector $\overrightarrow{AB}$, [2]

Therefore, the vector $\overrightarrow{AB}$ in terms of **a** and **c** is **c**

(ii) the position vector of M. [2]

the position vector of M in terms of **a** and **c** is 3**a**.

(b) Mark the point D on the diagram where $\overrightarrow{OD} = 3\mathbf{a} + \mathbf{c}$. [2]



NOT TO SCALE

O is the origin and $OABC$ is a parallelogram.
$CP = PB$ and $AQ = QB$.

$\overrightarrow{OA}$ = a and $\overrightarrow{OC}$ = c .
Find in terms of a and c, in their simplest form,

(a) $\overrightarrow{PQ}$, [2]

in terms of a and c, $\overrightarrow{PQ} = \mathrm{a} + 3\mathrm{c}$.

(b) the position vector of M, where M is the midpoint of PQ. [2]

The position vector of M in terms of **a** and **c** is:

$\mathbf{m} = \frac{\mathbf{p}+\mathbf{q}-\frac{\mathbf{a}}{2}-\frac{\mathbf{c}}{2}}{2}$

$\overrightarrow{AB} = \mathbf{a} + t\mathbf{b}$ and $\overrightarrow{CD} = \mathbf{a} + (3t - 5)\mathbf{b}$ where t is a number.

Find the value of t when $\overrightarrow{AB} = \overrightarrow{CD}$.

[2]

Setting $\overrightarrow{AB} = \overline{CD}$ and equating the corresponding components, we get:

$\mathbf{a} + t\mathbf{b} = \mathbf{a} + (3t - 5)\mathbf{b}$

$t\mathbf{b} = (3t - 5)\mathbf{b}$

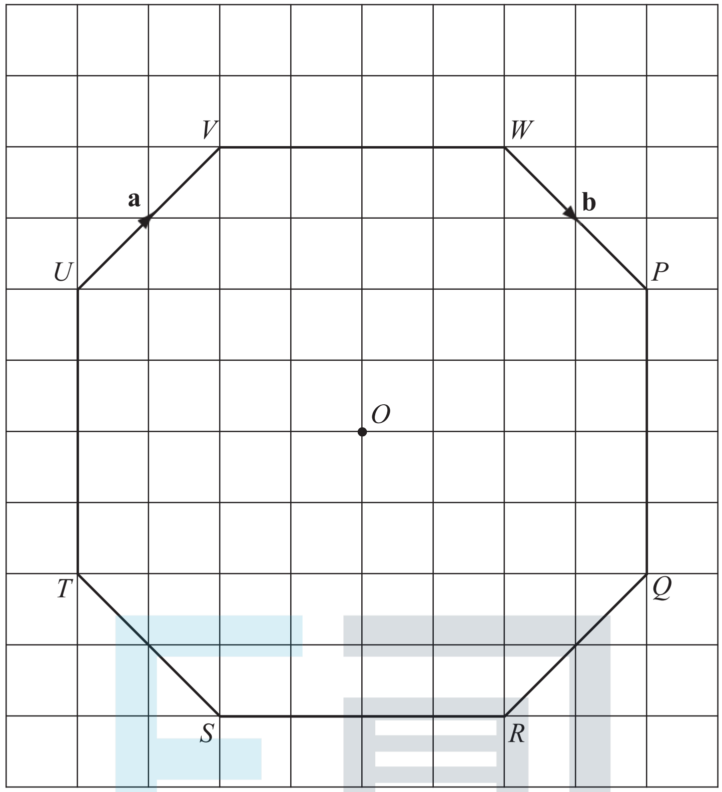
Since $\mathbf{b}$ is a vector and cannot be zero, we can cancel it from both sides of the equation, leaving:

$t = 3t - 5$

Solving for t :

$2t = 5 \Longrightarrow t = \frac{5}{2}$

So, the value of t is $\frac{5}{2}$.



The origin O is the centre of the octagon $PQRSTUVW$.
$\overrightarrow{UV} = \mathbf{a}$ and $\overrightarrow{WP} = \mathbf{b}$.

(a) Write down in terms of **a** and **b**

(i) $\overrightarrow{VW}$, [1]

In terms of **a** and **b** :

$$\overrightarrow{VW} = -\mathbf{a}$$

(ii) $\overrightarrow{TU}$, [1]

$$\overrightarrow{TU} = -\frac{1}{2}(\mathbf{a}+\mathbf{a})$$
$$= -\mathbf{a}$$

(iii) $\overrightarrow{TP}$, [2]

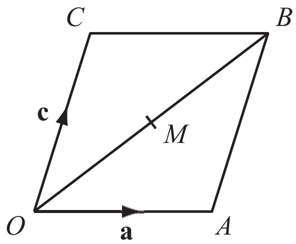
$$\overrightarrow{TP} = \overrightarrow{UQ} = \overrightarrow{WV} = \frac{1}{2}(\overrightarrow{UV}+\overrightarrow{WP}) = \frac{1}{2}(\mathbf{a}+\mathbf{b})$$

(iv) the position vector of the point P. [1]

the position vector of point P in terms of **a** and **b** is $\frac{1}{2}(\mathbf{a}+\mathbf{b})$.

(b) In the diagram, 1 centimetre represents 1 unit.
Write down the value of $|\mathbf{a}-\mathbf{b}|$. [1]

Given that 1 centimeter represents 1 unit, if $|\mathbf{a}| = 1$, then $|\mathbf{a}-\mathbf{b}| \leq 2$.



OABC is a parallelogram. $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OC} = \mathbf{c}$.
M is the mid-point of *OB*.
Find $\overrightarrow{MA}$ in terms of a and c.

[2]

In a parallelogram $OABC$, with $OA = \mathbf{a}$ and $OC = \mathbf{c}$, the midpoint M of OB is given by $\overrightarrow{MA} = \frac{3}{2}\mathbf{a} + \frac{1}{2}\mathbf{c}$.

(a) D is the point (2, –5) and $\overrightarrow{DE} = \begin{pmatrix} 7 \\ 1 \end{pmatrix}$.

Find the co-ordinates of the point E. [1]

It seems there might be some confusion in the question, as the information about points D and E is not directly related to the previous parallelogram.

(b) $\mathbf{v} = \begin{pmatrix} t \\ 12 \end{pmatrix}$ and $|\mathbf{v}| = 13$.

Work out the value of t, where t is negative. [2]

The magnitude (or norm) of a vector t
12 is given by:

$|\mathbf{v}| = \sqrt{t^2 + 12^2}$

You're given that $|\mathbf{v}| = 13$, so you can set up the equation:

$13 = \sqrt{t^2 + 12^2}$

Now, solve for t :

$13^2 = t^2 + 12^2$

$169 = t^2 + 144$

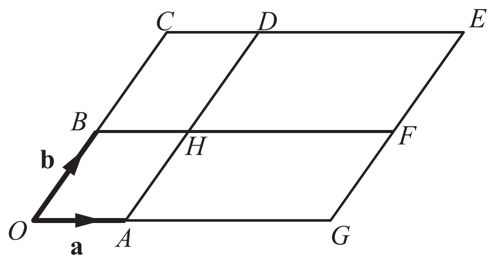
$t^2 = 25$

$t = \pm 5$

Given that t is negative, the solution is $t = -5$.

Question 9

The diagram shows a parallelogram *OCEG*.



NOT TO SCALE

O is the origin, $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$.
BHF and *AHD* are straight lines parallel to the sides of the parallelogram.
$\overrightarrow{OG} = 3\overrightarrow{OA}$ and $\overrightarrow{OC} = 2\overrightarrow{OB}$.

(a) Write the vector $\overrightarrow{HE}$ in terms of **a** and **b**. [1]

In terms of **a** and **b**, $\overrightarrow{HE} = 3\mathbf{b} - 4\mathbf{a}$.

(b) Complete this statement. [1]

$\mathbf{a} + 2\mathbf{b}$ is the position vector of point ....**E**................

(c) Write down two vectors that can be written as $3\mathbf{a} - \mathbf{b}$. [2]

Given that $\overrightarrow{OG} = 3\overrightarrow{OA}$ and $\overrightarrow{OC} = 2\overrightarrow{OB}$, we can express $\overrightarrow{OG}$ and $\overrightarrow{OC}$ in terms of and **b** :

$\overrightarrow{OG} = 3\mathbf{a}$

$\overrightarrow{OC} = 2\mathbf{b}$

Now, to find vectors in the form $3\mathbf{a} - \mathbf{b}$, we can subtract **b** from $3\mathbf{a}$:

$3\mathbf{a} - \mathbf{b} = 3\mathbf{a} - 1\mathbf{b}$

Therefore, two vectors that can be written as $3\mathbf{a} - \mathbf{b}$ are $3\mathbf{a} - \mathbf{b}$ itself and $2\mathbf{a} - \mathbf{b}$.

(a) $\overrightarrow{GH} = \begin{pmatrix} 6 \\ -4 \end{pmatrix}$

Find

(i) $5\overrightarrow{GH}$, [1]

To find $5\overrightarrow{GH}$, you simply multiply each component of 6
-4 by 5:

$$5\overrightarrow{GH} = 5 \times \begin{pmatrix} 6 \\ -4 \end{pmatrix} = \begin{pmatrix} 30 \\ -20 \end{pmatrix}$$

(ii) $\overrightarrow{HG}$. [1]

$$\overrightarrow{HG} = \begin{pmatrix} -6 \\ 4 \end{pmatrix}$$

(b) $\begin{pmatrix} 6 \\ 7 \end{pmatrix} + \begin{pmatrix} 2 \\ y \end{pmatrix} = \begin{pmatrix} 8 \\ 3 \end{pmatrix}$

Find the value of y. [1]

$$\begin{pmatrix} 6 \\ 7 \end{pmatrix} + \begin{pmatrix} 2 \\ y \end{pmatrix} = \begin{pmatrix} 8 \\ 3 \end{pmatrix}$$

Now, add the corresponding components:

$6 + 2 = 8$

$7 + y = 3$

Solving the second equation for y :

$y = 3 - 7 = -4$

So, the value of y is -4 .

$\overrightarrow{BC} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ $\overrightarrow{BA} = \begin{pmatrix} -5 \\ 6 \end{pmatrix}$

(a) Find $\overrightarrow{CA}$. [2]

So, $\overrightarrow{CA} = \begin{pmatrix} 7 \\ -3 \end{pmatrix}$.

(b) Work out $|\overrightarrow{BA}|$. [2]

The magnitude (or norm) of a vector $\overrightarrow{BA}$ is given by the formula:

$|\overrightarrow{BA}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

In this case, -5

6. The initial point B corresponds to $(0, 0)$, so $x_1 = 0$ and $y_1 = 0$. The final point A corresponds to $(-5, 6)$, so $x_2 = -5$ and $y_2 = 6$.

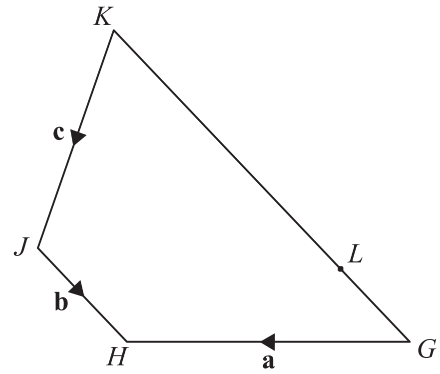
Now, plug these values into the formula:

$|\overrightarrow{BA}| = \sqrt{(-5 - 0)^2 + (6 - 0)^2}$

$|\overrightarrow{BA}| = \sqrt{25 + 36}$

$|\overrightarrow{BA}| = \sqrt{61}$

So, the magnitude of $\overrightarrow{BA}$ is $\sqrt{61}$.



GHJK is a quadrilateral.
$\overrightarrow{GH} = \mathbf{a}$, $\overrightarrow{JH} = \mathbf{b}$ and $\overrightarrow{KJ} = \mathbf{c}$.
L lies on *GK* so that $LK = 3GL$.

Find an expression, in terms of **a**, **b** and **c**, for $\overrightarrow{GL}$. [2]

Given that L lies on GK so that $LK = 3GL$, we can express $\overrightarrow{GL}$ in terms of $\overrightarrow{GK}$ using the fact that $LK = 3GL$.

$\overrightarrow{GL} = \frac{1}{3}\overrightarrow{GK}$

Now, express $\overrightarrow{GK}$ in terms of $\overrightarrow{GH}$, $\overrightarrow{HJ}$, and $\overrightarrow{JK}$:

$\overrightarrow{GK} = \overrightarrow{GH} + \overrightarrow{HJ} + \overrightarrow{JK}$

Substitute the given vectors:

$\overrightarrow{GK} = \mathbf{a} + \mathbf{b} + \mathbf{c}$

Now, substitute this into the expression for $\overrightarrow{GL}$:

$\overrightarrow{GL} = \frac{1}{3}(\mathbf{a} + \mathbf{b} + \mathbf{c})$

So, in terms of **a**, **b**, and **c**, the expression for $\overrightarrow{GL}$ is:

$\overrightarrow{GL} = \frac{1}{3}(\mathbf{a} + \mathbf{b} + \mathbf{c})$

NOT TO SCALE

A is the point (–1, 1) and B is the point (8, 7).

(a) Write $\overrightarrow{AB}$ as a column vector. [1]

$$\overrightarrow{AB} = \begin{pmatrix} 8-(-1) \\ 7-1 \end{pmatrix} = \begin{pmatrix} 9 \\ 6 \end{pmatrix}$$

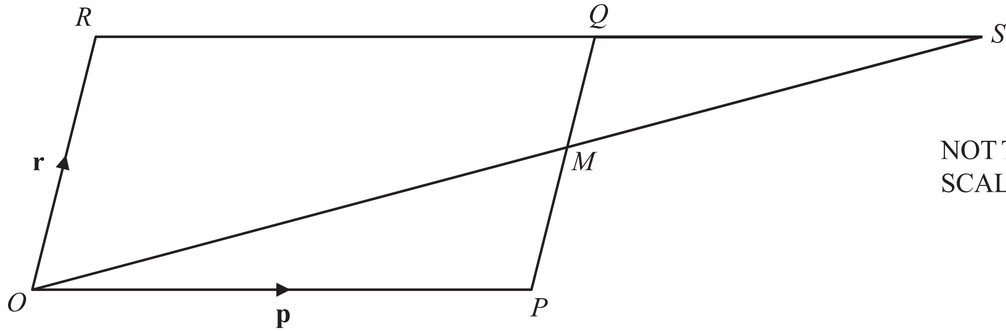
(b) Find $|\overrightarrow{AB}|$. [2]

$$|\overrightarrow{AB}| = \sqrt{117}$$

(c) $\overrightarrow{AC} = 2\overrightarrow{AB}$. [1]

Write down the co-ordinates of C.

$$C = A + \overrightarrow{AC} = \begin{pmatrix} -1 \\ 1 \end{pmatrix} + \begin{pmatrix} 18 \\ 12 \end{pmatrix} = \begin{pmatrix} 17 \\ 13 \end{pmatrix}$$



OPQR is a parallelogram, with *O* the origin.
M is the midpoint of *PQ*.
OM and *RQ* are extended to meet at *S*.
$\overrightarrow{OP} = \mathbf{p}$ and $\overrightarrow{OR} = \mathbf{r}$.

(a) Find, in terms of **p** and **r**, in its simplest form,

(i) $\overrightarrow{OM}$, [1]

$\overrightarrow{OM} = \mathbf{0}$, which is the vector at the origin.

(ii) the position vector of *S*. [1]

The position vector of S is $\frac{1}{2}\mathbf{0}$, which is simply $\mathbf{0}$.

(b) When $\overrightarrow{PT} = -\frac{1}{2}\mathbf{p} + \mathbf{r}$, what can you write down about the position of *T* ? [1]

Given that OM is the midpoint of PQ, the vector $\overrightarrow{OM}$ is half of the vector $\overrightarrow{OP}$:

$\overrightarrow{OM} = \frac{1}{2}\overrightarrow{OP} = \frac{1}{2}\mathbf{p}$

Now, you're given that $\overrightarrow{PT} = -\frac{1}{2}\mathbf{p} + \mathbf{r}$. Notice that $-\frac{1}{2}\mathbf{p} + \mathbf{r}$ is the vector that connects the midpoint M to point $T(\overrightarrow{TM})$.
So, you can express the position vector of T as:

$\overrightarrow{OT} = \overrightarrow{OM} + \overrightarrow{TM}$

Substitute the values:

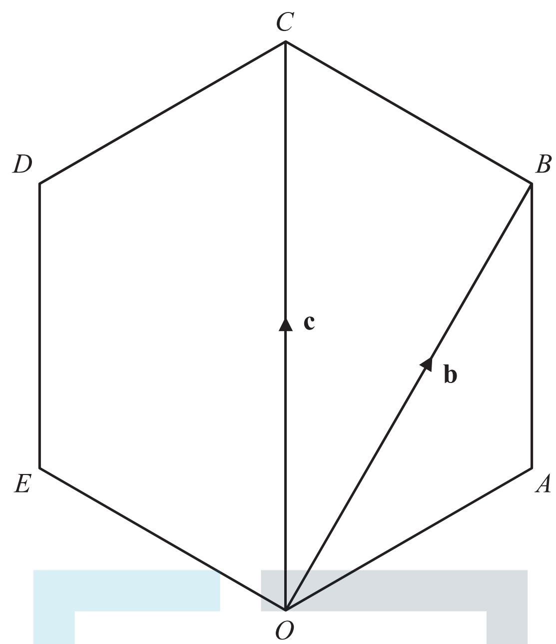
$\overrightarrow{OT} = \frac{1}{2}\mathbf{p} + (-\frac{1}{2}\mathbf{p} + \mathbf{r})$

Combine like terms:

$\overrightarrow{OT} = \frac{1}{2}\mathbf{p} - \frac{1}{2}\mathbf{p} + \mathbf{r}$

$\overrightarrow{OT} = \mathbf{r}$

So, the position vector of point T is $\mathbf{r}$.



OABCDE is a regular polygon.

(a) Write down the geometrical name for this polygon. [1]

Regular polygon $OABCDE$ is a hexagon.

(b) O is the origin. $\overrightarrow{OB} = \mathbf{b}$ and $\overrightarrow{OC} = \mathbf{c}$.

Find, in terms of **b** and **c**, in their simplest form,

(i) $\overrightarrow{BC}$, [1]

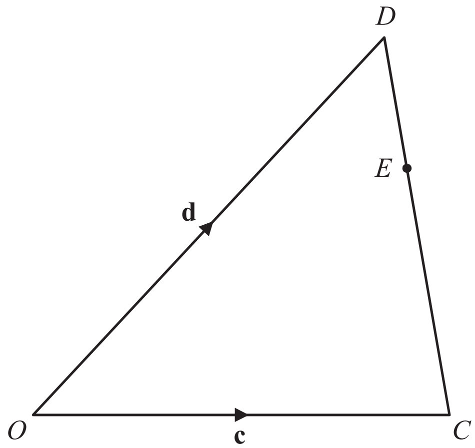
$$\overrightarrow{BC} = \mathbf{c} - \mathbf{b}$$

(ii) $\overrightarrow{OA}$, [2]

$$\overrightarrow{OA} = \overrightarrow{OB} + \overrightarrow{BC} + \overrightarrow{CD} + \overrightarrow{DE} + \overrightarrow{EA}$$
$$= \mathbf{b} + (\mathbf{c} - \mathbf{b}) + (\mathbf{c} - \mathbf{b}) + (\mathbf{c} - \mathbf{b}) - \overrightarrow{OE}$$
$$= 3\mathbf{c} - 5\mathbf{b} - \overrightarrow{OE}$$

(iii) the position vector of E. [1]

$$\overrightarrow{OE} = \overrightarrow{OB} + \overrightarrow{BC} + \overrightarrow{CD} + \overrightarrow{DE}$$
$$= \mathbf{b} + \mathbf{c} + \mathbf{c} + \mathbf{c}$$
$$= \mathbf{b} + 3\mathbf{c}$$



NOT TO SCALE

In the diagram, O is the origin.
$\overrightarrow{OC} = \mathbf{c}$ and $\overrightarrow{OD} = \mathbf{d}$.
E is on CD so that $CE = 2ED$.

Find, in terms of c and d, in their simplest forms,

(a) $\overrightarrow{DE}$, [2]

$$\overrightarrow{DE} = \frac{1}{3}(\mathrm{c} - \mathrm{d}) = \frac{1}{3}\mathrm{c} - \frac{1}{3}\,\mathrm{d}$$

(b) the position vector of E. [2]

Let's denote the position vector of E as OE. Since E is on CD and $CE = 2ED$, we can express $\overrightarrow{OE}$ in terms of $\overrightarrow{OC}$ and $\overrightarrow{OD}$.

$\overrightarrow{OE} = \overrightarrow{OC} + \overrightarrow{CE}$

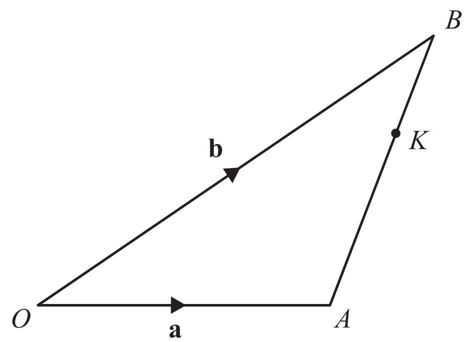
$\quad = \overrightarrow{OC} + 2\overrightarrow{ED}$

Now, substitute the given vectors:

$\overrightarrow{OE} = \mathrm{c} + 2\,\mathrm{d}$

So, in terms of c and d, the position vector of E is $\mathrm{c} + 2\,\mathrm{d}$.

Question 17



NOT TO SCALE

O is the origin and K is the point on AB so that $AK : KB = 2 : 1$.
$\overrightarrow{OA} =$ **a** and $\overrightarrow{OB} =$ **b**.

Find the position vector of K.
Give your answer in terms of **a** and **b** in its simplest form. [3]

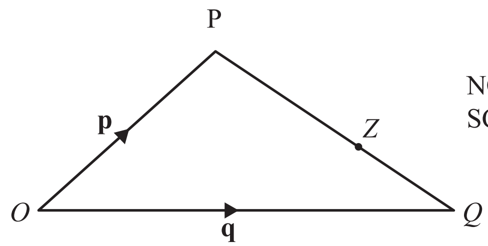
Since $AK : KB = 2 : 1$, we can express the position vector of $K(\overline{OK})$ in terms of $\overline{OA}$ and$\overrightarrow{OB}$:

$\overrightarrow{OK} = \frac{1}{3}\overrightarrow{OA} + \frac{2}{3}\overrightarrow{OB}$

Now, substitute the given vectors:

$\overrightarrow{OK} = \frac{1}{3}\mathbf{a} + \frac{2}{3}\mathbf{b}$

So, in terms of $\mathbf{a}$ and $\mathbf{b}$, the position vector of K is $\frac{1}{3}\mathbf{a} + \frac{2}{3}\mathbf{b}$.



O is the origin, $\overrightarrow{OP} = \mathbf{p}$ and $\overrightarrow{OQ} = \mathbf{q}$.
Z is a point on PQ such that $PZ : ZQ = 5 : 2$.

Work out, in terms of **p** and **q**, the position vector of Z.
Give your answer in its simplest form. [3]

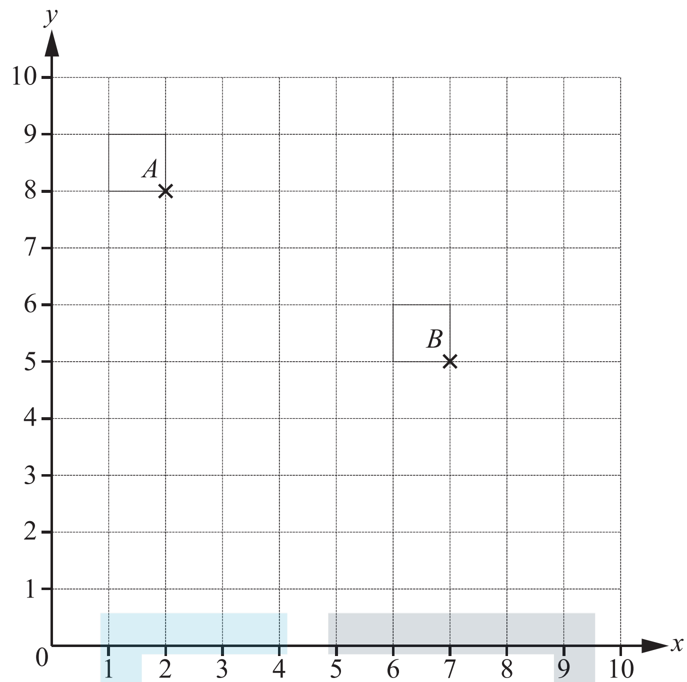
Given that $PZ : ZQ = 5 : 2$, the position vector of $Z(\overline{OZ})$ can be expressed as:

$\overrightarrow{OZ} = \frac{2}{7}\overrightarrow{OP} + \frac{5}{7}\overrightarrow{OQ}$

Now, substitute the given vectors:

$\overrightarrow{OZ} = \frac{2}{7}\mathbf{p} + \frac{5}{7}\mathbf{q}$

So, in terms of $\mathbf{p}$ and $\mathbf{q}$, the position vector of Z is $\frac{2}{7}\mathbf{p} + \frac{5}{7}\mathbf{q}$.



Points A and B are marked on the grid.

$\overrightarrow{BC} = \begin{pmatrix} -4 \\ 0 \end{pmatrix}$

(a) On the grid, plot the point C. [1]

The point C is plotted on the x-axis, 7 units to the right of the origin.

(b) Write $\overrightarrow{AC}$ as a column vector. [1]

(3, 3)

(c) $\overrightarrow{DE}$ is a vector that is perpendicular to $\overrightarrow{BC}$.
The magnitude of $\overrightarrow{DE}$ is equal to the magnitude of $\overrightarrow{BC}$. [2]

Write down a possible column vector for $\overrightarrow{DE}$.

Possible column vector for $\overrightarrow{DE}$ as $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$.

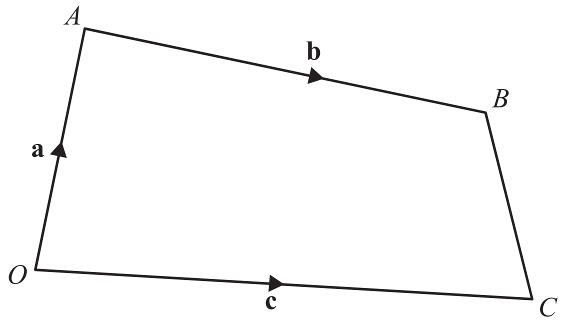
Work out

$$2\begin{pmatrix}3\\5\end{pmatrix}-\begin{pmatrix}1\\2\end{pmatrix}$$

[1]

$$2\begin{pmatrix}3\\5\end{pmatrix}-\begin{pmatrix}1\\2\end{pmatrix}=\begin{pmatrix}2\cdot 3-1\\2\cdot 5-2\end{pmatrix}=\begin{pmatrix}5\\8\end{pmatrix}$$

So, $2\begin{pmatrix}3\\5\end{pmatrix}-\begin{pmatrix}1\\2\end{pmatrix}=\begin{pmatrix}5\\8\end{pmatrix}$.



NOT TO SCALE

In the diagram, O is the origin, $\overrightarrow{OA} = \mathbf{a}$, $\overrightarrow{OC} = \mathbf{c}$ and $\overrightarrow{AB} = \mathbf{b}$.
P is on the line AB so that $AP : PB = 2 : 1$.
Q is the midpoint of BC.

Find, in terms of **a**, **b** and **c**, in its simplest form

(a) $\overrightarrow{CB}$, [1]

$$\overrightarrow{CB} = \mathbf{c}$$

(b) the position vector of Q, [2]

$$\overrightarrow{OQ} = \frac{1}{2}\mathbf{b} + \frac{1}{2}\mathbf{c}$$

(c) $\overrightarrow{PQ}$. [2]

$$\overrightarrow{PQ} = \frac{3}{2}((\mathbf{a}+\mathbf{b}) - \mathbf{a}) = \frac{3}{2}\mathbf{b}$$

$$\overrightarrow{AB} = \begin{pmatrix} -3 \\ 5 \end{pmatrix}$$

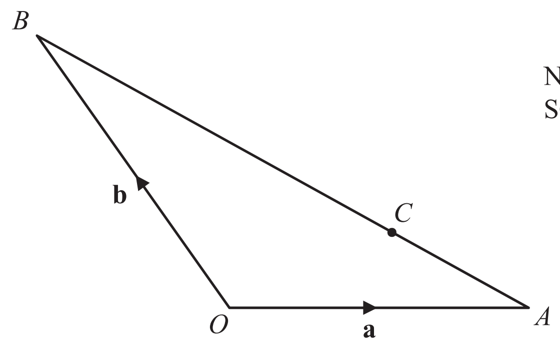
Find $|\overrightarrow{AB}|$. [2]

$|\overrightarrow{AB}| = \sqrt{(-3)^2 + 5^2}$

$|\overrightarrow{AB}| = \sqrt{9 + 25}$

$|\overrightarrow{AB}| = \sqrt{34}$

So, the magnitude of $\overrightarrow{AB}$ is $\sqrt{34}$.



NOT TO SCALE

In the diagram, O is the origin, $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$.
C is on the line AB so that $AC : CB = 1 : 2$.

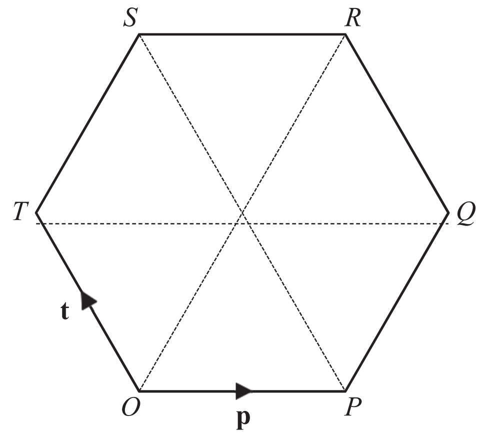
Find, in terms of **a** and **b**, in its simplest form,

(a) $\overrightarrow{AC}$, [2]

$$\overrightarrow{AC} = \frac{2}{3}\mathbf{a} + \frac{1}{3}\mathbf{b}$$

(b) the position vector of C. [2]

$$\overrightarrow{OC} = \frac{1}{3}\mathbf{a} + \frac{2}{3}\mathbf{b}$$



O is the origin and $OPQRST$ is a regular hexagon.

$\overrightarrow{OP} = \mathbf{p}$ and $\overrightarrow{OT} = \mathbf{t}$.

Find, in terms of **p** and **t**, in their simplest forms,

(a) $\overrightarrow{PT}$, [1]

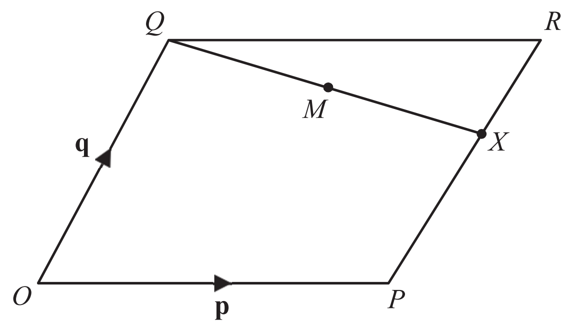
$$\overrightarrow{PT} = \frac{1}{6}(\mathbf{p} + \mathbf{t})$$
$$= \frac{1}{6}\mathbf{p} + \frac{1}{6}\mathbf{t}$$

(b) $\overrightarrow{PR}$, [2]

$$\overrightarrow{PR} = \frac{1}{6}\mathbf{p} - \mathbf{p} = -\frac{5}{6}\mathbf{p}$$

(c) the position vector of R. [2]

$$\overrightarrow{OR} = \begin{pmatrix} \frac{1}{2}\left(p_x - \sqrt{3}p_y\right) \\ \frac{\sqrt{3}}{2}p_x + \frac{1}{2}p_y \end{pmatrix}$$



NOT TO SCALE

O is the origin and $OPRQ$ is a parallelogram.
The position vectors of P and Q are p and q.
X is on PR so that $PX = 2XR$.

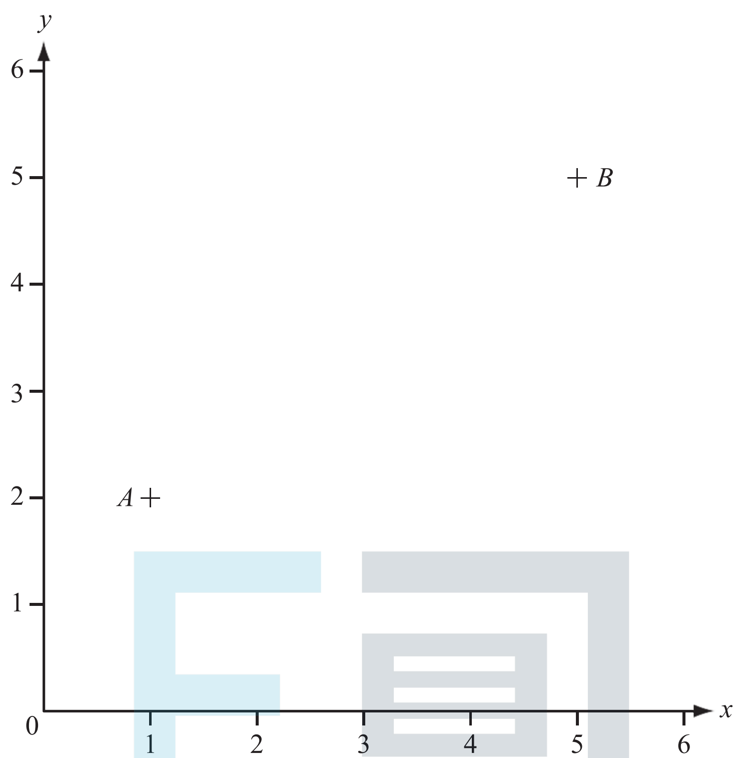
Find, in terms of p and q, in their simplest forms

(a) ,$\overrightarrow{QX}$ [2]

$$\overrightarrow{QX} = \mathrm{q} - \frac{2}{3}(\mathrm{q} - \mathrm{p}) = \frac{1}{3}(\mathrm{q} + \mathrm{p})$$

(b) the position vector of M, the midpoint of QX. [2]

$$\overrightarrow{OM} = \frac{1}{2}(\mathrm{q} + \mathrm{p})$$



The points $A(1, 2)$ and $B(5, 5)$ are shown on the diagram .

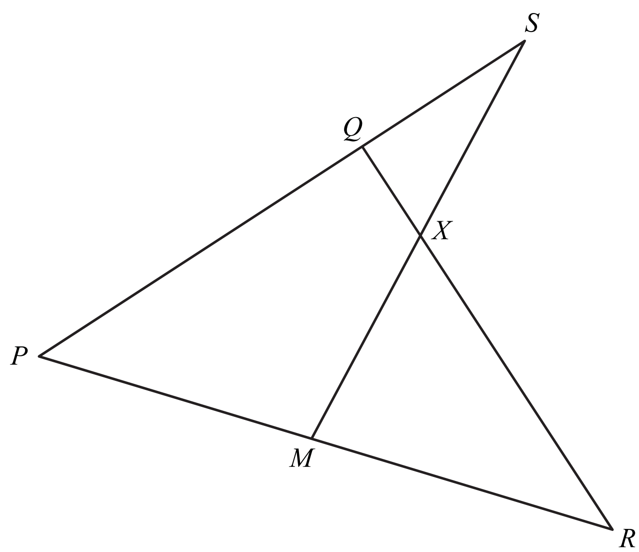
(a) Work out the co-ordinates of the midpoint of AB. [1]

The midpoint of AB is $(3, 3.5)$.

(b) Write down the column vector $\overrightarrow{AB}$. [1]

The column vector $\overrightarrow{AB}$ is:

$[4, 3]$



In the diagram, *PQS*, *PMR*, *MXS* and *QXR* are straight lines.

PQ = 2 *QS*.
M is the midpoint of *PR*.
QX : *XR* = 1 : 3.

$\overrightarrow{PQ} = \mathbf{q}$ and $\overrightarrow{PR} = \mathbf{r}$.

(a) Find, in terms of q and r,

(i) $\overrightarrow{RQ}$, [1]

$$\overrightarrow{RQ} = \begin{pmatrix}4\\3\end{pmatrix} - \begin{pmatrix}8\\5\end{pmatrix} = \begin{pmatrix}-4\\-2\end{pmatrix}$$

(ii) $\overrightarrow{MS}$. [1]

$$\overrightarrow{MS} = 2\begin{pmatrix}q\\0\end{pmatrix} = \begin{pmatrix}2q\\0\end{pmatrix}$$

(b) By finding $\overrightarrow{MX}$, show that *X* is the midpoint of *MS*. [3]

$$\overrightarrow{MX} = \vec{X} - \vec{M} = \frac{1}{2}\vec{S} - \frac{1}{2}\vec{M} = \frac{1}{2}\overrightarrow{MS}$$

The position vector r is given by $\mathbf{r} = 2\mathbf{p} + t(\mathbf{p} + \mathbf{q})$.

(a) Complete the table below for the given values of t.
Write each vector in its simplest form.
One result has been done for you. [3]

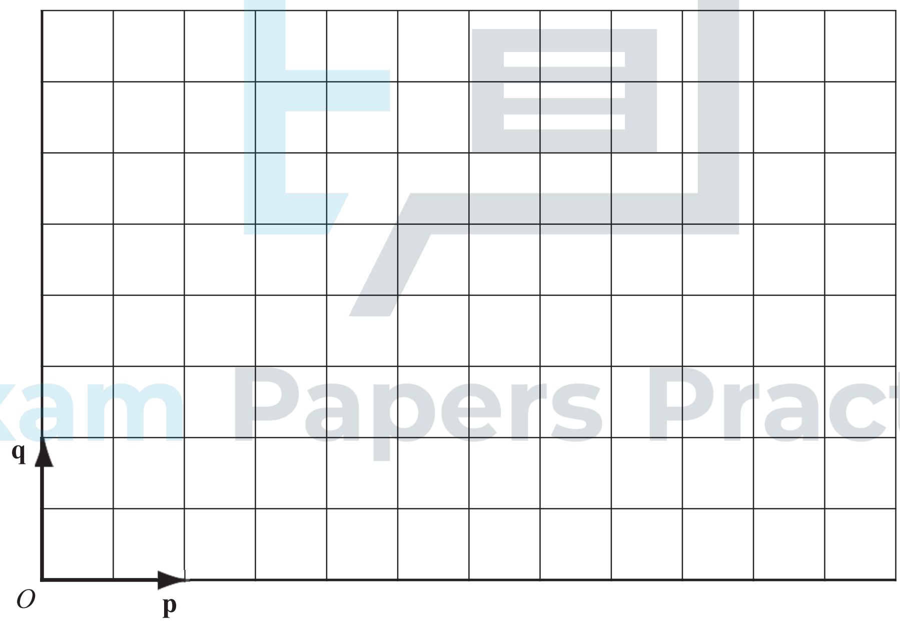
t	0	1	2	3
r	**2p**	**3p + q**	4p + 2q	**5p + 3q**

(b) O is the origin and **p** and **q** are shown on the diagram.

(i) Plot the 4 points given by the position vectors in the table.

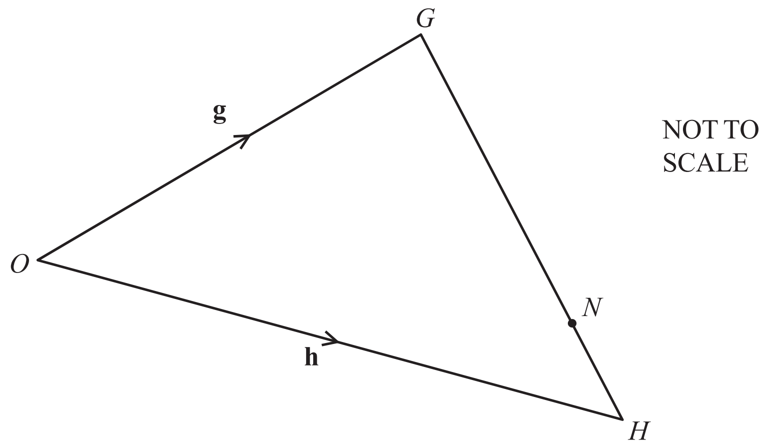
The points are labeled as follows:
- $P = (2, 1)$
- $Q = (4, 3)$
- $M = (3, 2)$
- $R = (5, 2)$

[2]



(ii) What can you say about these four points? [1]

The four points are collinear, evenly spaced, and divide the line segment joining **p** and **q** into four equal parts.



In triangle OGH, the ratio $GN : NH = 3 : 1$.

$\overrightarrow{OG} = \mathbf{g}$ and $\overrightarrow{OH} = \mathbf{h}$.

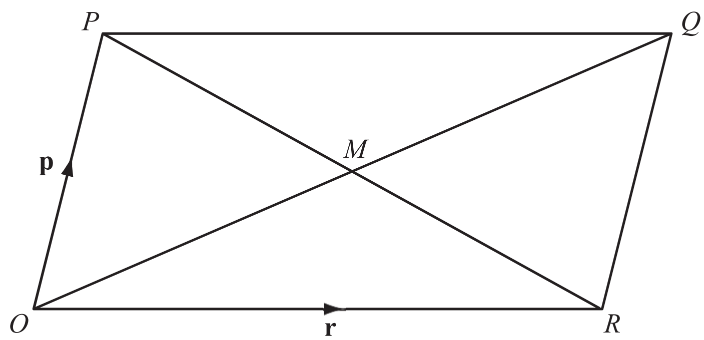
Find the following in terms of g and **h**, giving your answers in their simplest form.

(a) $\overrightarrow{HG}$ [1]

$\overrightarrow{GH} = \overrightarrow{GH}$. The vector $\overrightarrow{HG}$ is equal to $\overrightarrow{GH}$.

(b) $\overrightarrow{ON}$ [2]

$$\overrightarrow{ON} = \frac{1}{4}\mathbf{g} + \frac{3}{4}\mathbf{h}$$



O is the origin and $OPQR$ is a parallelogram whose diagonals intersect at M.

The vector $\overrightarrow{OP}$ is represented by p and the vector $\overrightarrow{OR}$ is represented by r.

(a) Write down a single vector which is represented by

(i) $\mathbf{p} + \mathbf{r}$, [1]

$\overrightarrow{OP} + \overrightarrow{OR} = \mathrm{p} + \mathrm{r}$

So, the single vector represented by $\mathrm{p} + \mathrm{r}$ is $\mathrm{p} + \mathrm{r}$.

(ii) $\frac{1}{2}\mathbf{p} - \frac{1}{2}\mathbf{r}$. [1]

$$\frac{1}{2}\mathbf{p} - \frac{1}{2}\mathbf{r} = \frac{1}{2}(\mathbf{p} - \mathbf{r})$$

(b) On the diagram, mark with a cross (x) and label with the letter S the point with position vector [2]

$\frac{1}{2}\mathbf{p} + \frac{3}{4}\mathbf{r}$.

To mark the point S with the given position vector $\frac{1}{2}\mathbf{p} + \frac{3}{4}\mathbf{r}$, we can find its coordinates in terms of the vectors $\mathbf{p}$ and $\mathbf{r}$.
The coordinates of S are given by $\left(\frac{1}{2}\mathbf{p} + \frac{3}{4}\mathbf{r}\right)$.
To find the coordinates, we can use the components of vectors $\mathbf{p}$ and $\mathbf{r}$. If p_x
p_y and $\mathbf{r} = r_x$
r_y, then the coordinates of S are:
$\left(\frac{1}{2}p_x + \frac{3}{4}r_x, \frac{1}{2}p_y + \frac{3}{4}r_y\right)$
Now, mark the point S on the diagram at these coordinates with a cross (x) and label it as S.