

IB Maths: AA HL

Vector Properties

Topic Questions

These practice questions can be used by students and teachers and is Suitable for IB Maths AA HL Topic Questions

Course	IB Maths
Section	3. Geometry & Trigonometry
Topic	3.9 Vector Properties
Difficulty	Medium

Level: IB Maths

Subject: IB Maths AA HL

Board: IB Maths

Topic: Vector Properties

Question 1

a)

Show that the vectors $\mathbf{a} = 2\mathbf{i} - 6\mathbf{j} + \mathbf{k}$ and $\mathbf{b} = -\mathbf{i} + 3\mathbf{j} - \mathbf{k}$ are not parallel.

[3 marks]

b)

Show that $|\mathbf{a} \cdot \mathbf{b}| < |\mathbf{a}| |\mathbf{b}|$

[3 marks]

c)

Show that $\mathbf{a} \cdot \mathbf{a} = |\mathbf{a}|^2$

[2 marks]

Question 2

Consider the two vectors $\mathbf{s} = 3\mathbf{i} + 4\mathbf{j} - \mathbf{k}$ and $\mathbf{t} = -2\mathbf{i} + 2\mathbf{j} - 3\mathbf{k}$.

(i)

Find the cross product of \mathbf{s} and \mathbf{t} .

(ii)

Hence, find the angle between \mathbf{s} and \mathbf{t} . Give your answer in radians.

[5 marks]

Question 3

The vectors \mathbf{a} and \mathbf{b} are defined by $\mathbf{a} = \begin{pmatrix} 1 \\ -3 \\ 1 \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} 5 \\ 2 \\ -2 \end{pmatrix}$.

By finding the scalar product of \mathbf{a} and \mathbf{b} , find the angle between them. Give your answer in degrees.

[4 marks]

Question 4

Let $\mathbf{v} = \begin{pmatrix} t \\ -3 \\ t+2 \end{pmatrix}$ and $\mathbf{w} = \begin{pmatrix} -6 \\ 7 \\ t \end{pmatrix}$.

a)

Given that \mathbf{v} and \mathbf{w} are perpendicular, find all possible values of t .

[4 marks]

b)

Show that the angle between \mathbf{v} and \mathbf{w} is acute for all $t > 7$.

[2 marks]

Question 5

Consider the vectors $\mathbf{a} = 3i - j + 4k$ and $\mathbf{b} = (2 + t)i - 2j + 2tk$.

By finding the vector product, determine the value of t , given that \mathbf{a} and \mathbf{b} are parallel.

[4 marks]

Question 6

Consider the vectors $\mathbf{a} = -2i - j + 3k$ and $\mathbf{b} = 3i + 5k$.

a)

Find a vector of length 7 that is parallel to \mathbf{a} .

[3 marks]

b)

Find the vector that is normal to both \mathbf{a} and \mathbf{b} .

[3 marks]

Question 7

a)

Given the vectors $\mathbf{r} = -i + 2j + k$, $\mathbf{s} = 5i + j - k$ and $\mathbf{t} = 2i + 2j + 4k$, show that

(i)

$$\mathbf{r} \cdot (\mathbf{s} + \mathbf{t}) = \mathbf{r} \cdot \mathbf{s} + \mathbf{r} \cdot \mathbf{t}$$

(ii)

$$\mathbf{r} \times (\mathbf{s} + \mathbf{t}) = \mathbf{r} \times \mathbf{s} + \mathbf{r} \times \mathbf{t}$$

[5 marks]

b)

Given any two non-zero vectors \mathbf{a} and \mathbf{b} , show that $\mathbf{a} \times \mathbf{b} = -\mathbf{b} \times \mathbf{a}$.

[4 marks]

Question 8

Consider the vectors $\mathbf{r} = \begin{pmatrix} 2 \\ 4 \\ -1 \end{pmatrix}$ and $\mathbf{t} = \begin{pmatrix} -3 \\ 5 \\ 3 \end{pmatrix}$.

a)

Show that $3\mathbf{r} \times \mathbf{t} = 3(\mathbf{r} \times \mathbf{t})$.

[3 marks]

b)

Find the area of a triangle which has vectors $3\mathbf{r}$ and \mathbf{t} as two of its sides.

[3 marks]

Question 9

On a calm day, a remote-controlled boat is being driven along a vector $\mathbf{u} = i + 3j$ from one side of a pond to the other.

The boat is retrieved and taken to the same starting point, to make the journey again but this time a steady wind causes the boat to travel in a direction represented by the vector $\mathbf{w} = 2i - j$.

a)

Calculate the angle, in degrees, between the direction of travel on its initial journey and the direction on its subsequent journey.

[3 marks]

During the first journey, the boat takes 6.3 seconds to travel the 7.56 m to the other side of the pond.

b)

Find the velocity vector of the boat.

[4 marks]

c)

Given that during the second journey the boat covers a distance of 5.1 m, find the distance between the end points for both journeys.

[4 marks]

Question 10

ABCD is a parallelogram with vertices $A(2, 3, 0)$, $B(3, 9, 4)$, $C(7, 4, 2)$ and $D(6, -2, -2)$.

a)

Find the vectors \overrightarrow{AB} and \overrightarrow{AD} .

[2 marks]

b)

Find the area of the parallelogram.

[3 marks]

c)

By finding the scalar product of \vec{BA} and \vec{BC} , determine if the angle \widehat{ABC} is acute or obtuse.

[4 marks]