Question 1

(a) Show that the vectors $\mathbf{a} = 2i - 6j + k$ and $\mathbf{b} = -i + 3j - k$ are not parallel. [3]

(b) Show that $|\mathbf{a} \cdot \mathbf{b}| < |\mathbf{a}||\mathbf{b}|$ [3]

(c) Show that $\mathbf{a} \cdot \mathbf{a} = |\mathbf{a}|^2$ [2]

a) For two vector to be parallel, then $ka = b$ or

$|a \cdot b| = |a||b|$ or $|a \times b| = 0$

$$k\begin{pmatrix} 2 \\ -6 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \\ 3 \\ -1 \end{pmatrix}$$

$2k = -1$, $-6k = 3$, $k = -1$

$\therefore k = -\frac{1}{2}$ $\therefore k = -\frac{1}{2}$

No consistent scale factor, so they are not parallel.

(a) Show that the vectors $\mathbf{a} = 2i - 6j + k$ and $\mathbf{b} = -i + 3j - k$ are not parallel. [3]

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(c) Show that $\mathbf{a} \cdot \mathbf{a} = |\mathbf{a}|^2$ [2]

b) scalar product

$v \cdot w = v_1 w_1 + v_2 w_2 + v_3 w_3$ (in formula booklet)

$|a \cdot b| = |(2)(-1) + (-6)(3) + (1)(-1)| = 21$

vector magnitude

$|v| = \sqrt{v_1^2 + v_2^2 + v_3^2}$ (in formula booklet)

$|a||b| = \sqrt{(2)^2 + (-6)^2 + (1)^2} \times \sqrt{(-1)^2 + (3)^2 + (-1)^2}$

$|a||b| = \sqrt{41}\sqrt{11} = \sqrt{451}$

$\therefore 21 < \sqrt{451}$, since $21^2 = 441$

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c) scalar product

$v \cdot w = v_1 w_1 + v_2 w_2 + v_3 w_3$ (in formula booklet)

vector magnitude

$|v| = \sqrt{v_1^2 + v_2^2 + v_3^2}$ (in formula booklet)

$a \cdot a = (2)(2) + (-6)(-6) + (1)(1) = 41$

$|a|^2 = \left(\sqrt{(2)^2 + (-6)^2 + (1)^2}\right)^2 = \left(\sqrt{41}\right)^2 = 41$

$\boxed{\therefore\ a \cdot a = |a|^2 = 41}$

Question 2

Consider the two vectors $\mathbf{s} = 3i + 4j - k$ and $\mathbf{t} = -2i + 2j - 3k$.

(i) Find the cross product of $\mathbf{s}$ and $\mathbf{t}$.

(ii) Hence, find the angle between $\mathbf{s}$ and $\mathbf{t}$. Give your answer in radians. [5]

cross/vector product (in formula booklet)

$$a \times b = \begin{pmatrix} a_2 b_3 - a_3 b_2 \\ a_3 b_1 - a_1 b_3 \\ a_1 b_2 - a_2 b_1 \end{pmatrix}$$

i) $$s \times t = \begin{pmatrix} (4)(-3) - (-1)(2) \\ (-1)(-2) - (-3)(3) \\ (3)(2) - (-2)(4) \end{pmatrix}$$

$$\boxed{s \times t = \begin{pmatrix} -10 \\ 11 \\ 14 \end{pmatrix}}$$

ii) $|s \times t| = \sqrt{(-10)^2 + (11)^2 + (14)^2} = \sqrt{417}$

$|s \times t| = |s||t| \sin\theta$ (in formula booklet)

$\sqrt{417} = \left|\sqrt{(3)^2 + (4)^2 + (-1)^2}\right|\left|\sqrt{(-2)^2 + (2)^2 + (-3)^2}\right| \sin\theta$

$\theta = \sin^{-1}\left(\frac{\sqrt{417}}{\sqrt{26}\sqrt{17}}\right) = 1.3306...$

$\boxed{\theta = 1.33 \text{ radians (3 s.f.)}}$

Question 3

The vectors **a** and **b** are defined by $\mathbf{a} = \begin{pmatrix} 1 \\ -3 \\ 1 \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} 5 \\ 2 \\ -2 \end{pmatrix}$.

By finding the scalar product of **a** and **b**, find the angle between them. Give your answer in degrees.

[4]

scalar product

$v \cdot w = v_1 w_1 + v_2 w_2 + v_3 w_3$ (in formula booklet)

$a \cdot b = (1)(5) + (-3)(2) + (1)(-2) = -3$

$a \cdot b = |a||b| \cos\theta$ (in formula booklet)

$-3 = \left(\sqrt{(1)^2 + (-3)^2 + (1)^2}\right)\left(\sqrt{(5)^2 + (2)^2 + (-2)^2}\right) \cos\theta$

$-3 = (\sqrt{11})(\sqrt{33}) \cos\theta$

$\theta = \cos^{-1}\left(-\frac{3}{\sqrt{11}\sqrt{33}}\right) = 99.059\ldots$

$\boxed{\theta = 99.1^\circ \text{ (3 s.f.)}}$

Question 4

Let $\mathbf{v} = \begin{pmatrix} t \\ -3 \\ t+2 \end{pmatrix}$ and $\mathbf{w} = \begin{pmatrix} -6 \\ 7 \\ t \end{pmatrix}$.

(a) Given that **v** and **w** are perpendicular, find all possible values of t.

[4]

(b) Show that the angle between **v** and **w** is acute for all $t > 7$.

[2]

a) Given v and w perpendicular, then $v \cdot w = 0$

scalar product

$v \cdot w = v_1 w_1 + v_2 w_2 + v_3 w_3$ (in formula booklet)

$v \cdot w = -6t - 21 + t^2 + 2t = 0$

$t^2 - 4t - 21 = 0$

$(t-7)(t+3) = 0$

$\boxed{\therefore t = 7 \text{ and } -3}$

Let $\mathbf{v} = \begin{pmatrix} t \\ -3 \\ t+2 \end{pmatrix}$ and $\mathbf{w} = \begin{pmatrix} -6 \\ 7 \\ t \end{pmatrix}$.

(a) Given that $\mathbf{v}$ and $\mathbf{w}$ are perpendicular, find all possible values of t.

[4]

(b) Show that the angle between $\mathbf{v}$ and $\mathbf{w}$ is acute for all $t > 7$.

[2]

b) For two vectors to have an acute angle between them, their scalar product > 0 or $\cos\theta > 0$.

$v.w = (t-7)(t+3) > 0 \quad \therefore t > 7$ and $t < -3$

$\boxed{\therefore \text{ angle between } v \text{ and } w \text{ is acute for all } t > 7.}$

Question 5

Consider the vectors $\mathbf{a} = 3i - j + 4k$ and $\mathbf{b} = (2+t)i - 2j + 2tk$.

By finding the vector product, determine the value of t, given that $\mathbf{a}$ and $\mathbf{b}$ are parallel.

[4]

Given a and b are parallel, then $|a \times b| = 0$

cross/vector product (in formula booklet)

$$a \times b = \begin{pmatrix} a_2 b_3 - a_3 b_2 \\ a_3 b_1 - a_1 b_3 \\ a_1 b_2 - a_2 b_1 \end{pmatrix}$$

vector magnitude

$|v| = \sqrt{v_1^2 + v_2^2 + v_3^2}$ (in formula booklet)

$$a \times b = \begin{pmatrix} (-1)(2t) - (-2)(4) \\ (4)(2+t) - (2t)(3) \\ (3)(-2) - (-1)(2+t) \end{pmatrix} = \begin{pmatrix} 8-2t \\ 8-2t \\ t-4 \end{pmatrix}$$

$$|a \times b| = \sqrt{(8-2t)^2 + (8-2t)^2 + (t-4)^2} = 0$$

$\therefore 9t^2 - 72t + 144 = 0$

$t^2 - 8t + 16 = 0$

$(t-4)^2 = 0$

$\boxed{t = 4}$

Question 6

Consider the vectors $\mathbf{a} = -2i - j + 3k$ and $\mathbf{b} = 3i + 5k$.

(a) Find a vector of length 7 that is parallel to $\mathbf{a}$. [3]

(b) Find the vector that is normal to both $\mathbf{a}$ and $\mathbf{b}$. [3]

a) unit vector for a 3D vector (not in formula booklet)

$$a = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \longrightarrow u = \frac{1}{\sqrt{a_1^2 + a_2^2 + a_3^2}} \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$$

$$7u = 7 \times \frac{1}{\sqrt{(-2)^2 + (-1)^2 + (3)^2}} (-2i - j + 3k)$$

$$7u = \frac{7}{\sqrt{14}} (-2i - j + 3k)$$

$$\boxed{7u = -\sqrt{14}\, i - \frac{\sqrt{14}}{2} j + \frac{3\sqrt{14}}{2} k}$$

Consider the vectors $\mathbf{a} = -2i - j + 3k$ and $\mathbf{b} = 3i + 5k$.

(a) Find a vector of length 7 that is parallel to $\mathbf{a}$. [3]

(b) Find the vector that is normal to both $\mathbf{a}$ and $\mathbf{b}$. [3]

b) The vector that is normal to a and b is equal to their vector product, $a \times b$.

cross/vector product (in formula booklet)

$$a \times b = \begin{pmatrix} a_2 b_3 - a_3 b_2 \\ a_3 b_1 - a_1 b_3 \\ a_1 b_2 - a_2 b_1 \end{pmatrix}$$

$$a \times b = \begin{pmatrix} (-1)(5) - (0)(3) \\ (3)(3) - (5)(-2) \\ (-2)(0) - (3)(-1) \end{pmatrix} = \begin{pmatrix} -5 \\ 19 \\ 3 \end{pmatrix}$$

$$\boxed{-5i + 19j + 3k}$$

Question 7

(a) Given the vectors $\mathbf{r} = -i + 2j + k$, $\mathbf{s} = 5i + j - k$ and $\mathbf{t} = 2i + 2j + 4k$, show that

(i) $\mathbf{r} \cdot (\mathbf{s} + \mathbf{t}) = \mathbf{r} \cdot \mathbf{s} + \mathbf{r} \cdot \mathbf{t}$

(ii) $\mathbf{r} \times (\mathbf{s} + \mathbf{t}) = \mathbf{r} \times \mathbf{s} + \mathbf{r} \times \mathbf{t}$

[5]

(b) Given any two non-zero vectors $\mathbf{a}$ and $\mathbf{b}$, show that $\mathbf{a} \times \mathbf{b} = -\mathbf{b} \times \mathbf{a}$.

[4]

a) i) $r \cdot (s + t) = r \cdot s + r \cdot t$

$$\begin{pmatrix}-1\\2\\1\end{pmatrix} \cdot \left(\begin{pmatrix}5\\1\\-1\end{pmatrix} + \begin{pmatrix}2\\2\\4\end{pmatrix}\right) = \begin{pmatrix}-1\\2\\1\end{pmatrix} \cdot \begin{pmatrix}5\\1\\-1\end{pmatrix} + \begin{pmatrix}-1\\2\\1\end{pmatrix} \cdot \begin{pmatrix}2\\2\\4\end{pmatrix}$$

$$\begin{pmatrix}-1\\2\\1\end{pmatrix} \cdot \begin{pmatrix}7\\3\\3\end{pmatrix} = \big((-1)(5) + (2)(1) + (1)(-1)\big) + \big((-1)(2) + (2)(2) + (1)(4)\big)$$

$$(-1)(7) + (2)(3) + (1)(3) = -4 + 6$$

$$\boxed{2 = 2}$$

ii) $r \times (s + t) = r \times s + r \times t$

$$\begin{pmatrix}-1\\2\\1\end{pmatrix} \times \left(\begin{pmatrix}5\\1\\-1\end{pmatrix} + \begin{pmatrix}2\\2\\4\end{pmatrix}\right) = \begin{pmatrix}-1\\2\\1\end{pmatrix} \times \begin{pmatrix}5\\1\\-1\end{pmatrix} + \begin{pmatrix}-1\\2\\1\end{pmatrix} \times \begin{pmatrix}2\\2\\4\end{pmatrix}$$

$$\begin{pmatrix}-1\\2\\1\end{pmatrix} \times \begin{pmatrix}7\\3\\3\end{pmatrix} = \begin{pmatrix}(2)(-1)-(1)(1)\\(1)(5)-(-1)(-1)\\(-1)(1)-(2)(5)\end{pmatrix} + \begin{pmatrix}(2)(4)-(1)(2)\\(1)(2)-(-1)(4)\\(-1)(2)-(2)(2)\end{pmatrix}$$

$$\begin{pmatrix}(2)(3)-(1)(3)\\(1)(7)-(-1)(3)\\(-1)(3)-(2)(7)\end{pmatrix} = \begin{pmatrix}-3\\4\\-11\end{pmatrix} + \begin{pmatrix}6\\6\\-6\end{pmatrix}$$

$$\boxed{\begin{pmatrix}3\\10\\-17\end{pmatrix} = \begin{pmatrix}3\\10\\-17\end{pmatrix}}$$

(a) Given the vectors $\mathbf{r} = -i + 2j + k$, $\mathbf{s} = 5i + j - k$ and $\mathbf{t} = 2i + 2j + 4k$, show that

(i) $\mathbf{r} \cdot (\mathbf{s} + \mathbf{t}) = \mathbf{r} \cdot \mathbf{s} + \mathbf{r} \cdot \mathbf{t}$

(ii) $\mathbf{r} \times (\mathbf{s} + \mathbf{t}) = \mathbf{r} \times \mathbf{s} + \mathbf{r} \times \mathbf{t}$

[5]

(b) Given any two non-zero vectors $\mathbf{a}$ and $\mathbf{b}$, show that $\mathbf{a} \times \mathbf{b} = -\mathbf{b} \times \mathbf{a}$.

[4]

b) Let $a = \begin{pmatrix}a_1\\a_2\\a_3\end{pmatrix}$ and $b = \begin{pmatrix}b_1\\b_2\\b_3\end{pmatrix}$

$$\begin{pmatrix}a_1\\a_2\\a_3\end{pmatrix} \times \begin{pmatrix}b_1\\b_2\\b_3\end{pmatrix} = -\begin{pmatrix}b_1\\b_2\\b_3\end{pmatrix} \times \begin{pmatrix}a_1\\a_2\\a_3\end{pmatrix}$$

$$\begin{pmatrix}a_2b_3 - a_3b_2\\a_3b_1 - a_1b_3\\a_1b_2 - a_2b_1\end{pmatrix} = -\begin{pmatrix}b_2a_3 - b_3a_2\\b_3a_1 - b_1a_3\\b_1a_2 - b_2a_1\end{pmatrix}$$

$$\boxed{\begin{pmatrix}a_2b_3 - a_3b_2\\a_3b_1 - a_1b_3\\a_1b_2 - a_2b_1\end{pmatrix} = \begin{pmatrix}b_3a_2 - b_2a_3\\b_1a_3 - b_3a_1\\b_2a_1 - b_1a_2\end{pmatrix}}$$

Question 8

Consider the vectors $\mathbf{r} = \begin{pmatrix} 2 \\ 4 \\ -1 \end{pmatrix}$ and $\mathbf{t} = \begin{pmatrix} -3 \\ 5 \\ 3 \end{pmatrix}$.

(a) Show that $3\mathbf{r} \times \mathbf{t} = 3(\mathbf{r} \times \mathbf{t})$ [3]

(b) Find the area of a triangle which has vectors $3\mathbf{r}$ and $\mathbf{t}$ as two of its sides. [3]

a) $3r \times t = 3(r \times t)$

$$3\begin{pmatrix} 2 \\ 4 \\ -1 \end{pmatrix} \times \begin{pmatrix} -3 \\ 5 \\ 3 \end{pmatrix} = 3\left(\begin{pmatrix} 2 \\ 4 \\ -1 \end{pmatrix} \times \begin{pmatrix} -3 \\ 5 \\ 3 \end{pmatrix}\right)$$

$$\begin{pmatrix} 6 \\ 12 \\ -3 \end{pmatrix} \times \begin{pmatrix} -3 \\ 5 \\ 3 \end{pmatrix} = 3\begin{pmatrix} (4)(3)-(-1)(5) \\ (-1)(-3)-(3)(2) \\ (2)(5)-(4)(-3) \end{pmatrix}$$

$$\begin{pmatrix} (12)(3)-(-3)(5) \\ (-3)(-3)-(3)(6) \\ (6)(5)-(12)(-3) \end{pmatrix} = 3\begin{pmatrix} 17 \\ -3 \\ 22 \end{pmatrix}$$

$$\boxed{\begin{pmatrix} 51 \\ -9 \\ 66 \end{pmatrix} = \begin{pmatrix} 51 \\ -9 \\ 66 \end{pmatrix}}$$

Consider the vectors $\mathbf{r} = \begin{pmatrix} 2 \\ 4 \\ -1 \end{pmatrix}$ and $\mathbf{t} = \begin{pmatrix} -3 \\ 5 \\ 3 \end{pmatrix}$.

(a) Show that $3\mathbf{r} \times \mathbf{t} = 3(\mathbf{r} \times \mathbf{t})$ [3]

(b) Find the area of a triangle which has vectors $3\mathbf{r}$ and $\mathbf{t}$ as two of its sides. [3]

b) $A = \frac{1}{2}|3r \times t|$

$$A = \frac{1}{2}\sqrt{(51)^2 + (-9)^2 + (66)^2} = \frac{3\sqrt{782}}{2} = 41.94\ldots$$

$$\boxed{A = 41.9 \text{ units}^2 \ (3 \text{ s.f.})}$$

Question 9

On a calm day, a remote-controlled boat is being driven along a vector $\mathbf{u} = i + 3j$ from one side of a pond to the other.

The boat is retrieved and taken to the same starting point, to make the journey again but this time a steady wind causes the boat to travel in a direction represented by the vector $\mathbf{w} = 2i - j$.

(a) Calculate the angle, in degrees, between the direction of travel on its initial journey and the direction on its subsequent journey.

[3]

During the first journey, the boat takes 6.3 seconds to travel the 7.56 m to the other side of the pond.

(b) Find the velocity vector of the boat.

[4]

(c) Given that during the second journey the boat covers a distance of 5.1 m, find the distance between the end points for both journeys.

[4]

a) Angle between two vectors

$$\cos\theta = \frac{v_1w_1 + v_2w_2 + v_3w_3}{|v||w|}$$ (in formula booklet)

$$\cos\theta = \frac{(1)(2) + (3)(-1)}{\left(\sqrt{(1)^2 + (3)^2}\right)\left(\sqrt{(2)^2 + (-1)^2}\right)}$$

$$\boxed{\theta = 98.13\ldots = 98.1^\circ \quad (3 \text{ s.f.})}$$

On a calm day, a remote-controlled boat is being driven along a vector $\mathbf{u} = i + 3j$ from one side of a pond to the other.

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During the first journey, the boat takes 6.3 seconds to travel the 7.56 m to the other side of the pond.

(b) Find the velocity vector of the boat.

[4]

(c) Given that during the second journey the boat covers a distance of 5.1 m, find the distance between the end points for both journeys.

[4]

b) $$\text{speed} = \frac{7.56}{6.3} = 1.2\text{ms}^{-1}$$

unit vector for a 3D vector (not in formula booklet)

$$a = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \rightarrow u = \frac{1}{\sqrt{a_1^2 + a_2^2 + a_3^2}} \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$$

$$\text{unit vector} = \frac{1}{\sqrt{(1)^2 + (3)^2}} \begin{pmatrix} 1 \\ 3 \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{10}} \\ \frac{3}{\sqrt{10}} \end{pmatrix} = \begin{pmatrix} \frac{\sqrt{10}}{10} \\ \frac{3\sqrt{10}}{10} \end{pmatrix}$$

velocity vector = speed × unit vector

$$\text{velocity vector} = 1.2 \begin{pmatrix} \frac{\sqrt{10}}{10} \\ \frac{3\sqrt{10}}{10} \end{pmatrix}$$

$$\boxed{\text{velocity vector} = \begin{pmatrix} \frac{3\sqrt{10}}{25} \\ \frac{9\sqrt{10}}{25} \end{pmatrix}}$$

On a calm day, a remote-controlled boat is being driven along a vector $\mathbf{u} = i + 3j$ from one side of a pond to the other.

The boat is retrieved and taken to the same starting point, to make the journey again but this time a steady wind causes the boat to travel in a direction represented by the vector $\mathbf{w} = 2i - j$.

(a) Calculate the angle, in degrees, between the direction of travel on its initial journey and the direction on its subsequent journey.

$\boxed{\theta = 98.13\ldots = 98.1^\circ \quad (3 \text{ s.f.})}$

[3]

During the first journey, the boat takes 6.3 seconds to travel the 7.56 m to the other side of the pond.

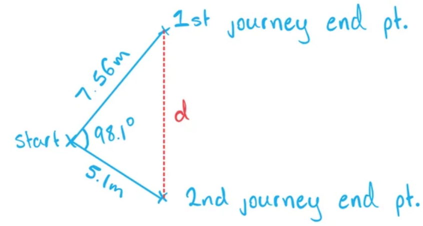
(b) Find the velocity vector of the boat.

[4]

(c) Given that during the second journey the boat covers a distance of 5.1 m, find the distance between the end points for both journeys.

[4]

c) Draw a diagram



Cosine rule (in formula booklet)

$c^2 = a^2 + b^2 - 2ab\cos C \; ; \quad \cos C = \dfrac{a^2 + b^2 - c^2}{2ab}$

$d^2 = (7.56)^2 + (5.1)^2 - 2(7.56)(5.1)\cos 98.13\ldots$

$\boxed{d = 9.698\ldots = 9.70\text{m} \quad (3 \text{ s.f.})}$

Question 10

ABCD is a parallelogram with vertices A(2, 3, 0), B(3, 9, 4), C(7, 4, 2) and D(6, −2, −2).

(a) Find the vectors $\overrightarrow{AB}$ and $\overrightarrow{AD}$.

[2]

(b) Find the area of the parallelogram.

[3]

(c) By finding the scalar product of $\overrightarrow{BA}$ and $\overrightarrow{BC}$, determine if the angle $A\hat{B}C$ is acute or obtuse.

[4]

a) $\overrightarrow{AB} = \begin{pmatrix} 3-2 \\ 9-3 \\ 4-0 \end{pmatrix} \longrightarrow \boxed{\overrightarrow{AB} = \begin{pmatrix} 1 \\ 6 \\ 4 \end{pmatrix}}$

$\overrightarrow{AD} = \begin{pmatrix} 6-2 \\ -2-3 \\ -2-0 \end{pmatrix} \longrightarrow \boxed{\overrightarrow{AD} = \begin{pmatrix} 4 \\ -5 \\ -2 \end{pmatrix}}$

ABCD is a parallelogram with vertices A(2, 3, 0), B(3, 9, 4), C(7, 4, 2) and D(6, −2, −2).

(a) Find the vectors $\overrightarrow{AB}$ and $\overrightarrow{AD}$. [2]

$\boxed{\overrightarrow{AB} = \begin{pmatrix} 1 \\ 6 \\ 4 \end{pmatrix}}$ $\boxed{\overrightarrow{AD} = \begin{pmatrix} 4 \\ -5 \\ -2 \end{pmatrix}}$

(b) Find the area of the parallelogram. [3]

(c) By finding the scalar product of $\overrightarrow{BA}$ and $\overrightarrow{BC}$, determine if the angle $A\hat{B}C$ is acute or obtuse. [4]

b) Parallelogram area (in formula booklet)

$A = |v \times w|$, where v and w are adjacent sides

$$|\overrightarrow{AB} \times \overrightarrow{AD}| = \left| \begin{pmatrix} (6)(-2) - (-5)(4) \\ (4)(4) - (-2)(1) \\ (1)(-5) - (4)(6) \end{pmatrix} \right|$$

$$|\overrightarrow{AB} \times \overrightarrow{AD}| = \left| \begin{pmatrix} 8 \\ 18 \\ -29 \end{pmatrix} \right| = \sqrt{(8)^2 + (18)^2 + (-29)^2} = 35.05\ldots$$

$$\boxed{A = 35.1 \text{ units}^2 \quad (3 \text{ s.f.})}$$

ABCD is a parallelogram with vertices A(2, 3, 0), B(3, 9, 4), C(7, 4, 2) and D(6, −2, −2).

(a) Find the vectors $\overrightarrow{AB}$ and $\overrightarrow{AD}$. [2]

(b) Find the area of the parallelogram. [3]

(c) By finding the scalar product of $\overrightarrow{BA}$ and $\overrightarrow{BC}$, determine if the angle $A\hat{B}C$ is acute or obtuse. [4]

c) $\overrightarrow{BA} = \begin{pmatrix} 2-3 \\ 3-9 \\ 0-4 \end{pmatrix} = \begin{pmatrix} -1 \\ -6 \\ -4 \end{pmatrix}$

$\overrightarrow{BC} = \begin{pmatrix} 7-3 \\ 4-9 \\ 2-4 \end{pmatrix} = \begin{pmatrix} 4 \\ -5 \\ -2 \end{pmatrix}$

scalar product

$v \cdot w = v_1 w_1 + v_2 w_2 + v_3 w_3$ (in formula booklet)

$\overrightarrow{BA} \cdot \overrightarrow{BC} = (-1)(4) + (-6)(-5) + (-4)(-2) = 34$

$$\boxed{\begin{array}{l} \overrightarrow{BA} \cdot \overrightarrow{BC} = 34, \text{ which is positive} \\ \therefore \cos\theta > 0 \quad \therefore \theta \text{ is acute} \end{array}}$$