

# IB Maths: AA HL

## Vector Properties

### Topic Questions

These practice questions can be used by students and teachers and is Suitable for IB Maths AA HL Topic Questions

Course	IB Maths
Section	3. Geometry & Trigonometry
Topic	3.7 Vector Properties
Difficulty	Medium

**Level: IB Maths**

**Subject: IB Maths AA HL**

**Board: IB Maths**

**Topic: Vector Properties**

## Question 1

a)

Show that the vectors  $a = 2i - 6j + k$  and  $b = -i + 3j - k$  are not parallel.

[3 marks]

b)

Show that  $|\mathbf{a} \cdot \mathbf{b}| < |\mathbf{a}| |\mathbf{b}|$

[3 marks]

c)

Show that  $\mathbf{a} \cdot \mathbf{a} = |\mathbf{a}|^2$

[2 marks]

## Question 2

Consider the two vectors  $\mathbf{s} = 3i + 4j - k$  and  $\mathbf{t} = -2i + 2j - 3k$ .

(i)

Find the cross product of  $\mathbf{s}$  and  $\mathbf{t}$ .

(ii)

Hence, find the angle between  $\mathbf{s}$  and  $\mathbf{t}$ . Give your answer in radians.

[5 marks]

## Question 3

The vectors  $\mathbf{a}$  and  $\mathbf{b}$  are defined by  $\mathbf{a} = \begin{pmatrix} 1 \\ -3 \\ 1 \end{pmatrix}$ ,  $\mathbf{b} = \begin{pmatrix} 5 \\ 2 \\ -2 \end{pmatrix}$ .

By finding the scalar product of  $\mathbf{a}$  and  $\mathbf{b}$ , find the angle between them. Give your answer in degrees.

[4 marks]

### Question 4

Let  $\mathbf{v} = \begin{pmatrix} t \\ -3 \\ t+2 \end{pmatrix}$  and  $\mathbf{w} = \begin{pmatrix} -6 \\ 7 \\ t \end{pmatrix}$ .

a)

Given that  $\mathbf{v}$  and  $\mathbf{w}$  are perpendicular, find all possible values of  $t$ .

[4 marks]

b)

Show that the angle between  $\mathbf{v}$  and  $\mathbf{w}$  is acute for all  $t > 7$ .

[2 marks]

### Question 5

Consider the vectors  $\mathbf{a} = 3i - j + 4k$  and  $\mathbf{b} = (2 + t)i - 2j + 2tk$ .

Given that  $\mathbf{a}$  and  $\mathbf{b}$  are parallel and hence the vector product is equal to zero, determine the value of  $t$ .

[1 mark]

### Question 6

Consider the vectors  $\mathbf{a} = -2i - j + 3k$  and  $\mathbf{b} = 3i + 5k$ .

a)

Find a vector of length 7 that is parallel to  $\mathbf{a}$ .

[3 marks]

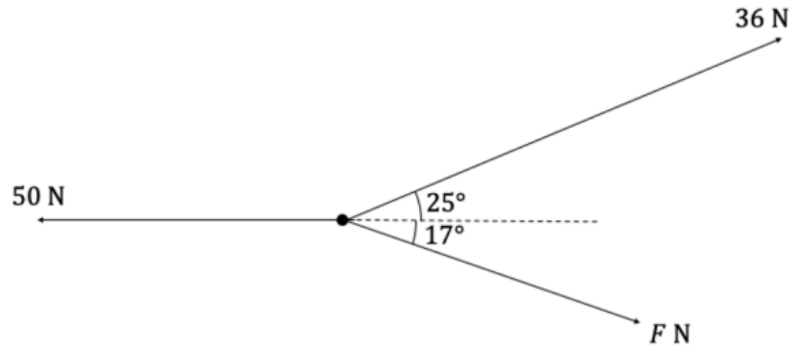
b)

Find the vector that is normal to both  $\mathbf{a}$  and  $\mathbf{b}$ .

[3 marks]

### Question 7

A particle is subjected to a force of 36 N acting at an angle of  $25^\circ$  above the horizontal and a second force  $F$  at an angle of  $17^\circ$  below the horizontal. There is also a resistive force of 50 N acting horizontally on the particle. This information can be seen in the diagram below.



(a) Given that the resultant horizontal force acting on the particle is 0 N, find the value of  $F$ .

[3 marks]

(b) Show that the vertical component of the resultant force is 9.9 N.

[2 marks]

### Question 8

Consider the vectors  $\mathbf{r} = \begin{pmatrix} 2 \\ 4 \\ -1 \end{pmatrix}$  and  $\mathbf{t} = \begin{pmatrix} -3 \\ 5 \\ 3 \end{pmatrix}$ .

a)  
Show that  $3\mathbf{r} \times \mathbf{t} = 3(\mathbf{r} \times \mathbf{t})$ .

[3 marks]

b)  
Find the area of a triangle which has vectors  $3\mathbf{r}$  and  $\mathbf{t}$  as two of its sides.

[3 marks]

## Question 9

On a calm day, a remote-controlled boat is being driven along a vector  $\mathbf{u} = i + 3j$  from one side of a pond to the other.

The boat is retrieved and taken to the same starting point, to make the journey again but this time a steady wind causes the boat to travel in a direction represented by the vector  $\mathbf{w} = 2i - j$ .

a)

Calculate the angle, in degrees, between the direction of travel on its initial journey and the direction on its subsequent journey.

[3 marks]

During the first journey, the boat takes 6.3 seconds to travel the 7.56 m to the other side of the pond.

b)

Find the velocity vector of the boat.

[4 marks]

c)

Given that during the second journey the boat covers a distance of 5.1 m, find the distance between the end points for both journeys.

[4 marks]

## Question 10

ABCD is a parallelogram with vertices  $A(2, 3, 0)$ ,  $B(3, 9, 4)$ ,  $C(7, 4, 2)$  and  $D(6, -2, -2)$ .

a)

Find the vectors  $\overrightarrow{AB}$  and  $\overrightarrow{AD}$ .

[2 marks]

b)

Find the area of the parallelogram.

[3 marks]

c)

By finding the scalar product of  $\vec{BA}$  and  $\vec{BC}$ , determine if the angle  $\widehat{ABC}$  is acute or obtuse.

[4 marks]

### Question 11

The velocity of a river can be described by the vector  $\mathbf{a} = 2i - 3j \text{ kmh}^{-1}$  and a swimmer moves through the river with velocity  $\mathbf{b} = -4i + j \text{ kmh}^{-1}$ .

(a) Find the speed at which the river is flowing and the swimmer is swimming.

[2 marks]

(b) Find the resultant vector of the swimmer and the river.

[2 marks]

(c) Find the bearing along which the swimmer actually moves.

[2 marks]

(d) The swimmer is attempting to complete a 5 km race for charity. Given that the velocity vectors for the river and the swimmer do not change, determine how long it will take the swimmer to complete the challenge.

[2 marks]