IB Maths: AA HL

Vector Equations of Lines

Topic Questions

These practice questions can be used by students and teachers and is Suitable for IB Maths AA HL Topic Questions

Course	IB Maths
Section	3. Geometry & Trigonometry
Topic	3.10 Vector Equations of Lines
Difficulty	Medium

Level: IB Maths

Subject: IB Maths AA HL

Board: IB Maths

Topic: Vector Equations of Lines

Question 1

The points A and B are given by $A(4, 2, -3)$ and $B(0, 5, 1)$.

a)
Find a vector equation of the line L that passes through points A and B.

[3 marks]

b)
Determine if the point $C(-1, 3, 2)$ does not lie on the line L.

[3 marks]

Question 2

Find the Cartesian equations of a line that is parallel to the vector $\boldsymbol{a} = 3\boldsymbol{i} - 4\boldsymbol{j} + \boldsymbol{k}$ and passes through the point $X(3, -2, 0)$.

[5 marks]

Question 3

Find the equation of the line that is normal to the vector $4\boldsymbol{i} + 5\boldsymbol{j}$ and passes through the point $P(7, -1)$, leaving your answer in the form $ax + by + c = 0$, where a, b and $c \in \mathbb{Z}$.

[6 marks]

Question 4

Consider the two lines l_1 and l_2 defined by the equations:

$$l_1 : \boldsymbol{a} = \begin{pmatrix} 4 \\ 1 \\ 6 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -3 \\ -5 \end{pmatrix}$$

$$l_2 : \boldsymbol{b} = \begin{pmatrix} 5 \\ -11 \\ 10 \end{pmatrix} + \mu \begin{pmatrix} -1 \\ 6 \\ 2 \end{pmatrix}$$

a)
Find the scalar product of the direction vectors.

[2 marks]

b)
Hence, find the angle, in radians, between the l_1 and l_2.

[4 marks]

Question 5

Consider the lines l_1 and l_2 defined by:

$$l_1:\begin{cases} x = 3 - \mu \\ y = -2 + 5\mu \\ z = 4 + 2\mu \end{cases}$$

$$l_2: \mathbf{r} = \begin{pmatrix} 3 \\ -1 \\ 0 \end{pmatrix} + \lambda\begin{pmatrix} 4 \\ 2 \\ 2 \end{pmatrix}.$$

a)
Show that the lines are not parallel.

[2 marks]

b)
Hence, show that the lines l_1 and l_2 are skew.

[5 marks]

Question 6

Consider the lines l_1 and l_2 defined by the equations $\boldsymbol{r}_1 = \begin{pmatrix} t \\ -2 \\ 5 \end{pmatrix} + \alpha\begin{pmatrix} -5 \\ 2 \\ 1 \end{pmatrix}$ and $\boldsymbol{r}_2 = \begin{pmatrix} -3 \\ 6 \\ 9 \end{pmatrix} + \beta\begin{pmatrix} 15 \\ 3k \\ -3 \end{pmatrix}$.

a)
Given that l_1 and l_2 are coincident, find the value of k.

[2 marks]

b)
Find the value of t.

[4 marks]

Question 7

Two ships A and B are travelling so that their position relative to a fixed point O at time t, in hours, can be defined by the position vectors $\boldsymbol{r}_{\mathrm{A}}=(2-t)\boldsymbol{i}+(4+3t)\boldsymbol{j}$ and $\boldsymbol{r}_{\mathrm{B}}=(t-8)\boldsymbol{i}+(29-2t)\boldsymbol{j}$.

The unit vectors $\boldsymbol{i}$ and $\boldsymbol{j}$ are a displacement of 1 km due East and North of O respectively.

a)
Find the coordinates of the initial position of the two ships.

[2 marks]

b)
Show that the two ships will collide and find the time at which this will occur.

[3 marks]

c)
Find the coordinates of the point of collision.

[2 marks]

Question 8

The lines l_1 and l_2 can be defined by:

$$l_1: \boldsymbol{r}=\begin{pmatrix}2\\-5\\1\end{pmatrix}+\alpha\begin{pmatrix}3\\2\\k\end{pmatrix}$$

$$l_2: \boldsymbol{s}=\begin{pmatrix}-3\\-4\\2\end{pmatrix}+\beta\begin{pmatrix}-11\\-3\\5\end{pmatrix}$$

a)
Write down the parametric equations for l_1.

[2 marks]

b)
Given that l_1 and l_2 intersect at point T,

(i)
find the value of k.

(ii)
determine the coordinates of the point of intersection, T.

[7 marks]

Question 9

Consider the triangle ABC. The points A, B and C have coordinates $(4, 0, -3)$, $(2, -2, -1)$ and $(7, 1, 5)$ respectively.

M is the midpoint of [AB].

a)
Find the coordinates of the midpoint M.

[2 marks]

b)
Hence, find a vector equation of the line that passes through points C and M.

[2 marks]

The point P is the midpoint of [BC]. The line passing through points A and P can be defined by

$$\boldsymbol{a} = \begin{pmatrix} 4 \\ 0 \\ -3 \end{pmatrix} + \mu \begin{pmatrix} \frac{1}{2} \\ -\frac{1}{2} \\ 5 \end{pmatrix}.$$

c) Show that the line AP intersects CM at the point $\left(\frac{13}{3}, -\frac{1}{3}, \frac{1}{3}\right)$.

[5 marks]

Question 10

A car, moving at constant speed, takes 4 minutes to drive in a straight line from point $\mathrm{A}(-3, 5)$ to point $\mathrm{B}(7, 11)$.

At time t, in minutes, the position vector of the car relative to the origin can be given in the form $\boldsymbol{p} = a + tb$.

a)
Find the vectors a and b.

[3 marks]

A cat has decided to take a nap at point $\mathrm{X}(4, 9)$.

b)
Show that the cat does not lie on the route along which the car drives.

[3 marks]

c)
Find the shortest distance between the car and the cat during the movement of the car.

[6 marks]