Question 1

The points A and B are given by A(4, 2, −3) and B(0, 5, 1).

(a) Find a vector equation of the line L that passes through points A and B. [3]

(b) Determine if the point C(−1, 3, 2) does not lie on the line L. [3]

a) Find $\overrightarrow{AB}$

$$\overrightarrow{AB} = \begin{pmatrix} 0-4 \\ 5-2 \\ 1-(-3) \end{pmatrix} = \begin{pmatrix} -4 \\ 3 \\ 4 \end{pmatrix}$$

$\therefore$ Equation of L is

$$\boxed{r = \begin{pmatrix} 4 \\ 2 \\ -3 \end{pmatrix} + \lambda \begin{pmatrix} -4 \\ 3 \\ 4 \end{pmatrix} \text{ or } r = \begin{pmatrix} 0 \\ 5 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} -4 \\ 3 \\ 4 \end{pmatrix}}$$

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b) $$\begin{pmatrix} 4 \\ 2 \\ -3 \end{pmatrix} + \lambda \begin{pmatrix} -4 \\ 3 \\ 4 \end{pmatrix} = \begin{pmatrix} -1 \\ 3 \\ 2 \end{pmatrix}$$

λ must be the same for each line

$4 - 4\lambda = -1 \quad \therefore \lambda = \frac{5}{4}$

$2 + 3\lambda = 3 \quad \therefore \lambda = \frac{1}{3}$

$-3 + 4\lambda = 2 \quad \therefore \lambda = \frac{5}{4}$

$\boxed{\therefore C \text{ does not lie on } L.}$

Question 2

Find the Cartesian equations of a line that is parallel to the vector $\boldsymbol{a} = 3\boldsymbol{i} - 4\boldsymbol{j} + \boldsymbol{k}$ and passes through the point $X(3, -2, 0)$.

[5]

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ -2 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ -4 \\ 1 \end{pmatrix}$$

$\therefore x = 3 + 3\lambda$
$y = -2 - 4\lambda$
$z = \lambda$

Cartesian form (in formula booklet)

$$\frac{x - x_0}{l} = \frac{y - y_0}{m} = \frac{z - z_0}{n}$$

$$\frac{x-3}{3} = -\frac{y+2}{4} = \frac{z-0}{1}$$

$$\boxed{\frac{x}{3} - 1 = -\frac{y}{4} - \frac{1}{2} = z}$$

Question 3

Find the equation of the line that is normal to the vector $4\boldsymbol{i} + 5\boldsymbol{j}$ and passes through the point $P(7, -1)$, leaving your answer in the form $ax + by + c = 0$, where a, b and $c \in \mathbb{Z}$.

[6]

Vectors that are normal to each other have a scalar product of 0.

scalar product

$v \cdot w = v_1 w_1 + v_2 w_2 + v_3 w_3$ (in formula booklet)

$\therefore$ normal vector: $5i - 4j$ or $-5i + 4j$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 7 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 5 \\ -4 \end{pmatrix}$$

$\therefore x = 7 + 5\lambda$
$y = -1 - 4\lambda$

$$\therefore \frac{x-7}{5} = \frac{-1-y}{4}$$

cross multiply

$$4(x-7) = 5(-1-y)$$

$$4x - 28 = -5 - 5y$$

$$\boxed{4x + 5y - 23 = 0}$$

Question 4

Consider the two lines l_1 and l_2 defined by the equations:

$$l_1: \boldsymbol{a} = \begin{pmatrix} 4 \\ 1 \\ 6 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -3 \\ -5 \end{pmatrix}$$

$$l_2: \boldsymbol{b} = \begin{pmatrix} 5 \\ -11 \\ 10 \end{pmatrix} + \mu \begin{pmatrix} -1 \\ 6 \\ 2 \end{pmatrix}.$$

(a) Find the scalar product of the direction vectors.

[2]

(b) Hence, find the angle, in radians, between the l_1 and l_2.

[4]

a) scalar product

$v \cdot w = v_1w_1 + v_2w_2 + v_3w_3$ (in formula booklet)

scalar product $= (1)(-1) + (-3)(6) + (-5)(2)$

$\boxed{\text{scalar product} = -29}$

Consider the two lines l_1 and l_2 defined by the equations:

$$l_1: \boldsymbol{a} = \begin{pmatrix} 4 \\ 1 \\ 6 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -3 \\ -5 \end{pmatrix}$$

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(a) Find the scalar product of the direction vectors.

$\boxed{\text{scalar product} = -29}$

[2]

(b) Hence, find the angle, in radians, between the l_1 and l_2.

[4]

b) $v \cdot w = |v||w| \cos\theta$ (in formula booklet)

$|-29| = \sqrt{(1)^2 + (-3)^2 + (-5)^2}\sqrt{(-1)^2 + (6)^2 + (2)^2} \cos\theta$

$\therefore \theta = \cos^{-1}\left(\frac{29}{\sqrt{35}\sqrt{41}}\right) = 0.6989...$

$\boxed{\theta = 0.699 \text{ radians}}$

Note: There is always an acute and an obtuse angle between two lines. In this question, we want the acute angle, that is why we use the absolute value of the scalar product.

Question 5

Consider the lines l_1 and l_2 defined by:

$$l_1: \begin{cases} x = 3 - \mu \\ y = -2 + 5\mu \\ z = 4 + 2\mu \end{cases}$$

$$l_2: \boldsymbol{r} = \begin{pmatrix} 3 \\ -1 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ 2 \\ 2 \end{pmatrix}.$$

(a) Show that the lines are not parallel.

[2]

(b) Hence, show that the lines l_1 and l_2 are skew.

[5]

a) Use the direction vectors to show they are not parallel.

$$l_1: r = \begin{pmatrix} 3 \\ -2 \\ 4 \end{pmatrix} + \mu \begin{pmatrix} -1 \\ 5 \\ 2 \end{pmatrix}$$

Direction vectors are not the same or colinear.

$\therefore$ they are not parallel.

Consider the lines l_1 and l_2 defined by:

$$l_1: \begin{cases} x = 3 - \mu \\ y = -2 + 5\mu \\ z = 4 + 2\mu \end{cases}$$

$$l_2: \boldsymbol{r} = \begin{pmatrix} 3 \\ -1 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ 2 \\ 2 \end{pmatrix}.$$

(a) Show that the lines are not parallel.

[2]

(b) Hence, show that the lines l_1 and l_2 are skew.

[5]

b) To show the lines are skew, we must show they don't intersect.

$$l_1: \begin{cases} x = 3 - \mu \\ y = -2 + 5\mu \\ z = 4 + 2\mu \end{cases} \qquad l_2: \begin{cases} x = 3 + 4\lambda \\ y = -1 + 2\lambda \\ z = 2\lambda \end{cases}$$

$$\therefore \begin{aligned} x &= 3 - \mu = 3 + 4\lambda \\ y &= -2 + 5\mu = -1 + 2\lambda \\ z &= 4 + 2\mu = 2\lambda \longrightarrow \lambda = 2 + \mu \end{aligned}$$

sub $\lambda = 2 + \mu$ into x and y lines.

$$3 - \mu = 3 + 4(2 + \mu) = 11 + 4\mu \qquad \therefore \mu = -\frac{8}{5}$$

$$-2 + 5\mu = -1 + 2(2 + \mu) = 3 + 2\mu \qquad \therefore \mu = \frac{5}{3}$$

inconsistent values for μ, so the lines do not intersect.

$\therefore$ the lines are skew.

Question 6

Consider the lines l_1 and l_2 defined by the equations $\boldsymbol{r_1} = \begin{pmatrix} t \\ -2 \\ 5 \end{pmatrix} + \alpha \begin{pmatrix} -5 \\ 2 \\ 1 \end{pmatrix}$ and $\boldsymbol{r_2} = \begin{pmatrix} -3 \\ 6 \\ 9 \end{pmatrix} + \beta \begin{pmatrix} 15 \\ 3k \\ -3 \end{pmatrix}$.

(a) Given that l_1 and l_2 are coincident, find the value of k.

[2]

(b) Find the value of t.

[4]

a) Given they are coincident, the direction vectors must be equal or have a constant scale factor.

$$q\begin{pmatrix} -5 \\ 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 15 \\ 3k \\ -3 \end{pmatrix} \longrightarrow \begin{array}{l} -5q = 15 \\ 2q = 3k \\ q = -3 \end{array}$$

$$\boxed{\therefore k = -2}$$

Consider the lines l_1 and l_2 defined by the equations $\boldsymbol{r_1} = \begin{pmatrix} t \\ -2 \\ 5 \end{pmatrix} + \alpha \begin{pmatrix} -5 \\ 2 \\ 1 \end{pmatrix}$ and $\boldsymbol{r_2} = \begin{pmatrix} -3 \\ 6 \\ 9 \end{pmatrix} + \beta \begin{pmatrix} 15 \\ 3k \\ -3 \end{pmatrix}$.

(a) Given that l_1 and l_2 are coincident, find the value of k.

$$\boxed{\therefore k = -2}$$

[2]

(b) Find the value of t.

[4]

b) $x = t - 5\alpha = -3 + 15\beta$

$y = -2 + 2\alpha = 6 - 6\beta \longrightarrow \alpha = 4 - 3\beta$

$z = 5 + \alpha = 9 - 3\beta \longrightarrow \alpha = 4 - 3\beta$

All variables should generate the same equation.

$$\alpha = \frac{-3 + 15\beta - t}{-5} = 4 - 3\beta$$

$$\therefore \frac{-3 - t}{-5} = 4 \longrightarrow -3 - t = -20$$

$$\boxed{t = 17}$$

Question 7

Two ships A and B are travelling so that their position relative to a fixed point O at time t, in hours, can be defined by the position vectors $\boldsymbol{r_A} = (2 - t)\boldsymbol{i} + (4 + 3t)\boldsymbol{j}$ and $\boldsymbol{r_B} = (t - 8)\boldsymbol{i} + (29 - 2t)\boldsymbol{j}$.

The unit vectors i and j are a displacement of 1 km due East and North of O respectively.

(a) Find the coordinates of the initial position of the two ships.

[2]

(b) Show that the two ships will collide and find the time at which this will occur.

[3]

(c) Find the coordinates of the point of collision.

[2]

a) The initial position is when $t = 0$.

$r_A = (2 - (0))i + (4 + 3(0))j = 2i + 4j$

$r_B = ((0) - 8)i + (29 - 2(0))j = -8i + 29j$

$\boxed{A(2,4) \text{ and } B(-8, 29)}$

Two ships A and B are travelling so that their position relative to a fixed point O at time t, in hours, can be defined by the position vectors $\boldsymbol{r_A} = (2 - t)\boldsymbol{i} + (4 + 3t)\boldsymbol{j}$ and $\boldsymbol{r_B} = (t - 8)\boldsymbol{i} + (29 - 2t)\boldsymbol{j}$.

The unit vectors i and j are a displacement of 1 km due East and North of O respectively.

(a) Find the coordinates of the initial position of the two ships.

$\boxed{A(2,4) \text{ and } B(-8, 29)}$

[2]

(b) Show that the two ships will collide and find the time at which this will occur.

[3]

(c) Find the coordinates of the point of collision.

[2]

b) $r_A = \begin{pmatrix} 2 \\ 4 \end{pmatrix} + t\begin{pmatrix} -1 \\ 3 \end{pmatrix}$ $\quad r_B = \begin{pmatrix} -8 \\ 29 \end{pmatrix} + t\begin{pmatrix} 1 \\ -2 \end{pmatrix}$

$x = 2 - t = -8 + t \longrightarrow 10 = 2t$

$y = 4 + 3t = 29 - 2t \longrightarrow 25 = 5t$

$\boxed{\therefore \text{ the ships collide when } t = 5 \text{ hours}}$

Two ships A and B are travelling so that their position relative to a fixed point O at time t, in hours, can be defined by the position vectors $\boldsymbol{r_A} = (2-t)\boldsymbol{i} + (4+3t)\boldsymbol{j}$ and $\boldsymbol{r_B} = (t-8)\boldsymbol{i} + (29-2t)\boldsymbol{j}$.

The unit vectors i and j are a displacement of 1 km due East and North of O respectively.

(a) Find the coordinates of the initial position of the two ships.

[2]

(b) Show that the two ships will collide and find the time at which this will occur.

$\boxed{\therefore \text{ the ships collide when } t = 5 \text{ hours}}$

[3]

(c) Find the coordinates of the point of collision.

[2]

c) sub in $t=5$ into r_A or r_B

$r_A = (2-(5))i + (4+3(5))j = -3i + 19j$

$\boxed{\therefore \text{ collision pt is } (-3, 19)}$

Question 8

The lines l_1 and l_2 can be defined by:

$$l_1: \boldsymbol{r} = \begin{pmatrix} 2 \\ -5 \\ 1 \end{pmatrix} + \alpha \begin{pmatrix} 3 \\ 2 \\ k \end{pmatrix}$$

$$l_2: \boldsymbol{s} = \begin{pmatrix} -3 \\ -4 \\ 2 \end{pmatrix} + \beta \begin{pmatrix} -11 \\ -3 \\ 5 \end{pmatrix}$$

(a) Write down the parametric equations for l_1.

[2]

(b) Given that l_1 and l_2 intersect at point T,

(i) find the value of k.

(ii) determine the coordinates of the point of intersection, T.

[7]

a) Parametric form of a line (in formula booklet)

$$\begin{cases} x = x_0 + \lambda l \\ y = y_0 + \lambda m \\ z = z_0 + \lambda n \end{cases}$$

$$\boxed{l_1: \begin{cases} x = 2 + 3\alpha \\ y = -5 + 2\alpha \\ z = 1 + k\alpha \end{cases}}$$

The lines l_1 and l_2 can be defined by:

$$l_1 : \mathbf{r} = \begin{pmatrix} 2 \\ -5 \\ 1 \end{pmatrix} + \alpha \begin{pmatrix} 3 \\ 2 \\ k \end{pmatrix}$$

$$l_2 : \mathbf{s} = \begin{pmatrix} -3 \\ -4 \\ 2 \end{pmatrix} + \beta \begin{pmatrix} -11 \\ -3 \\ 5 \end{pmatrix}$$

(a) Write down the parametric equations for l_1.

$$l_1 : \begin{cases} x = 2 + 3\alpha \\ y = -5 + 2\alpha \\ z = 1 + k\alpha \end{cases}$$

[2]

(b) Given that l_1 and l_2 intersect at point T,

(i) find the value of k.

(ii) determine the coordinates of the point of intersection, T.

[7]

b) $l_1 : \begin{cases} x = 2 + 3\alpha \\ y = -5 + 2\alpha \\ z = 1 + k\alpha \end{cases}$ $\quad l_2 : \begin{cases} x = -3 - 11\beta \\ y = -4 - 3\beta \\ z = 2 + 5\beta \end{cases}$

Equate the x, y and z lines

$x = 2 + 3\alpha = -3 - 11\beta$

$y = -5 + 2\alpha = -4 - 3\beta$

$z = 1 + k\alpha = 2 + 5\beta$

$2 + 3\alpha = -3 - 11\beta$ $\quad -5 + 2\alpha = -4 - 3\beta$

$\alpha = \dfrac{-5 - 11\beta}{3}$ $\quad \alpha = \dfrac{1 - 3\beta}{2}$

$\dfrac{-5 - 11\beta}{3} = \dfrac{1 - 3\beta}{2}$

cross multiply

$2(-5 - 11\beta) = 3(1 - 3\beta)$

$-10 - 22\beta = 3 - 9\beta \longrightarrow -13\beta = 13 \quad \therefore \beta = -1$

$\alpha = \dfrac{1 - 3(-1)}{2} = 2$

sub $\alpha = 2$ and $\beta = -1$ into the z line.

$z = 1 + 2k = 2 - 5 = -3$

i) $\boxed{\therefore k = -2}$

sub $\alpha = 2$, $\beta = -1$ and $k = -2$ into the x, y, z equations

$x = 2 + 3(2) = -3 - 11(-1) = 8$

$y = -5 + 2(2) = -4 - 3(-1) = -1$

$z = 1 + (-2)(2) = 2 + 5(-1) = -3$

ii) $\boxed{T(8, -1, -3)}$

Question 9

Consider the triangle ABC. The points A, B and C have coordinates $(4, 0, -3)$, $(2, -2, -1)$ and $(7, 1, 5)$ respectively.

M is the midpoint of [AB].

(a) Find the coordinates of the midpoint M.

[2]

(b) Hence, find a vector equation of the line that passes through points C and M.

[2]

The point P is the midpoint of [BC]. The line passing through points A and P can be defined by $\boldsymbol{a} = \begin{pmatrix} 4 \\ 0 \\ -3 \end{pmatrix} + \mu \begin{pmatrix} \frac{1}{2} \\ -\frac{1}{2} \\ 5 \end{pmatrix}$.

(c) Show that the line AP intersects CM at the point $\left(\frac{13}{3}, -\frac{1}{3}, \frac{1}{3}\right)$.

[5]

a) Midpoint, M (in formula booklet)

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2}\right)$$

$$M = \left(\frac{4+2}{2}, \frac{0+(-2)}{2}, \frac{(-3)+(-1)}{2}\right)$$

$$\boxed{M = (3, -1, -2)}$$

Consider the triangle ABC. The points A, B and C have coordinates $(4, 0, -3)$, $(2, -2, -1)$ and $(7, 1, 5)$ respectively.

M is the midpoint of [AB].

(a) Find the coordinates of the midpoint M.

$\boxed{M = (3, -1, -2)}$

[2]

(b) Hence, find a vector equation of the line that passes through points C and M.

[2]

The point P is the midpoint of [BC]. The line passing through points A and P can be defined by $\boldsymbol{a} = \begin{pmatrix} 4 \\ 0 \\ -3 \end{pmatrix} + \mu \begin{pmatrix} \frac{1}{2} \\ -\frac{1}{2} \\ 5 \end{pmatrix}$.

(c) Show that the line AP intersects CM at the point $\left(\frac{13}{3}, -\frac{1}{3}, \frac{1}{3}\right)$.

[5]

b) Find $\overrightarrow{CM}$ or $\overrightarrow{MC}$

$$\overrightarrow{CM} = \begin{pmatrix} 3-7 \\ -1-1 \\ -2-5 \end{pmatrix} = \begin{pmatrix} -4 \\ -2 \\ -7 \end{pmatrix} \quad \text{or} \quad \overrightarrow{MC} = \begin{pmatrix} 7-3 \\ 1-(-1) \\ 5-(-2) \end{pmatrix} = \begin{pmatrix} 4 \\ 2 \\ 7 \end{pmatrix}$$

There are 4 possible solutions.

$$\boxed{\begin{gathered} r = \begin{pmatrix} 3 \\ -1 \\ -2 \end{pmatrix} + \lambda \begin{pmatrix} -4 \\ -2 \\ -7 \end{pmatrix} \text{ or } r = \begin{pmatrix} 3 \\ -1 \\ -2 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ 2 \\ 7 \end{pmatrix} \text{ or} \\ r = \begin{pmatrix} 7 \\ 1 \\ 5 \end{pmatrix} + \lambda \begin{pmatrix} -4 \\ -2 \\ -7 \end{pmatrix} \text{ or } r = \begin{pmatrix} 7 \\ 1 \\ 5 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ 2 \\ 7 \end{pmatrix} \end{gathered}}$$

Consider the triangle ABC. The points A, B and C have coordinates $(4, 0, -3)$, $(2, -2, -1)$ and $(7, 1, 5)$ respectively.

M is the midpoint of [AB].

(a) Find the coordinates of the midpoint M.

[2]

(b) Hence, find a vector equation of the line that passes through points C and M.

$\boxed{r = \begin{pmatrix} 3 \\ -1 \\ -2 \end{pmatrix} + \lambda \begin{pmatrix} -4 \\ -2 \\ -7 \end{pmatrix}}$

[2]

The point P is the midpoint of [BC]. The line passing through points A and P can be defined by $\boldsymbol{a} = \begin{pmatrix} 4 \\ 0 \\ -3 \end{pmatrix} + \mu \begin{pmatrix} \frac{1}{2} \\ -\frac{1}{2} \\ 5 \end{pmatrix}$.

(c) Show that the line AP intersects CM at the point $\left(\frac{13}{3}, -\frac{1}{3}, \frac{1}{3}\right)$.

[5]

c) $\begin{cases} x = 3 - 4\lambda = 4 + \frac{1}{2}\mu \\ y = -1 - 2\lambda = -\frac{1}{2}\mu \\ z = -2 - 7\lambda = -3 + 5\mu \end{cases}$

Rearrange x, y or z for either λ or μ.

Rearranging x for μ.

$x = 3 - 4\lambda = 4 + \frac{1}{2}\mu \longrightarrow \mu = -2 - 8\lambda$

sub $\mu = -2 - 8\lambda$ into y or z.

$y = -1 - 2\lambda = -\frac{1}{2}(-2 - 8\lambda) = 1 + 4\lambda$

$6\lambda = -2 \longrightarrow \lambda = -\frac{1}{3}$

sub in $\lambda = -\frac{1}{3}$ to find x, y and z

$x = 3 - 4\left(-\frac{1}{3}\right) = \frac{13}{3}$

$y = -1 - 2\left(-\frac{1}{3}\right) = -\frac{1}{3}$

$z = -2 - 7\left(-\frac{1}{3}\right) = \frac{1}{3}$

$\boxed{\therefore \text{ intersection at } \left(\frac{13}{3}, -\frac{1}{3}, \frac{1}{3}\right)}$

Question 10

A car, moving at constant speed, takes 4 minutes to drive in a straight line from point A$(-3, 5)$ to point B$(7, 11)$.

At time t, in minutes, the position vector of the car relative to the origin can be given in the form $\boldsymbol{p} = \boldsymbol{a} + t\boldsymbol{b}$.

(a) Find the vectors $\boldsymbol{a}$ and $\boldsymbol{b}$.

[3]

A cat has decided to take a nap at point X$(4, 9)$.

(b) Show that the cat does not lie on the route along which the car drives.

[3]

(c) Find the shortest distance between the car and the cat during the movement of the car.

[6]

a) vector a represents the initial position and vector b represents the direction vector.

$b = \frac{1}{4}\begin{pmatrix} 7 - (-3) \\ 11 - 5 \end{pmatrix}$

4 minutes to travel between A and B.

$\boxed{\therefore\ a = \begin{pmatrix} -3 \\ 5 \end{pmatrix}, \quad b = \begin{pmatrix} \frac{5}{2} \\ \frac{3}{2} \end{pmatrix}}$

A car, moving at constant speed, takes 4 minutes to drive in a straight line from point A$(-3, 5)$ to point B$(7, 11)$.

At time t, in minutes, the position vector of the car relative to the origin can be given in the form $\boldsymbol{p} = a + tb$.

(a) Find the vectors a and b.

$\boxed{\therefore\ a = \begin{pmatrix} -3 \\ 5 \end{pmatrix}, \quad b = \begin{pmatrix} \frac{5}{2} \\ \frac{3}{2} \end{pmatrix}}$

[3]

A cat has decided to take a nap at point X$(4, 9)$.

(b) Show that the cat does not lie on the route along which the car drives.

[3]

(c) Find the shortest distance between the car and the cat during the movement of the car.

[6]

b) show that t is inconsistent at X.

$$\begin{pmatrix} 4 \\ 9 \end{pmatrix} = \begin{pmatrix} -3 \\ 5 \end{pmatrix} + t\begin{pmatrix} \frac{5}{2} \\ \frac{3}{2} \end{pmatrix}$$

$4 = -3 + \frac{5}{2}t \qquad \therefore t = \frac{14}{5}$

$9 = 5 + \frac{3}{2}t \qquad \therefore t = \frac{8}{3}$

t is inconsistent, so $(4,9)$ does not lie on the car's route.

A car, moving at constant speed, takes 4 minutes to drive in a straight line from point A$(-3, 5)$ to point B$(7, 11)$.

At time t, in minutes, the position vector of the car relative to the origin can be given in the form $\boldsymbol{p} = a + tb$.

(a) Find the vectors a and b.

[3]

A cat has decided to take a nap at point X$(4, 9)$.

(b) Show that the cat does not lie on the route along which the car drives.

[3]

(c) Find the shortest distance between the car and the cat during the movement of the car.

[6]

c) shortest distance is the perpendicular distance from X to the line

direction vector $= \begin{pmatrix} \frac{5}{2} \\ \frac{3}{2} \end{pmatrix}$

$\therefore$ perpendicular vector $= \begin{pmatrix} -\frac{3}{2} \\ \frac{5}{2} \end{pmatrix}$

$\therefore r = \begin{pmatrix} 4 \\ 9 \end{pmatrix} + \lambda\begin{pmatrix} -\frac{3}{2} \\ \frac{5}{2} \end{pmatrix}$ (equation from cat to route)

write both equations in parametric form and rearrange x or y for t or λ

$$\begin{cases} x = -3 + \frac{5}{2}t = 4 - \frac{3}{2}\lambda \longrightarrow t = \frac{14}{5} - \frac{3}{5}\lambda \\ y = 5 + \frac{3}{2}t = 9 + \frac{5}{2}\lambda \end{cases}$$

sub $t = \frac{14}{5} - \frac{3}{5}\lambda$ into y.

$5 + \frac{3}{2}\left(\frac{14}{5} - \frac{3}{5}\lambda\right) = 9 + \frac{5}{2}\lambda \longrightarrow \lambda = \frac{1}{17}$

$x = 4 - \frac{3}{2}\left(\frac{1}{17}\right) = \frac{133}{34} \qquad y = 9 + \frac{5}{2}\left(\frac{1}{17}\right) = \frac{311}{34}$

distance $= \sqrt{\left(\frac{133}{34} - 4\right)^2 + \left(\frac{311}{34} - 9\right)^2} = \frac{\sqrt{34}}{34}$

distance $= 0.171$ units (3 s.f.)