

Trigonometry Toolkit

Mark Schemes

Question 1

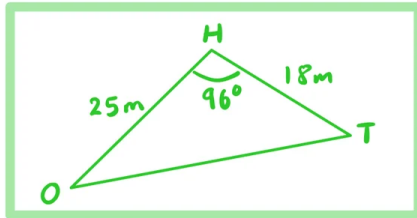
Owen, Henry and Tom are rugby players passing a ball in a park. Owen is at point O, Henry is at point H and Tom is at point T. The distance between Owen and Henry is 25 m and the distance between Henry and Tom is 18 m. The angle \widehat{OHT} is 96° .

- (a) (i) Draw and label a diagram to represent the situation described above.
 (ii) Find the length of the line OT.

(b) Find the size of the angle \widehat{OTH} .

(c) Find the area of the section of the park the players are using to pass the ball.

(a) (i) THREE POINTS CREATE TRIANGLE OHT



ORIENTATION OF TRIANGLE MAY DIFFER

Owen, Henry and Tom are rugby players passing a ball in a park. Owen is at point O, Henry is at point H and Tom is at point T. The distance between Owen and Henry is 25 m and the distance between Henry and Tom is 18 m. The angle \widehat{OHT} is 96° .

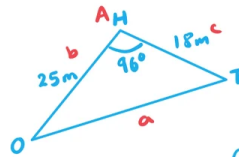
- (a) (i) Draw and label a diagram to represent the situation described above.
 (ii) Find the length of the line OT.

$OT = 32.3 \text{ m (3sf)}$

(b) Find the size of the angle \widehat{OTH} .

(c) Find the area of the section of the park the players are using to pass the ball.

(ii) OT = SIDE OPPOSITE GIVEN ANGLE



USING COSINE RULE
 $a^2 = b^2 + c^2 - 2bc \cos A$

$$OT^2 = OH^2 + HT^2 - 2(OH)(HT) \cos(\widehat{OHT})$$

SUB IN VALUES

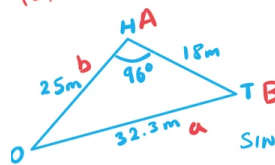
$$OT^2 = 25^2 + 18^2 - 2(25)(18) \cos(96)$$

$$OT = \sqrt{25^2 + 18^2 - 2(25)(18) \cos(96)}$$

$$OT = 32.29668121$$

$OT = 32.3 \text{ m (3sf)}$

(b) $\widehat{OTH} = \text{ANGLE OPPOSITE SIDE OH}$



TWO PAIRS OF OPPOSITE SIDES AND ANGLES = SINE RULE

$$\frac{\sin B}{b} = \frac{\sin A}{a}$$

$$\frac{\sin(\widehat{OTH})}{OH} = \frac{\sin(\widehat{OHT})}{OT}$$

SUB IN VALUES

$$\frac{\sin(\widehat{OTH})}{25} = \frac{\sin(96)}{32.2966...} \quad (\text{USE ANSWER FROM a})$$

$$\sin(\widehat{OTH}) = \frac{\sin(96)}{32.2966...} \times 25$$

$$\widehat{OTH} = \sin^{-1}\left(\frac{\sin(96)}{32.2966...} \times 25\right)$$

$$\widehat{OTH} = 50.33888476$$

$\widehat{OTH} = 50.3^\circ \text{ (3sf)}$

SAME ANSWER FROM OT=32.3

Owen, Henry and Tom are rugby players passing a ball in a park. Owen is at point O, Henry is at point H and Tom is at point T. The distance between Owen and Henry is 25 m and the distance between Henry and Tom is 18 m. The angle \widehat{OHT} is 96° .

(a) (i) Draw and label a diagram to represent the situation described above.

(ii) Find the length of the line OT.

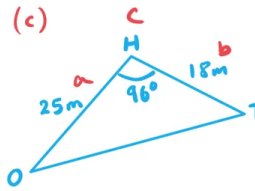
(b) Find the size of the angle \widehat{OTH} .

(c) Find the area of the section of the park the players are using to pass the ball.

[4]

[3]

[3]



SUB IN VALUES

$$\text{AREA} = \frac{1}{2} ab \sin C$$

$$A = \frac{1}{2} (OH)(HT) \sin(\widehat{OHT})$$

$$A = \frac{1}{2} (25)(18) \sin(96)$$

$$A = 223.7674265$$

$$\text{AREA} = 224 \text{ m}^2 \text{ (3sf)}$$

Question 2

A sailboat race takes place annually for under 18's on a large lake. The competitors must sail around five flagged buoys at the points A, B, C, D and E, in a clockwise direction.

B is due east of A, C is due south of B and A is due north of E.

The bearing from A to C is 110° and the bearing from C to D is 220° .

The distance $AB = 1200$ m, the distance $BC = 600$ m, the distance $CD = 800$ m and the distances $DE = EA = 1000$ m.

(a) Draw and label a diagram to show the buoys A, B, C, D and E and clearly mark the bearings and distances given above.

[3]

The boats all start at A and must complete the course 5 times. A support motorboat is present and can travel across the course from A to C and A to D in case of an emergency.

(b) Calculate the distance from A to C.

[2]

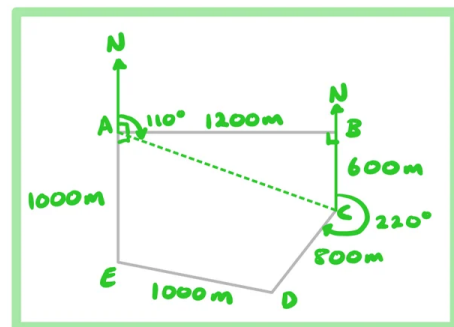
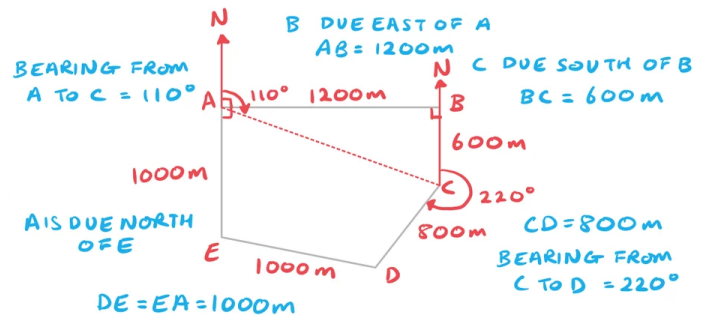
(c) Calculate the distance from A to D.

[3]

(d) Calculate the bearing the support boat must follow to travel from A to D.

[4]

(a) LOOK OUT FOR RIGHT ANGLES WHEN BEARINGS ARE USED, WORK THROUGH STATEMENTS SYSTEMATICALLY BEARINGS ARE MEASURED CLOCKWISE FROM NORTH



A sailboat race takes place annually for under 18's on a large lake. The competitors must sail around five flagged buoys at the points A, B, C, D and E, in a clockwise direction.

B is due east of A, C is due south of B and A is due north of E.
 The bearing from A to C is 110° and the bearing from C to D is 220° .
 The distance AB = 1200 m, the distance BC = 600 m, the distance CD = 800 m
 and the distances DE = EA = 1000 m.

(a) Draw and label a diagram to show the buoys A, B, C, D and E and clearly mark the bearings and distances given above.

[3]

The boats all start at A and must complete the course 5 times. A support motorboat is present and can travel across the course from A to C and A to D in case of an emergency.

(b) Calculate the distance from A to C.

[2]

(c) Calculate the distance from A to D.

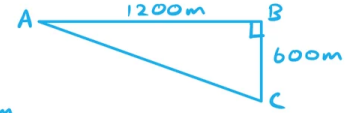
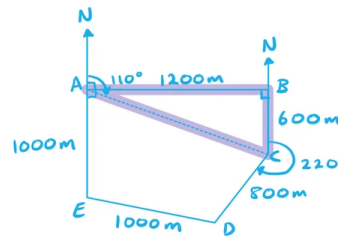
[3]

(d) Calculate the bearing the support boat must follow to travel from A to D.

[4]

(b) USE DIAGRAM CONSTRUCTED IN PART (a)

AC = HYPOTENUSE OF RIGHT ANGLED TRIANGLE ABC



USING PYTHAGORAS' THEOREM

$$a^2 + b^2 = c^2$$

$$AB^2 + BC^2 = AC^2$$

$$AC^2 = 1200^2 + 600^2$$

$$AC = \sqrt{1200^2 + 600^2}$$

$$AC = 1341.640786$$

$$AC = 1340 \text{ m (3sf)}$$

A sailboat race takes place annually for under 18's on a large lake. The competitors must sail around five flagged buoys at the points A, B, C, D and E, in a clockwise direction.

B is due east of A, C is due south of B and A is due north of E.
 The bearing from A to C is 110° and the bearing from C to D is 220° .
 The distance AB = 1200 m, the distance BC = 600 m, the distance CD = 800 m
 and the distances DE = EA = 1000 m.

(a) Draw and label a diagram to show the buoys A, B, C, D and E and clearly mark the bearings and distances given above.

[3]

The boats all start at A and must complete the course 5 times. A support motorboat is present and can travel across the course from A to C and A to D in case of an emergency.

(b) Calculate the distance from A to C.

$$AC = 1340 \text{ m (3sf)}$$

[2]

(c) Calculate the distance from A to D.

[3]

(d) Calculate the bearing the support boat must follow to travel from A to D.

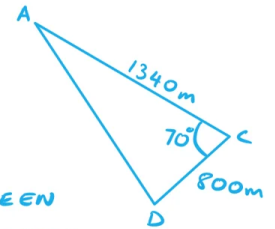
[4]

(c) USE DIAGRAM CONSTRUCTED IN PART (a)

$$\textcircled{1} 110 - 90 = 20^\circ$$

$$\textcircled{2} 180 - (90 + 20) = 70^\circ$$

$$\textcircled{3} 360 - (220 + 70) = 70^\circ$$



WE NOW HAVE ANGLE BETWEEN
TWO SIDES SO CAN USE COSINE RULE

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$AD^2 = AC^2 + CD^2 - 2(AC)(CD) \cos C$$

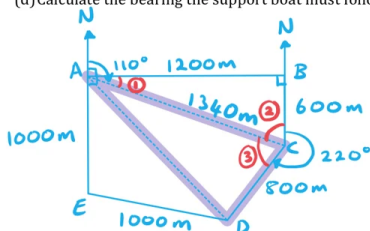
SUB IN VALUES

$$AD^2 = 1340^2 + 800^2 - 2(1340)(800) \cos(70)$$

$$AD = \sqrt{1340^2 + 800^2 - 2(1340)(800) \cos(70)}$$

$$AD = 1304.72557$$

$$AD = 1300 \text{ m (3sf)}$$



NEED TO FIND MORE INFO
FOR TRIANGLE ACD

USE BEARINGS TO LABEL
MORE ANGLES ON DIAGRAM

A sailboat race takes place annually for under 18's on a large lake. The competitors must sail around five flagged buoys at the points A, B, C, D and E, in a clockwise direction.

B is due east of A, C is due south of B and A is due north of E.
 The bearing from A to C is 110° and the bearing from C to D is 220° .
 The distance AB = 1200 m, the distance BC = 600 m, the distance CD = 800 m and the distances DE = EA = 1000 m.

(a) Draw and label a diagram to show the buoys A, B, C, D and E and clearly mark the bearings and distances given above.

[3]

The boats all start at A and must complete the course 5 times. A support motorboat is present and can travel across the course from A to C and A to D in case of an emergency.

(b) Calculate the distance from A to C.

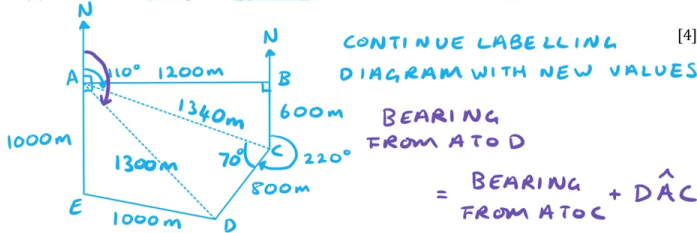
[2]

(c) Calculate the distance from A to D.

AD = 1300 m (3sf)

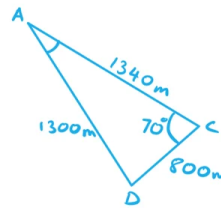
[3]

(d) Calculate the bearing the support boat must follow to travel from A to D.



[4]

(d) $\hat{D}AC$ CAN BE FOUND USING SINE RULE



$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

$$\frac{\sin \hat{D}AC}{DC} = \frac{\sin \hat{A}CD}{AD}$$

SUB IN VALUES

$$\frac{\sin \hat{D}AC}{800} = \frac{\sin 70}{1300}$$

$$\sin \hat{D}AC = \frac{\sin 70}{1300} \times 800$$

$$\hat{D}AC = \sin^{-1} \left(\frac{\sin 70}{1300} \times 800 \right)$$

$$\hat{D}AC = 35.32912271$$

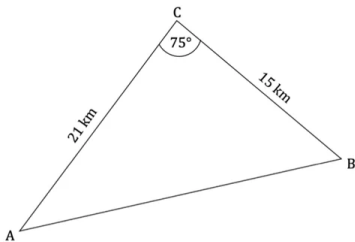
$$\text{BEARING FROM A TO D} = 110 + 35.3 \dots = 145.3 \dots$$

BEARINGS ARE ALWAYS GIVEN AS 3 FIGURES

BEARING A TO D = 145°

Question 3

The following diagram shows triangle ABC. AC = 21 km, CB = 15 km, $\hat{A}CB = 75^\circ$.



(a) Find the area of triangle ABC.

[2]

(b) Find AB.

[3]

(c) Given that it is acute, find $\hat{C}AB$.

[2]

(a) $\text{AREA} = \frac{1}{2} ab \sin C$

$$A = \frac{1}{2} (AC)(CB) \sin(\hat{A}CB)$$

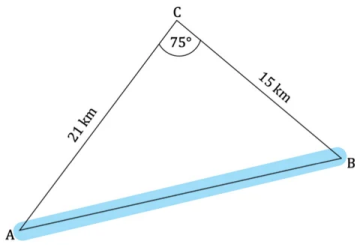
SUB IN VALUES

$$A = \frac{1}{2} (21)(15) \sin(75)$$

$$A = 152.133 \dots$$

AREA = 152 km² (3sf)

The following diagram shows triangle ABC. AC = 21 km, CB = 15 km, $\hat{A}CB = 75^\circ$.



(a) Find the area of triangle ABC.

(b) Find AB.

(c) Given that it is acute, find $\hat{C}AB$.

(b) AB = SIDE OPPOSITE GIVEN ANGLE

USING COSINE RULE $a^2 = b^2 + c^2 - 2bc \cos A$

$$AB^2 = AC^2 + CB^2 - 2(AC)(CB) \cos(\hat{A}CB)$$

SUB IN VALUES

$$AB^2 = 21^2 + 15^2 - 2(21)(15) \cos(75)$$

$$AB = \sqrt{21^2 + 15^2 - 2(21)(15) \cos(75)}$$

$$AB = 22.4264\dots$$

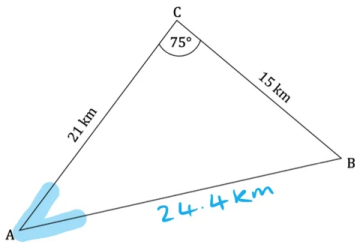
[2]

[3]

$AB = 22.4 \text{ km (3sf)}$

[2]

The following diagram shows triangle ABC. AC = 21 km, CB = 15 km, $\hat{A}CB = 75^\circ$.



(a) Find the area of triangle ABC.

(b) Find AB.

$AB = 22.4264\dots \text{ km}$

(c) Given that it is acute, find $\hat{C}AB$.

(c) $\hat{C}AB$ = ANGLE OPPOSITE SIDE CB

TWO PAIRS OF OPPOSITE SIDES AND ANGLES

$$= \text{SINE RULE } \frac{\sin B}{b} = \frac{\sin A}{a}$$

$$\frac{\sin(\hat{C}AB)}{CB} = \frac{\sin(\hat{A}CB)}{AB}$$

SUB IN VALUES

$$\frac{\sin(\hat{C}AB)}{15} = \frac{\sin(75)}{22.4264\dots} \text{ (USE ANSWER FROM b)}$$

[2]

[3]

$$\sin(\hat{C}AB) = \frac{\sin(75)}{22.4264\dots} \times 15$$

[2]

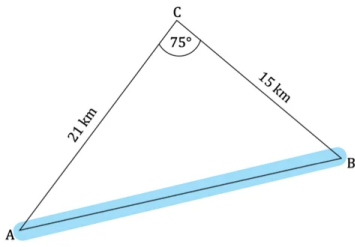
$$\hat{C}AB = \sin^{-1}\left(\frac{\sin(75)}{22.4264\dots} \times 15\right)$$

$$\hat{C}AB = 40.2454\dots$$

$\hat{C}AB = 40.2^\circ \text{ (3sf)}$

$\hat{C}AB = 40.3$
USING $AB = 22.4$

The following diagram shows triangle ABC. AC = 21 km, CB = 15 km, $\hat{A}CB = 75^\circ$.



(a) Find the area of triangle ABC.

(b) Find AB.

(c) Given that it is acute, find $\hat{C}AB$.

(b) AB = SIDE OPPOSITE GIVEN ANGLE

USING COSINE RULE $a^2 = b^2 + c^2 - 2bc \cos A$

$$AB^2 = AC^2 + CB^2 - 2(AC)(CB) \cos(\hat{A}CB)$$

SUB IN VALUES

$$AB^2 = 21^2 + 15^2 - 2(21)(15) \cos(75)$$

$$AB = \sqrt{21^2 + 15^2 - 2(21)(15) \cos(75)}$$

$$AB = 22.4264\dots$$

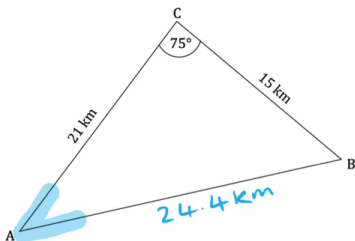
[2]

[3]

$$AB = 22.4 \text{ km (3sf)}$$

[2]

The following diagram shows triangle ABC. AC = 21 km, CB = 15 km, $\hat{A}CB = 75^\circ$.



(a) Find the area of triangle ABC.

(b) Find AB.

(c) Given that it is acute, find $\hat{C}AB$.

(c) $\hat{C}AB$ = ANGLE OPPOSITE SIDE CB

TWO PAIRS OF OPPOSITE SIDES AND ANGLES

$$= \text{SINE RULE } \frac{\sin B}{b} = \frac{\sin A}{a}$$

$$\frac{\sin(\hat{C}AB)}{CB} = \frac{\sin(\hat{A}CB)}{AB}$$

SUB IN VALUES

$$\frac{\sin(\hat{C}AB)}{15} = \frac{\sin(75)}{22.4264\dots} \text{ (USE ANSWER FROM b)}$$

[2]

$$\sin(\hat{C}AB) = \frac{\sin(75)}{22.4264\dots} \times 15$$

[3]

$$\hat{C}AB = \sin^{-1}\left(\frac{\sin(75)}{22.4264\dots} \times 15\right)$$

[2]

$$\hat{C}AB = 40.2454\dots$$

$$\hat{C}AB = 40.2^\circ \text{ (3sf)}$$

$\hat{C}AB = 40.3$
USING $AB = 22.4$

Question 4

Triangle ABC has an area of 122 cm^2 , $AB = 24 \text{ cm}$ and $BC = 11 \text{ cm}$.

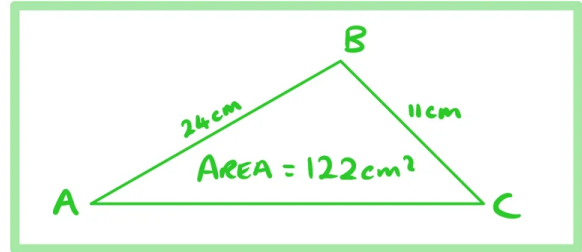
(a) Draw and label a diagram to show triangle ABC and clearly mark the distances given.

(b) Given that $\hat{A}BC$ is acute, find

- (i) $\hat{A}BC$
- (ii) AC.

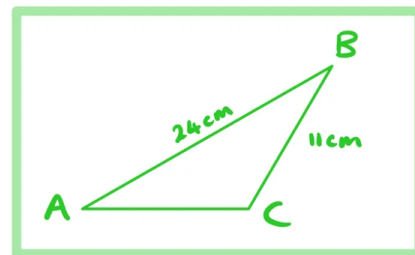
(a) Two possible valid diagrams

[1]



[4]

OR

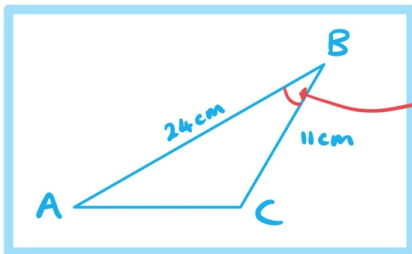


Triangle ABC has an area of 122 cm^2 , $AB = 24 \text{ cm}$ and $BC = 11 \text{ cm}$.

(a) Draw and label a diagram to show triangle ABC and clearly mark the distances given.

(b) Given that $\hat{A}BC$ is acute, find

- (i) $\hat{A}BC$ **ANGLE**
- (ii) AC. **LENGTH**



$\hat{A}BC$ IS ACUTE

[1]

(b)(i) $AREA = \frac{1}{2} ab \sin C$ $A = \frac{1}{2} (AB)(BC) \sin(\hat{A}BC)$

SUB IN VALUES AND REARRANGE

$$122 = \frac{1}{2} (24)(11) \sin(\hat{A}BC)$$

$$\sin(\hat{A}BC) = \frac{122}{132}$$

[4]

$$\hat{A}BC = \sin^{-1}\left(\frac{122}{132}\right) = 67.55439\dots$$

$$\hat{A}BC = 67.6^\circ \text{ (3sf)}$$

(ii) TWO SIDES GIVEN, USE COSINE RULE FOR THIRD SIDE $a^2 = b^2 + c^2 - 2bc \cos A$

$$AC^2 = AB^2 + BC^2 - 2(AB)(BC) \cos \hat{A}BC$$

$$AC^2 = 24^2 + 11^2 - 2(24)(11) \cos 67.55439\dots$$

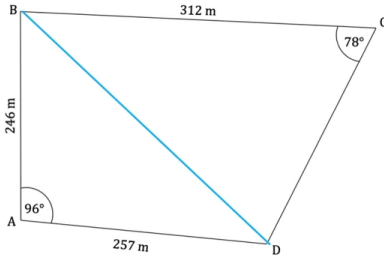
$$AC = \sqrt{24^2 + 11^2 - 2(24)(11) \cos 67.55439\dots}$$

$$AC = 22.25772561$$

$$AC = 22.3 \text{ cm (3sf)}$$

Question 5

The quadrilateral ABCD shown below represents a farm paddock, where AB = 246 m, BC = 312 m and AD = 257 m. Angle DAB = 96° and angle BCD = 78°.



A fence is built connecting points B and D to split the paddock into two.

(a) Find the length of the fence.

[3]

(b) Find the area of the paddock ABCD.

[5]

(a) AB = SIDE OPPOSITE GIVEN ANGLE

USING COSINE RULE $a^2 = b^2 + c^2 - 2bc \cos A$

$$BD^2 = AB^2 + AD^2 - 2(AB)(AD) \cos(\hat{DAB})$$

SUB IN VALUES

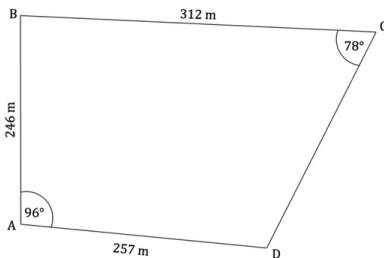
$$BD^2 = 246^2 + 257^2 - 2(246)(257) \cos(96)$$

$$BD = \sqrt{246^2 + 257^2 - 2(246)(257) \cos(96)}$$

$$BD = 373.8743064$$

$$BD = 374 \text{ m (3sf)}$$

The quadrilateral ABCD shown below represents a farm paddock, where AB = 246 m, BC = 312 m and AD = 257 m. Angle DAB = 96° and angle BCD = 78°.



A fence is built connecting points B and D to split the paddock into two.

(a) Find the length of the fence.

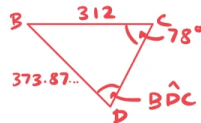
$$BD = 373.8743064$$

[3]

(b) Find the area of the paddock ABCD.

[5]

(b) AREA = $\frac{1}{2} ab \sin C$ SO FIRST NEED TO CALCULATE \hat{DBC}



SINE RULE TO FIND \hat{BDC}

$$\frac{\sin \hat{BDC}}{312} = \frac{\sin 78}{373.87...}$$

ANSWER FROM (a)

$$\hat{BDC} = \sin^{-1} \left(312 \times \frac{\sin 78}{373.87...} \right)$$

$$\hat{BDC} = 54.71304365 \quad 54.685... \text{ IF USING } 374 \text{ m}$$

$$\hat{DBC} = 180 - 54.173... - 78 = 47.286...$$

$$AREA_{ABCD} = AREA_{ABD} + AREA_{DBC}$$

$$AREA_{ABCD} =$$

$$\frac{1}{2} (246)(257) \sin 96 + \frac{1}{2} (312)(373.87...) \sin 47.286...$$

$$= 74292.27283 \quad \text{USING VALUES TO 3SF AREA} = 74315$$

$$AREA = 74300 \text{ m}^2 \text{ (3sf)}$$

Question 6

A 38 m high cliff is perpendicular to the sea and the angle of depression from the cliff to a boat at sea is 24° . Climbing the cliff is a rock climber and the angle of elevation from the boat to the climber 14° .

(a) Draw and label a diagram to show the top of the cliff, T, the foot of the cliff, F, the climber, C, the boat, B, labelling all the angles and distances given above.

[2]

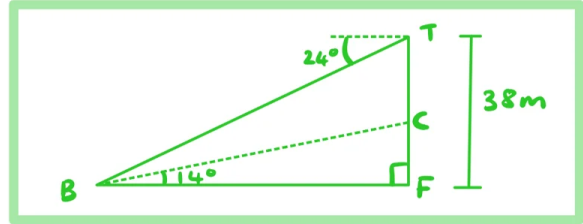
(b) Find the distance from the boat to the foot of the cliff.

[2]

(c) Find how far the climber must climb to reach the top of the cliff.

[4]

(a) DEPRESSION = DOWN FROM HORIZONTAL
ELEVATION = UP FROM HORIZONTAL



A 38 m high cliff is perpendicular to the sea and the angle of depression from the cliff to a boat at sea is 24° . Climbing the cliff is a rock climber and the angle of elevation from the boat to the climber 14° .

(a) Draw and label a diagram to show the top of the cliff, T, the foot of the cliff, F, the climber, C, the boat, B, labelling all the angles and distances given above.

[2]

(b) Find the distance from the boat to the foot of the cliff.

[2]

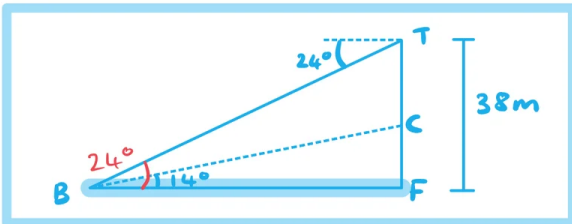
(c) Find how far the climber must climb to reach the top of the cliff.

[4]

(b) RIGHT ANGLED TRIG USING PARALLEL SEA AND DEPRESSION GIVES $\hat{TBF} = 24^\circ$

$$\begin{aligned} \text{Diagram: } \triangle TBF \text{ with } TF = 38\text{m}, \angle TBF = 24^\circ \\ \tan \theta = \frac{\text{O}}{\text{A}} \quad \tan \hat{TBF} = \frac{TF}{BF} \\ BF = \frac{TF}{\tan \hat{TBF}} \\ BF = \frac{38}{\tan 24} = 85.34939741 \end{aligned}$$

$BF = 85.3 \text{ m}$

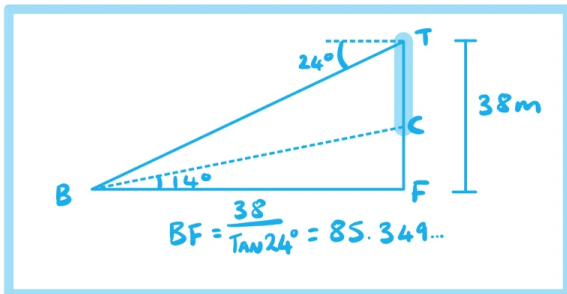


A 38 m high cliff is perpendicular to the sea and the angle of depression from the cliff to a boat at sea is 24° . Climbing the cliff is a rock climber and the angle of elevation from the boat to the climber 14° .

(a) Draw and label a diagram to show the top of the cliff, T, the foot of the cliff, F, the climber, C, the boat, B, labelling all the angles and distances given above.

(b) Find the distance from the boat to the foot of the cliff.

(c) Find how far the climber must climb to reach the top of the cliff.



(c) TO FIND CLIMB DISTANCE CT

$$CT = 38 - FC$$

USE RIGHT ANGLED TRIANGLE ON BFC



$$FC = \tan 14^\circ \times \frac{38}{\tan 24^\circ}$$

BF FROM (b) TO RETAIN ACCURACY

$$FC = 21.2799948$$

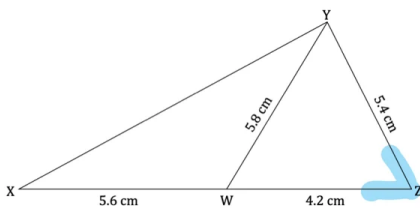
$$CT = 38 - 21.2799948$$

$$CT = 16.7200052$$

$$CT = 16.7 \text{ m (3sf)}$$

Question 7

The diagram below shows triangle XYZ with side length YZ = 5.4 cm. The point W is placed such that XW = 5.6 cm and WZ = 4.2 cm and YW = 5.8 cm.



(a) Find the angle $\hat{Y}Z\text{W}$.

(b) Find the area of triangle XYZ.

(c) Find the area of triangle XYW.

(a) THREESIDES FINDING ANGLE = COSINE RULE

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

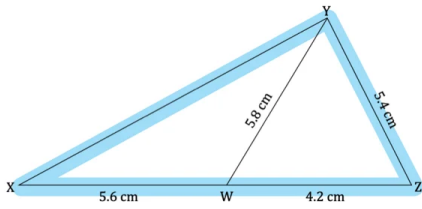
$$\cos \hat{Y}Z\text{W} = \frac{WZ^2 + YZ^2 - YW^2}{2(WZ)(YZ)}$$

$$\hat{Y}Z\text{W} = \cos^{-1} \left(\frac{4.2^2 + 5.4^2 - 5.8^2}{2(4.2)(5.4)} \right)$$

$$\hat{Y}Z\text{W} = 73.13465266$$

$$\hat{Y}Z\text{W} = 73.1^\circ \text{ (3sf)}$$

The diagram below shows triangle XYZ with side length YZ = 5.4 cm. The point W is placed such that XW = 5.6 cm and WZ = 4.2 cm and YW = 5.8 cm.



(a) Find the angle $Y\hat{Z}W$.

$$Y\hat{Z}W = 73.13465266$$

[2]

(b) Find the area of triangle XYZ.

[2]

(c) Find the area of triangle XYW.

[3]

(b)

$$AREA = \frac{1}{2} ab \sin C$$

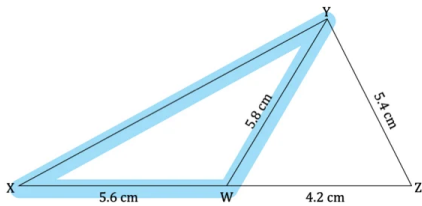
$$= \frac{1}{2} (XW + WZ) (YZ) \sin(Y\hat{Z}W)$$

$$AREA_{XYZ} = \frac{1}{2} (5.6 + 4.2) (5.4) \sin(73.13465266)$$

$$AREA_{XYZ} = 25.32193494$$

$$AREA_{XYZ} = 25.3 \text{ cm}^2 \text{ (3sf)}$$

The diagram below shows triangle XYZ with side length YZ = 5.4 cm. The point W is placed such that XW = 5.6 cm and WZ = 4.2 cm and YW = 5.8 cm.



(a) Find the angle $Y\hat{Z}W$.

$$Y\hat{Z}W = 73.13465266$$

[2]

(b) Find the area of triangle XYZ.

$$AREA_{XYZ} = 25.32193494$$

[2]

(c) Find the area of triangle XYW.

[3]

(c)

$$AREA = \frac{1}{2} ab \sin C$$

$$AREA_{XYW} = AREA_{XYZ} - AREA_{WYZ}$$

$$AREA_{XYW} = AREA_{XYZ} - \frac{1}{2} (4.2) (5.4) \sin(Y\hat{Z}W)$$

USING VALUES FOR $AREA_{XYZ}$ AND $Y\hat{Z}W$ FROM (a) AND (b) TO KEEP ACCURACY

$$AREA_{XYW} = 25.321... - \frac{1}{2} (4.2) (5.4) \sin(73.134...)$$

$$AREA_{XYW} = 14.46967711$$

$$AREA_{XYW} = 14.5 \text{ cm}^2 \text{ (3sf)}$$

Question 8

The distance between towns X and Y is 134.2 km. The bearing of town X from town Y is 119° . Town Z is 54 km south of town X. The bearing of town Z from town X is 207° .

(a) Draw and label a diagram to show towns X, Y and Z, clearly marking the bearings and distances given above.

(b) Calculate the distance between towns X and Z.

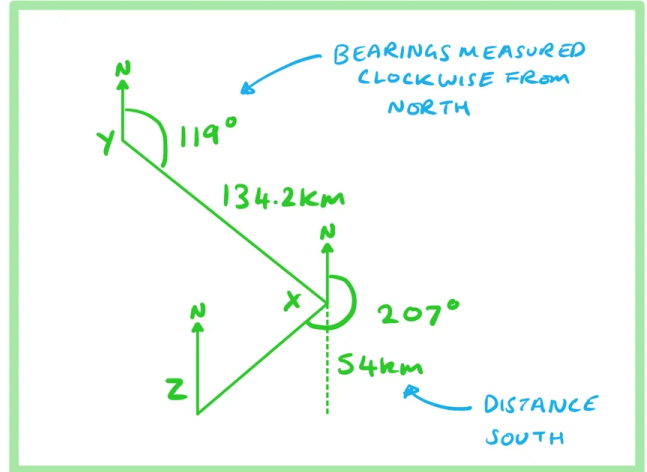
(c) Calculate the distance between towns Y and Z.

(a)

[2]

[2]

[4]



The distance between towns X and Y is 134.2 km. The bearing of town X from town Y is 119° . Town Z is 54 km south of town X. The bearing of town Z from town X is 207° .

(a) Draw and label a diagram to show towns X, Y and Z, clearly marking the bearings and distances given above.

(b) Calculate the distance between towns X and Z.

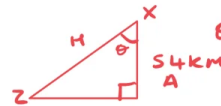
(c) Calculate the distance between towns Y and Z.

(b) DISTANCE XZ USES RIGHT ANGLED TRIG

[2]

[2]

[4]

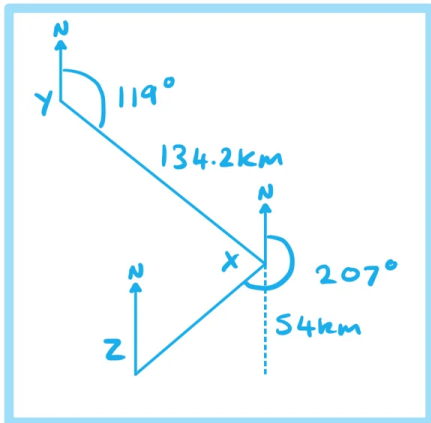


$$\theta = 207 - 180 = 27^\circ$$

$$\cos \theta = \frac{A}{H} \leftarrow XZ$$

$$XZ = \frac{54}{\cos 27^\circ} = 60.60561683$$

$$XZ = 60.6 \text{ km (3sf)}$$



The distance between towns X and Y is 134.2 km. The bearing of town X from town Y is 119° . Town Z is 54 km south of town X. The bearing of town Z from town X is 207° .

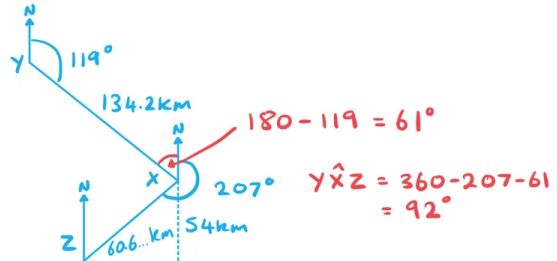
(a) Draw and label a diagram to show towns X, Y and Z, clearly marking the bearings and distances given above.

(b) Calculate the distance between towns X and Z.

$$XZ = \frac{54}{\cos 27^\circ} = 60.6 \dots \text{km}$$

(c) Calculate the distance between towns Y and Z.

(c) USE BEARINGS TO FIND \hat{YXZ}



[2]

[2]

[4]

USE COSINE RULE TO FIND YZ

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$YZ^2 = YX^2 + XZ^2 - 2(YX)(XZ) \cos(\hat{YXZ})$$

$$YZ^2 = (134.2)^2 + \left(\frac{54}{\cos 27^\circ}\right)^2 - 2(134.2)\left(\frac{54}{\cos 27^\circ}\right) \cos(92)$$

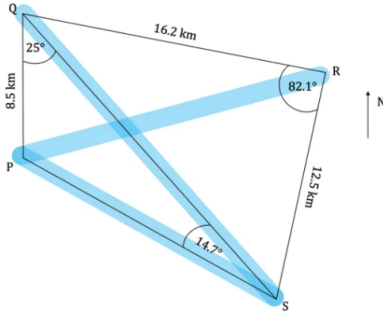
$$YZ = \sqrt{(134.2)^2 + \left(\frac{54}{\cos 27^\circ}\right)^2 - 2(134.2)\left(\frac{54}{\cos 27^\circ}\right) \cos(92)}$$

$$YZ = 149.1655963$$

$$YZ = 149 \text{ km (3 sf)}$$

Question 9

The diagram below shows four Islands P, Q, R and S. $PQ = 8.5$ km, $QR = 16.2$ km and $RS = 12.5$ km. Angle $P\hat{Q}S = 25^\circ$, angle $Q\hat{S}P = 14.7^\circ$ and angle $Q\hat{R}S = 82.1^\circ$. Island Q is due north from island P.



Mark is making deliveries around the Islands. He takes milk from Island Q to Island S, then takes wood from Island S to Island P, finally he delivers fruit from Island P to Island R.

Find the total distance Mark travels.

FOR EACH DISTANCE, FIRST FIND CORRESPONDING TRIANGLE TO USE

COSINE RULE $a^2 = b^2 + c^2 - 2bc \cos A$

$QS^2 = (16.2)^2 + (12.5)^2 - 2(16.2)(12.5) \cos(82.1)$

$QS = \sqrt{(16.2)^2 + (12.5)^2 - 2(16.2)(12.5) \cos(82.1)}$

$QS = 19.05321387$ km

SINE RULE $\frac{a}{\sin A} = \frac{b}{\sin B}$

$\frac{PS}{\sin 25} = \frac{8.5}{\sin 14.7}$

$PS = \frac{8.5}{\sin 14.7} \times \sin 25$

$PS = 14.15622762$ km

TO FIND PR FIRST FIND $S\hat{Q}R$ USING SINE RULE

[8] $\frac{\sin S\hat{Q}R}{12.5} = \frac{\sin 82.1}{QS}$

$S\hat{Q}R = \sin^{-1} \left(\frac{\sin 82.1}{19.05321387} \times 12.5 \right)$

$S\hat{Q}R = 40.52885831$

THEN USE $\Theta = P\hat{Q}S + S\hat{Q}R$ TO FIND PR

$\Theta = 25 + 40.52885831$

$\Theta = 65.52885831$

$PR^2 = 8.5^2 + 16.2^2 - 2(8.5)(16.2) \cos \Theta$

$PR = \sqrt{8.5^2 + 16.2^2 - 2(8.5)(16.2) \cos(65.5288...)}$

$PR = 14.85293632$ km

TOTAL DISTANCE = $QS + SP + PR$

$19.05321387 + 14.15622762 + 14.85293632$

$TOTAL = 48.06237781$ km

TOTAL = 48.1 km (3sf)

Question 10

Nathan (N) stands 10 m above the ground on the second-floor balcony of an apartment building and can see Melissa (M) in the car park. The angle of elevation from Melissa to Nathan is 21.6° .

(a) Calculate the distance from M to N.

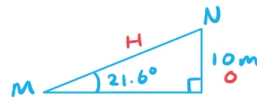
[2]

Louisa (L) is standing on the other side of the car park. The distance between Louisa and Nathan is 1.5 times the distance between Melissa and Nathan.

(b) Calculate the angle of depression from N to L.

[3]

(a) DRAW DIAGRAM



$$\sin \theta = \frac{o}{h}$$

$$MN = \frac{10}{\sin 21.6}$$

$$MN = 27.16471892$$

$$MN = 21.2 \text{ m (3sf)}$$

Nathan (N) stands 10 m above the ground on the second-floor balcony of an apartment building and can see Melissa (M) in the car park. The angle of elevation from Melissa to Nathan is 21.6° .

(a) Calculate the distance from M to N.

[2]

$$MN = \frac{10}{\sin 21.6} = 27.2 \dots$$

Louisa (L) is standing on the other side of the car park. The distance between Louisa and Nathan is 1.5 times the distance between Melissa and Nathan.

(b) Calculate the angle of depression from N to L.

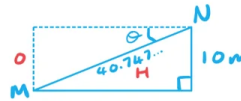
[3]

(b) $LN = 1.5 \times MN$

$$LN = 1.5 \times 27.16471892$$

$$LN = 40.74707837$$

DRAW DIAGRAM DEPRESSION IS DOWN FROM HORIZONTAL



$$\sin \theta = \frac{o}{h} \text{ LN}$$

$$\theta = \sin^{-1} \left(\frac{10}{LN} \right) = \sin^{-1} \left(\frac{10}{40.74707837} \right)$$

$$\theta = 14.20644114$$

$$\theta = 14.2^\circ \text{ (3sf)}$$