Question 1

Complete the table.

Degrees	Radians	sin	cos	tan
$30°$	$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$
45°	$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1
60°	$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
$120°$	$\frac{2\pi}{3}$	$\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$-\sqrt{3}$
270°	$\frac{3\pi}{2}$	-1	0	

[5]

Notes

① $\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$ and $\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

② The tangent of 270° (like the tangent of every multiple of 90°) is undefined. Sometimes '∞' or '$\pm\infty$' is used to indicate this.

Question 2

Given that $\sin\theta = \frac{3}{5}$, where $\frac{\pi}{2} < \theta < \pi$, find the possible values of $\cos\theta$ and $\tan\theta$.

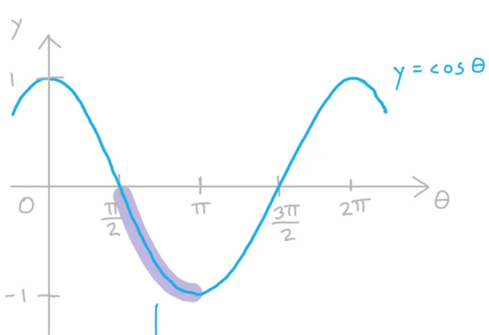
[3]

$\cos^2\theta + \sin^2\theta = 1$ } Pythagorean identity

$\cos^2\theta + \left(\frac{3}{5}\right)^2 = \cos^2\theta + \frac{9}{25} = 1$

$\cos^2\theta = 1 - \frac{9}{25} = \frac{16}{25}$

$\cos\theta = \pm\sqrt{\frac{16}{25}} = \pm\frac{4}{5}$



But for $\frac{\pi}{2} < \theta < \pi$, $\cos\theta$ is negative so

$$\boxed{\cos\theta = -\frac{4}{5}}$$

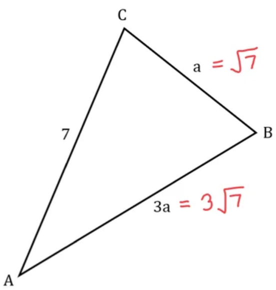
$\tan\theta = \frac{\sin\theta}{\cos\theta}$ } Identity for $\tan\theta$

$\tan\theta = \frac{3/5}{-4/5} = \frac{3}{5}\times\left(-\frac{5}{4}\right)$

$$\boxed{\tan\theta = -\frac{3}{4}}$$

Question 3

The following triangle shows triangle ABC, with AB = $3a$, BC = a and AC = 7.



Given that $\cos A\hat{B}C = \frac{1}{2}$, find the area of the triangle. Give your answer in the form $\frac{p\sqrt{3}}{r}$, where $p, q \in \mathbb{R}$.

[7]

$c^2 = a^2 + b^2 - 2ab\cos C$ } Cosine rule

$\cos^2\theta + \sin^2\theta = 1$ } Pythagorean identity

$\text{Area} = \frac{1}{2}ab\sin C$ } area of a triangle

Use cosine rule to find value of a

$7^2 = (a)^2 + (3a)^2 - 2(a)(3a)\left(\frac{1}{2}\right)$

$49 = a^2 + 9a^2 - 3a^2$

$7a^2 = 49 \implies a^2 = 7 \implies a = \sqrt{7}$

Use identity to find $\sin A\hat{B}C$

$\sin A\hat{B}C = \sqrt{1 - \left(\frac{1}{2}\right)^2} = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}$

Now use formula to find area of triangle

$\text{Area} = \frac{1}{2}(\sqrt{7})(3\sqrt{7})\left(\frac{\sqrt{3}}{2}\right)$

$= \frac{1}{2}(21)\left(\frac{\sqrt{3}}{2}\right) = \frac{21\sqrt{3}}{4}$

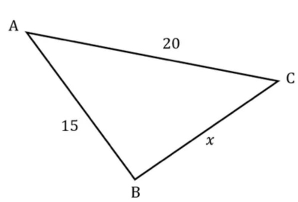
$\boxed{\text{Area} = \frac{21\sqrt{3}}{4} \text{ units}^2}$

$p = 21$

$r = 4$

Question 4

The following triangle shows triangle ABC, with AB = 15, AC = 20, BC = x.



(a) Given that $\cos B\hat{A}C = \frac{2}{3}$, find the value of $\sin B\hat{A}C$.

[3]

(b) Find the exact area of triangle ABC.

[3]

(c) By finding the value of x, show that triangle ABC is isosceles.

[3]

a) $\cos^2\theta + \sin^2\theta = 1$ } Pythagorean identity

Use identity to find $\sin B\hat{A}C$

$\left(\frac{2}{3}\right)^2 + \sin^2 B\hat{A}C = 1$

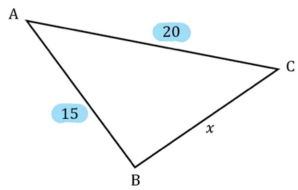
$\sin^2 B\hat{A}C + \frac{4}{9} = 1$

$\sin^2 B\hat{A}C = 1 - \frac{4}{9} = \frac{5}{9}$

$\sin B\hat{A}C = \sqrt{\frac{5}{9}}$

$\boxed{\sin B\hat{A}C = \frac{\sqrt{5}}{3}}$

The following triangle shows triangle ABC, with $AB = 15$, $AC = 20$, $BC = x$.



(a) Given that $\cos B\hat{A}C = \frac{2}{3}$, find the value of $\sin B\hat{A}C$. [3]

$\boxed{\sin B\hat{A}C = \frac{\sqrt{5}}{3}}$

(b) Find the exact area of triangle ABC. [3]

(c) By finding the value of x, show that triangle ABC is isosceles. [3]

b) $\text{Area} = \frac{1}{2}ab\sin C$ } area of a triangle

Use formula to find area of triangle

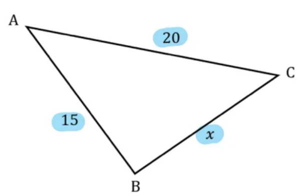
$\text{Area} = \frac{1}{2}(15)(20)\left(\frac{\sqrt{5}}{3}\right)$

$= \frac{1}{2}(300)\left(\frac{\sqrt{5}}{3}\right)$

$= \frac{300\sqrt{5}}{6} = 50\sqrt{5}$

$\boxed{\text{Area} = 50\sqrt{5} \text{ units}^2}$

The following triangle shows triangle ABC, with $AB = 15$, $AC = 20$, $BC = x$.



(a) Given that $\cos B\hat{A}C = \frac{2}{3}$, find the value of $\sin B\hat{A}C$. [3]

(b) Find the exact area of triangle ABC. [3]

(c) By finding the value of x, show that triangle ABC is isosceles. [3]

c) $c^2 = a^2 + b^2 - 2ab\cos C$ } Cosine rule

Use cosine rule to find value of x

$x^2 = (15)^2 + (20)^2 - 2(15)(20)\left(\frac{2}{3}\right)$

$= 225 + 400 - 600\left(\frac{2}{3}\right)$

$= 225 + 400 - 400 = 225$

$\Longrightarrow x = \sqrt{225} = 15$

$x = 15$, so $AB = BC = 15$.

Two sides are equal, therefore triangle ABC is isosceles.

Question 5

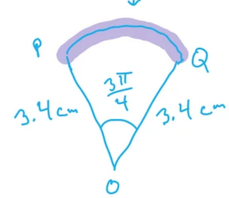
A sector of a circle, OPQ, is such that it has radius 3.4 cm and the angle at its centre, O, is $\frac{3\pi}{4}$ radians.

(i) Find the length of the arc PQ.

(ii) Find the area of the sector OPQ.

[4]

This is the arc PQ



A sketch can be helpful, but isn't needed to get the marks

Arc length : $l = r\theta$

Area of sector : $A = \frac{1}{2}r^2\theta$

θ must be in radians!

(i) $l = r\theta = (3.4)\left(\frac{3\pi}{4}\right) = \boxed{\frac{51\pi}{20} \text{ cm}}$

$(\approx 8.0 \text{ cm})$

(ii) $A = \frac{1}{2}r^2\theta = \frac{1}{2}(3.4)^2\left(\frac{3\pi}{4}\right) = \boxed{\frac{867\pi}{200} \text{ cm}^2}$

$(\approx 13.6 \text{ cm}^2)$

Question 6

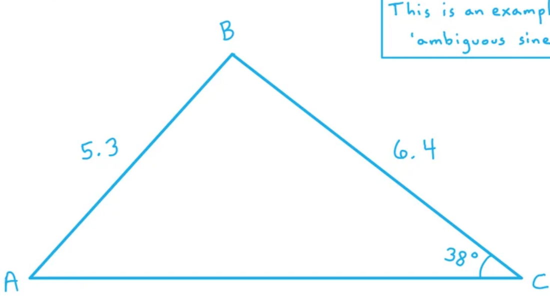
Two non-congruent triangles both have sides AB = 5.3 cm, BC = 6.4 cm and $A\hat{C}B$ = 38°.

(i) Show that the angle $B\hat{A}C$ for one of the triangles is 132°, to 3 significant figures.

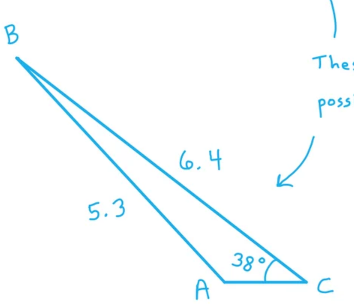
(ii) Find the angle $A\hat{B}C$ for the other triangle.

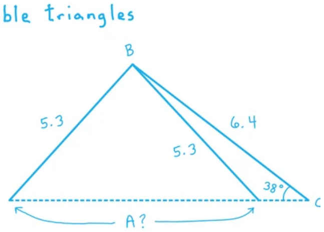
[4]

This is an example of the 'ambiguous sine rule'



These are the two possible triangles





(i) $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ Sine rule

$\frac{6.4}{\sin B\hat{A}C} = \frac{5.3}{\sin 38°} \implies \sin B\hat{A}C = \frac{6.4}{5.3}\sin 38°$

$B\hat{A}C = \sin^{-1}\left(\frac{6.4}{5.3}\sin 38°\right) = 48.025304...°$

$\sin\theta = \sin(180° - \theta)$ [Property of sine function]*

or $B\hat{A}C = 180 - \sin^{-1}\left(\frac{6.4}{5.3}\sin 38°\right) = 131.974695...°$

$B\hat{A}C = 132°$ (3 s.f.)

* This can be seen in the symmetry of the sine graph.

(ii) In the other triangle,

$B\hat{A}C = \sin^{-1}\left(\frac{6.4}{5.3}\sin 38°\right) = 48.025304...°$

Therefore

$A\hat{B}C = 180 - 38 - \sin^{-1}\left(\frac{6.4}{5.3}\sin 38°\right) = 93.974695...°$

$\boxed{A\hat{B}C = 94.0° \text{ (3 s.f.)}}$

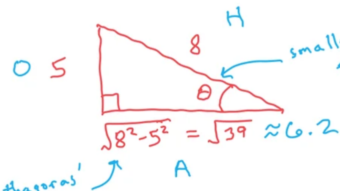
Question 7

A right-angled triangle has hypotenuse 8 cm. One of its other sides is 5 cm.

Find exact values for $\sin\theta$, $\cos\theta$ and $\tan\theta$, where θ is the smallest angle in the triangle.

[6]

Be sure to use surd form for the third side!



smallest angle is opposite the smallest side

$\sqrt{8^2-5^2} = \sqrt{39} \approx 6.2$

Pythagoras' Theorem

$$\boxed{\sin\theta = \frac{5}{8} \quad \cos\theta = \frac{\sqrt{39}}{8} \quad \tan\theta = \frac{5}{\sqrt{39}}}$$

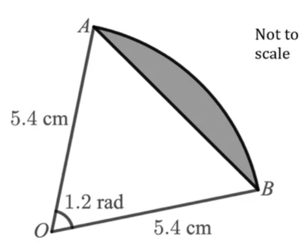
SOH

CAH

TOA

Question 8

The diagram below shows the sector of a circle OAB.



(a) (i) Find the area of the sector OAB, giving your answer to 3 significant figures.

(ii) Find the area of the triangle OAB, giving your answer to 3 significant figures.

(iii) Find the area of the shaded segment, giving your answer to 3 significant figures.

[5]

(b) (i) Find the length of the arc AB.

(ii) Find the perimeter of the sector OAB.

[3]

Arc length: $l = r\theta$

Area of sector: $A = \frac{1}{2}r^2\theta$

θ must be in radians!

$\text{Area} = \frac{1}{2}ab\sin\theta$

a) (i) area $= \frac{1}{2}(5.4)^2(1.2) = 17.496 \text{ cm}^2$

$\boxed{= 17.5 \text{ cm}^2 \ (3 \text{ s.f.})}$

(ii) area $= \frac{1}{2}(5.4)(5.4)\sin(1.2) = 13.589\ldots \text{ cm}^2$

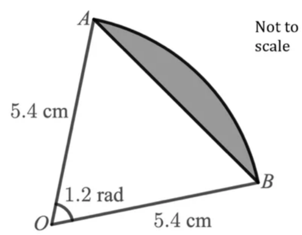
$\boxed{= 13.6 \text{ cm}^2 \ (3 \text{ s.f.})}$

(iii) area $= 17.496 - 13.589\ldots$

$= 3.906\ldots \text{ cm}^2$

$\boxed{= 3.91 \text{ cm}^2 \ (3 \text{ s.f.})}$

The diagram below shows the sector of a circle OAB.



Arc length: $l = r\theta$

Area of sector: $A = \frac{1}{2}r^2\theta$

θ must be in radians!

(a) (i) Find the area of the sector OAB, giving your answer to 3 significant figures.

(ii) Find the area of the triangle OAB, giving your answer to 3 significant figures.

(iii) Find the area of the shaded segment, giving your answer to 3 significant figures.

[5]

(b) (i) Find the length of the arc AB.

(ii) Find the perimeter of the sector OAB.

[3]

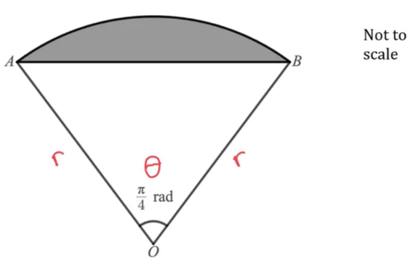
b) (i) $l = (5.4)(1.2) = \boxed{6.48 \text{ cm}}$

(ii) Perimeter $= 5.4 + 5.4 + 6.48$

$\boxed{= 17.28 \text{ cm}}$

Question 9

The canopy of a parachute and the outermost connecting cords form a sector of a circle as shown in the diagram below, with the parachutist modelled as a particle at point O.



The area of the sector OAB is $\frac{81\pi}{200}$ m^2.
Find the length of one of the connecting cords on the parachute.

[3]

$$\text{area} = \frac{81\pi}{200} = \frac{1}{2}r^2\left(\frac{\pi}{4}\right)$$

$$\frac{\cancel{\pi}}{8}r^2 = \frac{81\cancel{\pi}}{200}$$

$$\frac{1}{8}r^2 = \frac{81}{200}$$

$$r^2 = \frac{81}{25}$$

$$\text{radius} = \sqrt{\frac{81}{25}} = \frac{9}{5} \text{ m}$$

$\boxed{\text{length of connecting cord is } \frac{9}{5} \text{ m } (1.8 \text{ m})}$

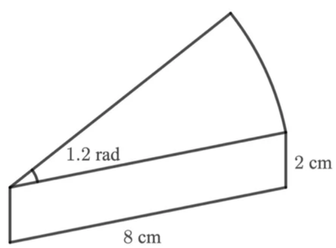
Arc length: $l = r\theta$

Area of sector: $A = \frac{1}{2}r^2\theta$

θ must be in radians!

Question 10

A plastic puzzle piece is in the form of a prism with a cross-section that is the sector of a circle, as shown in the diagram below. The radius of the sector is 8 cm, and the angle at the centre is 1.2 radians.
The height of the puzzle piece is 2 cm.



(i) Work out the area of the cross-section.

(ii) Hence, or otherwise, work out the volume of the puzzle piece.

[3]

(i) $\text{Area} = \frac{1}{2}(8)^2(1.2) = \boxed{38.4 \text{ cm}^2}$

(ii) $\text{Volume} = 38.4 \times 2 = \boxed{76.8 \text{ cm}^3}$

Volume of Prism = Area of Cross-section × Height

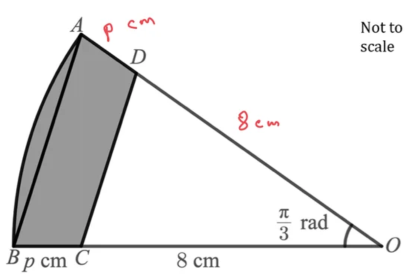
Arc length : $l = r\theta$

Area of sector : $A = \frac{1}{2}r^2\theta$

θ must be in radians!

Question 11

The circle sector OAB is shown in the diagram below.
The angle at the centre is $\frac{\pi}{3}$ radians, and the line segments OC and BC have lengths of 8 cm and p cm respectively.
Additionally, CD is parallel to AB, so that $AD = BC$ and $OD = OC$.



(a) Show that the area of the sector OAB is $\frac{\pi}{6}(p+8)^2$ cm^2.

[2]

(b) Show that the area of the triangle OCD is $16\sqrt{3}$ cm^2.

[2]

(c) Given that the area of the shaded shape $ABCD$ is $\left(\frac{50\pi}{3} - 16\sqrt{3}\right)$ cm^2, find the value of p.

[4]

Arc length : $l = r\theta$

Area of sector : $A = \frac{1}{2}r^2\theta$

θ must be in radians!

a) radius of sector OAB = $(p+8)$ cm

$\text{Area} = \frac{1}{2}(p+8)^2\left(\frac{\pi}{3}\right)$

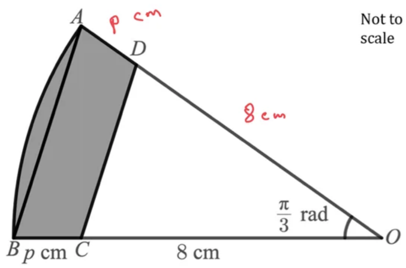
$= \frac{\pi}{6}(p+8)^2 \text{ cm}^2$

EXAM PAPERS PRACTICE

The circle sector OAB is shown in the diagram below.

The angle at the centre is $\frac{\pi}{3}$ radians, and the line segments OC and BC have lengths of 8 cm and p cm respectively.

Additionally, CD is parallel to AB, so that $AD = BC$ and $OD = OC$.



(a) Show that the area of the sector OAB is $\frac{\pi}{6}(p+8)^2$ cm^2.

[2]

(b) Show that the area of the triangle OCD is $16\sqrt{3}$ cm^2.

[2]

(c) Given that the area of the shaded shape $ABCD$ is $\left(\frac{50\pi}{3} - 16\sqrt{3}\right)$ cm^2, find the value of p.

[4]

$\text{Area} = \frac{1}{2}ab\sin\theta$

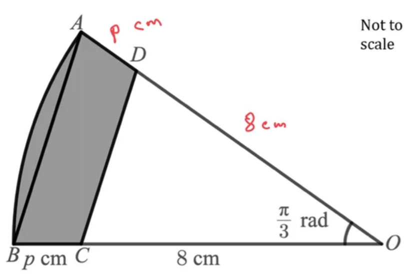
b)

$$\text{Area} = \frac{1}{2}(8)(8)\sin\left(\frac{\pi}{3}\right)$$
$$= 32\sin\left(\frac{\pi}{3}\right)$$
$$= 32\left(\frac{\sqrt{3}}{2}\right)$$
$$\boxed{= 16\sqrt{3} \text{ cm}^2}$$

The circle sector OAB is shown in the diagram below.

The angle at the centre is $\frac{\pi}{3}$ radians, and the line segments OC and BC have lengths of 8 cm and p cm respectively.

Additionally, CD is parallel to AB, so that $AD = BC$ and $OD = OC$.



(a) Show that the area of the sector OAB is $\frac{\pi}{6}(p+8)^2$ cm^2.

[2]

(b) Show that the area of the triangle OCD is $16\sqrt{3}$ cm^2.

[2]

(c) Given that the area of the shaded shape $ABCD$ is $\left(\frac{50\pi}{3} - 16\sqrt{3}\right)$ cm^2, find the value of p.

[4]

c) area of shape ABCD = area of sector OAB − area of triangle OCD

$$= \frac{\pi}{6}(p+8)^2 - 16\sqrt{3}$$

Therefore

$$\frac{\pi}{6}(p+8)^2 \cancel{- 16\sqrt{3}} = \frac{50\pi}{3} \cancel{- 16\sqrt{3}}$$
$$\frac{\cancel{\pi}}{6}(p+8)^2 = \frac{50\cancel{\pi}}{3}$$
$$\frac{1}{6}(p+8)^2 = \frac{50}{3}$$
$$(p+8)^2 = 100$$
$$p+8 = \pm 10$$
$$p = 2 \text{ or } -18$$

But p can't be negative

because it is the length of a line segment!

$$\boxed{\text{So } p = 2 \text{ cm}}$$