Question 1

Owen, Henry and Tom are rugby players passing a ball in a park. Owen is at point O, Henry is at point H and Tom is at point T. The distance between Owen and Henry is 25 m and the distance between Henry and Tom is 18 m.The angle $O\hat{H}T$ is 96°.

(a) (i) Draw and label a diagram to represent the situation described above.

(ii) Find the length of the line OT.

[4]

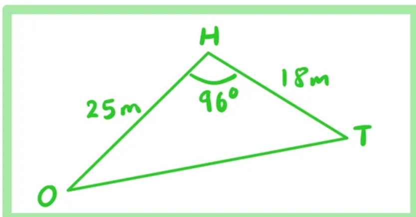
(b) Find the size of the angle $O\hat{T}H$.

[3]

(c) Find the area of the section of the park the players are using to pass the ball.

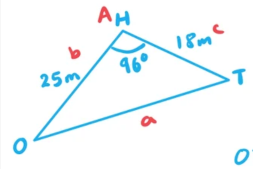
[3]

(a) (i) THREE POINTS CREATE TRIANGLE OHT



ORIENTATION OF TRIANGLE MAY DIFFER

(ii) OT = SIDE OPPOSITE GIVEN ANGLE



USING COSINE RULE

$a^2 = b^2 + c^2 - 2bc \cos A$

$OT^2 = OH^2 + HT^2 - 2(OH)(HT)\cos(O\hat{H}T)$

SUB IN VALUES

$OT^2 = 25^2 + 18^2 - 2(25)(18)\cos(96)$

$OT = \sqrt{25^2 + 18^2 - 2(25)(18)\cos(96)}$

$OT = 32.29668121$

$\boxed{OT = 32.3 \text{ m } (3SF)}$

Owen, Henry and Tom are rugby players passing a ball in a park. Owen is at point O, Henry is at point H and Tom is at point T. The distance between Owen and Henry is 25 m and the distance between Henry and Tom is 18 m.The angle $O\hat{H}T$ is 96°.

(a) (i) Draw and label a diagram to represent the situation described above.

(ii) Find the length of the line OT.

$\boxed{OT = 32.3 \text{ m } (3SF)}$

[4]

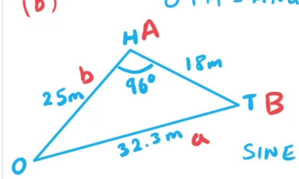
(b) Find the size of the angle $O\hat{T}H$.

[3]

(c) Find the area of the section of the park the players are using to pass the ball.

[3]

(b) $O\hat{T}H$ = ANGLE OPPOSITE SIDE OH



TWO PAIRS OF OPPOSITE SIDES AND ANGLES = SINE RULE

SINE RULE $\frac{\sin B}{b} = \frac{\sin A}{a}$

$\frac{\sin(O\hat{T}H)}{OH} = \frac{\sin(O\hat{H}T)}{OT}$

SUB IN VALUES

$\frac{\sin(O\hat{T}H)}{25} = \frac{\sin(96)}{32.2966...}$ (USE ANSWER FROM a)

$\sin(O\hat{T}H) = \frac{\sin(96)}{32.2966...} \times 25$

$O\hat{T}H = \sin^{-1}\left(\frac{\sin(96)}{32.2966...} \times 25\right)$

$O\hat{T}H = 50.3388476$

$\boxed{O\hat{T}H = 50.3° \ (3SF)}$

SAME ANSWER FROM OT=32.3

EXAM PAPERS PRACTICE

Owen, Henry and Tom are rugby players passing a ball in a park. Owen is at point O, Henry is at point H and Tom is at point T. The distance between Owen and Henry is 25 m and the distance between Henry and Tom is 18 m.The angle $O\hat{H}T$ is 96°.

(a) (i) Draw and label a diagram to represent the situation described above.

(ii) Find the length of the line OT.

[4]

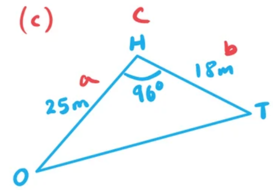
(b) Find the size of the angle $O\hat{T}H$.

[3]

(c) Find the area of the section of the park the players are using to pass the ball.

[3]

(c)



AREA = $\frac{1}{2}ab\sin C$

$A = \frac{1}{2}(OH)(HT)\sin(O\hat{H}T)$

SUB IN VALUES

$A = \frac{1}{2}(25)(18)\sin(96)$

$A = 223.7674265$

$\boxed{\text{AREA} = 224\ m^2\ (3sf)}$

Question 2

A sailboat race takes place annually for under 18's on a large lake. The competitors must sail around five flagged buoys at the points A, B, C, D and E, in a clockwise direction.

B is due east of A, C is due south of B and A is due north of E.
The bearing from A to C is 110° and the bearing from C to D is 220°.
The distance AB = 1200 m, the distance BC = 600 m, the distance CD = 800 m and the distances DE = EA = 1000 m.

(a) Draw and label a diagram to show the buoys A, B, C, D and E and clearly mark the bearings and distances given above.

[3]

The boats all start at A and must complete the course 5 times. A support motorboat is present and can travel across the course from A to C and A to D in case of an emergency.

(b) Calculate the distance from A to C.

[2]

(c) Calculate the distance from A to D.

[3]

(d) Calculate the bearing the support boat must follow to travel from A to D.

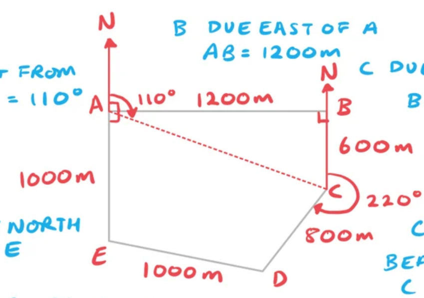
[4]

(a) LOOK OUT FOR RIGHT ANGLES WHEN BEARINGS ARE USED, WORK THROUGH STATEMENTS SYSTEMATICALLY
BEARINGS ARE MEASURED CLOCKWISE FROM NORTH

B DUE EAST OF A
AB = 1200m

BEARING FROM A TO C = 110°

C DUE SOUTH OF B
BC = 600m

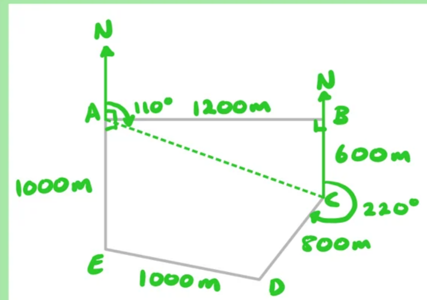


CD = 800m

A IS DUE NORTH OF E

BEARING FROM C TO D = 220°

DE = EA = 1000m



A sailboat race takes place annually for under 18's on a large lake. The competitors must sail around five flagged buoys at the points A, B, C, D and E, in a clockwise direction.

B is due east of A, C is due south of B and A is due north of E.
The bearing from A to C is 110° and the bearing from C to D is 220°.
The distance AB = 1200 m, the distance BC = 600 m, the distance CD = 800 m and the distances DE = EA = 1000 m.

(a) Draw and label a diagram to show the buoys A, B, C, D and E and clearly mark the bearings and distances given above.

[3]

The boats all start at A and must complete the course 5 times. A support motorboat is present and can travel across the course from A to C and A to D in case of an emergency.

(b) Calculate the distance from A to C.

[2]

(c) Calculate the distance from A to D.

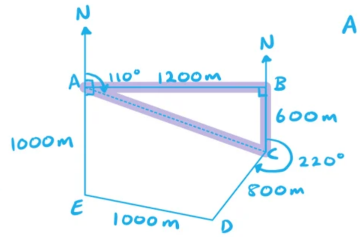
[3]

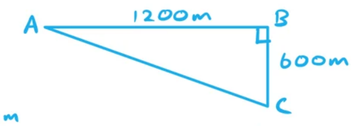
(d) Calculate the bearing the support boat must follow to travel from A to D.

[4]

(b) USE DIAGRAM CONSTRUCTED IN PART (a)

AC = HYPOTENUSE OF RIGHT ANGLED TRIANGLE ABC





USING PYTHAGORAS' THEOREM

$a^2 + b^2 = c^2$

$AB^2 + BC^2 = AC^2$

$AC^2 = 1200^2 + 600^2$

$AC = \sqrt{1200^2 + 600^2}$

$AC = 1341.640786$

$\boxed{AC = 1340 \text{ m (3sf)}}$

A sailboat race takes place annually for under 18's on a large lake. The competitors must sail around five flagged buoys at the points A, B, C, D and E, in a clockwise direction.

B is due east of A, C is due south of B and A is due north of E.
The bearing from A to C is 110° and the bearing from C to D is 220°.
The distance AB = 1200 m, the distance BC = 600 m, the distance CD = 800 m and the distances DE = EA = 1000 m.

(a) Draw and label a diagram to show the buoys A, B, C, D and E and clearly mark the bearings and distances given above.

[3]

The boats all start at A and must complete the course 5 times. A support motorboat is present and can travel across the course from A to C and A to D in case of an emergency.

(b) Calculate the distance from A to C.

$\boxed{AC = 1340 \text{ m (3sf)}}$

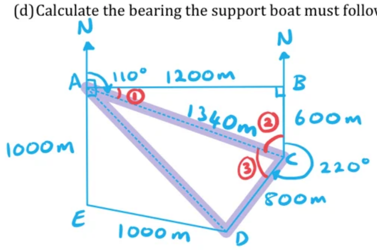
[2]

(c) Calculate the distance from A to D.

[3]

(d) Calculate the bearing the support boat must follow to travel from A to D.

[4]



NEED TO FIND MORE INFO FOR TRIANGLE ACD

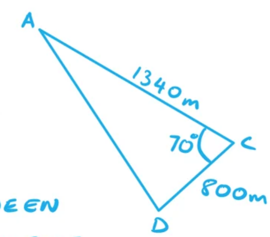
USE BEARINGS TO LABEL MORE ANGLES ON DIAGRAM

(c) USE DIAGRAM CONSTRUCTED IN PART (a)

① $110 - 90 = 20°$

② $180 - (90 + 20) = 70°$

③ $360 - (220 + 70) = 70°$



WE NOW HAVE ANGLE BETWEEN TWO SIDES SO CAN USE COSINE RULE

$a^2 = b^2 + c^2 - 2bc\cos A$

$AD^2 = AC^2 + CD^2 - 2(AC)(CD)\cos C$

SUB IN VALUES

$AD^2 = 1340^2 + 800^2 - 2(1340)(800)\cos(70)$

$AD = \sqrt{1340^2 + 800^2 - 2(1340)(800)\cos(70)}$

$AD = 1304.72557$

$\boxed{AD = 1300 \text{ m (3sf)}}$

A sailboat race takes place annually for under 18's on a large lake. The competitors must sail around five flagged buoys at the points A, B, C, D and E, in a clockwise direction.

B is due east of A, C is due south of B and A is due north of E.
The bearing from A to C is 110° and the bearing from C to D is 220°.
The distance AB = 1200 m, the distance BC = 600 m, the distance CD = 800 m and the distances DE = EA = 1000 m.

(a) Draw and label a diagram to show the buoys A, B, C, D and E and clearly mark the bearings and distances given above.

[3]

The boats all start at A and must complete the course 5 times. A support motorboat is present and can travel across the course from A to C and A to D in case of an emergency.

(b) Calculate the distance from A to C.

[2]

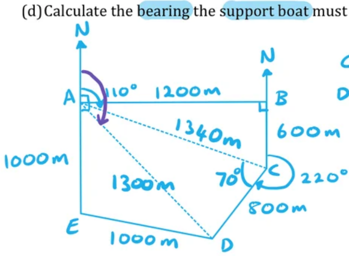
(c) Calculate the distance from A to D.

$\boxed{AD = 1300\text{ m (3sf)}}$

[3]

(d) Calculate the bearing the support boat must follow to travel from A to D.

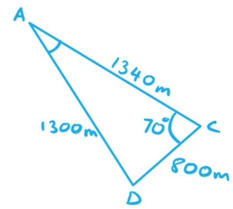
[4]



CONTINUE LABELLING DIAGRAM WITH NEW VALUES

BEARING FROM A TO D $= $ BEARING FROM A TO C $+ D\hat{A}C$

(d) $D\hat{A}C$ CAN BE FOUND USING SINE RULE



$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

$$\frac{\sin D\hat{A}C}{DC} = \frac{\sin A\hat{C}D}{AD}$$

SUB IN VALUES

$$\frac{\sin D\hat{A}C}{800} = \frac{\sin 70}{1300}$$

$$\sin D\hat{A}C = \frac{\sin 70}{1300} \times 800$$

$$D\hat{A}C = \sin^{-1}\left(\frac{\sin 70}{1300} \times 800\right)$$

$$D\hat{A}C = 35.32912271$$

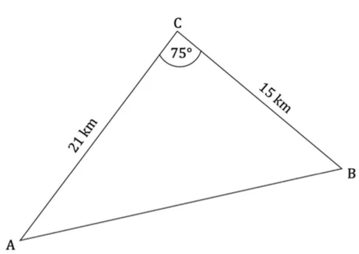
BEARING FROM A TO D $= 110 + 35.3\ldots = 145.3\ldots$

BEARINGS ARE ALWAYS GIVEN AS 3 FIGURES

$\boxed{\text{BEARING A TO D} = 145°}$

Question 3

The following diagram shows triangle ABC. AC = 21 km, CB = 15 km, $A\hat{C}B = 75°$.



(a) Find the area of triangle ABC.

[2]

(b) Find AB.

[3]

(c) Given that it is acute, find $C\hat{A}B$.

[2]

(a) AREA $= \frac{1}{2} ab \sin C$

$$A = \frac{1}{2}(AC)(CB)\sin(A\hat{C}B)$$

SUB IN VALUES

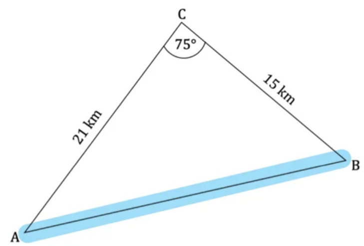
$$A = \frac{1}{2}(21)(15)\sin(75)$$

$$A = 152.133\ldots$$

$\boxed{\text{AREA} = 152\text{ km}^2\text{ (3sf)}}$

EXAM PAPERS PRACTICE

The following diagram shows triangle ABC. AC = 21 km, CB = 15 km, $A\hat{C}B = 75°$.



(a) Find the area of triangle ABC. [2]

(b) Find AB. [3]

(c) Given that it is acute, find $C\hat{A}B$. [2]

(b) AB = SIDE OPPOSITE GIVEN ANGLE

USING COSINE RULE $a^2 = b^2 + c^2 - 2bc\cos A$

$$AB^2 = AC^2 + CB^2 - 2(AC)(CB)\cos(A\hat{C}B)$$

SUB IN VALUES

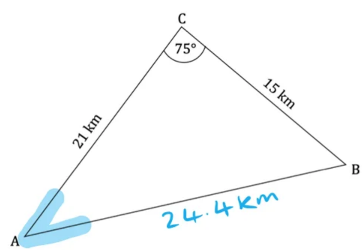
$$AB^2 = 21^2 + 15^2 - 2(21)(15)\cos(75)$$

$$AB = \sqrt{21^2 + 15^2 - 2(21)(15)\cos(75)}$$

$$AB = 22.4264\ldots$$

$$\boxed{AB = 22.4\text{ km (3sf)}}$$

The following diagram shows triangle ABC. AC = 21 km, CB = 15 km, $A\hat{C}B = 75°$.



(a) Find the area of triangle ABC. [2]

(b) Find AB. $\boxed{AB = 22.4264\ldots\text{ km}}$ [3]

(c) Given that it is acute, find $C\hat{A}B$. [2]

(c) $C\hat{A}B$ = ANGLE OPPOSITE SIDE CB

TWO PAIRS OF OPPOSITE SIDES AND ANGLES

= SINE RULE $\dfrac{\sin B}{b} = \dfrac{\sin A}{a}$

$$\frac{\sin(C\hat{A}B)}{CB} = \frac{\sin(A\hat{C}B)}{AB}$$

SUB IN VALUES

$$\frac{\sin(C\hat{A}B)}{15} = \frac{\sin(75)}{22.4264\ldots}$$ (USE ANSWER FROM b)

$$\sin(C\hat{A}B) = \frac{\sin(75)}{22.4264\ldots} \times 15$$

$$C\hat{A}B = \sin^{-1}\left(\frac{\sin(75)}{22.4264\ldots} \times 15\right)$$

$$C\hat{A}B = 40.2454\ldots$$

$$\boxed{C\hat{A}B = 40.2° \text{ (3sf)}}$$

$C\hat{A}B = 40.3$ USING AB = 22.4

Question 4

Triangle ABC has an area of 122 cm², AB = 24 cm and BC = 11 cm.

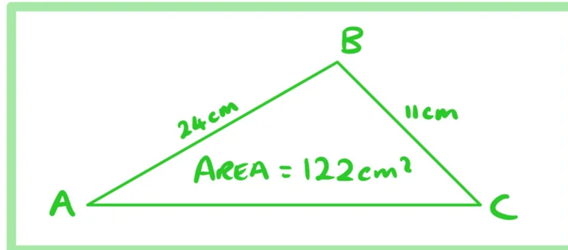
(a) Draw and label a diagram to show triangle ABC and clearly mark the distances given. [1]

(b) Given that $A\hat{B}C$ is acute, find

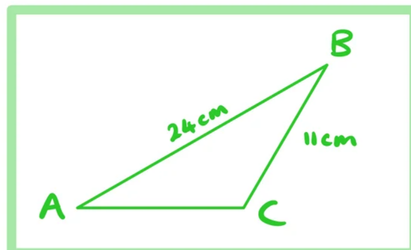
(i) $A\hat{B}C$

(ii) AC. [4]

(a) TWO POSSIBLE VALID DIAGRAMS



OR



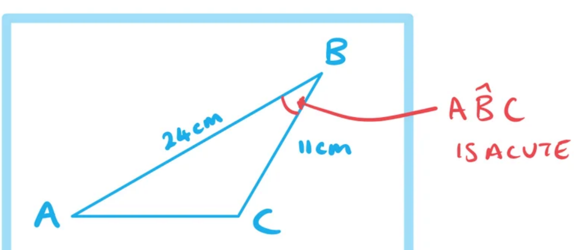
Triangle ABC has an area of 122 cm², AB = 24 cm and BC = 11 cm.

(a) Draw and label a diagram to show triangle ABC and clearly mark the distances given. [1]

(b) Given that $A\hat{B}C$ is acute, find

(i) $A\hat{B}C$ ANGLE

(ii) AC. LENGTH [4]



(b)(i) AREA = $\frac{1}{2}ab\sin C$ $\quad A = \frac{1}{2}(AB)(BC)\sin(A\hat{B}C)$

SUB IN VALUES AND REARRANGE

$$122 = \frac{1}{2}(24)(11)\sin(A\hat{B}C)$$

$$\sin(A\hat{B}C) = \frac{122}{132}$$

$$A\hat{B}C = \sin^{-1}\left(\frac{122}{132}\right) = 67.55439\ldots$$

$$\boxed{A\hat{B}C = 67.6° \text{ (3sf)}}$$

(ii) TWO SIDES GIVEN, USE COSINE RULE FOR THIRD SIDE $\quad a^2 = b^2 + c^2 - 2bc\cos A$

$$AC^2 = AB^2 + BC^2 - 2(AB)(BC)\cos A\hat{B}C$$

FROM (i)

$$AC^2 = 24^2 + 11^2 - 2(24)(11)\cos 67.55439\ldots$$

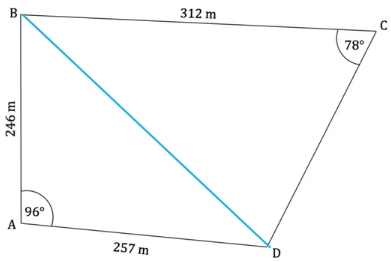
$$AC = \sqrt{24^2 + 11^2 - 2(24)(11)\cos 67.55439\ldots}$$

$$AC = 22.25772561$$

$$\boxed{AC = 22.3 \text{ cm (3sf)}}$$

Question 5

The quadrilateral ABCD shown below represents a farm paddock, where AB = 246 m, BC = 312 m and AD = 257 m. Angle $D\hat{A}B = 96°$ and angle $B\hat{C}D = 78°$.



A fence is built connecting points B and D to split the paddock into two.

(a) Find the length of the fence.

[3]

(b) Find the area of the paddock ABCD.

[5]

(a) AB = SIDE OPPOSITE GIVEN ANGLE

USING COSINE RULE $a^2 = b^2 + c^2 - 2bc \cos A$

$BD^2 = AB^2 + AD^2 - 2(AB)(AD)\cos(D\hat{A}B)$

SUB IN VALUES

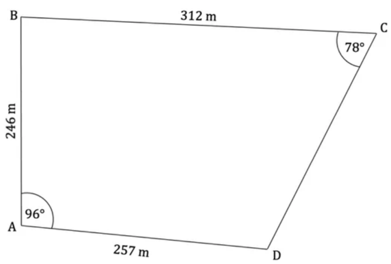
$BD^2 = 246^2 + 257^2 - 2(246)(257)\cos(96)$

$BD = \sqrt{246^2 + 257^2 - 2(246)(257)\cos(96)}$

$BD = 373.8743064$

$\boxed{BD = 374\text{ m } (3sf)}$

The quadrilateral ABCD shown below represents a farm paddock, where AB = 246 m, BC = 312 m and AD = 257 m. Angle $D\hat{A}B = 96°$ and angle $B\hat{C}D = 78°$.



A fence is built connecting points B and D to split the paddock into two.

(a) Find the length of the fence.

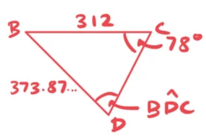
$\boxed{BD = 373.8743064}$

[3]

(b) Find the area of the paddock ABCD.

[5]

(b) AREA = $\frac{1}{2}ab\sin C$ SO FIRST NEED TO CALCULATE $D\hat{B}C$



SINE RULE TO FIND $B\hat{D}C$

$\frac{\sin B\hat{D}C}{312} = \frac{\sin 78}{373.87...}$ ANSWER FROM (a)

$B\hat{D}C = \sin^{-1}\left(312 \times \frac{\sin 78}{373.87...}\right)$

$B\hat{D}C = 54.71304365$ 54.685... IF USING 374m

$D\hat{B}C = 180 - 54.173... - 78 = 47.286...$

$AREA_{ABCD} = AREA_{ABD} + AREA_{DBC}$

$AREA_{ABCD} =$

$\frac{1}{2}(246)(257)\sin 96 + \frac{1}{2}(312)(373.87...)\sin 47.286...$

$= 74292.27283$ USING VALUES TO 3SF AREA = 74315

$\boxed{AREA = 74300\text{ m}^2 \ (3sf)}$

Question 6

A 38 m high cliff is perpendicular to the sea and the angle of depression from the cliff to a boat at sea is 24°. Climbing the cliff is a rock climber and the angle of elevation from the boat to the climber 14°.

(a) Draw and label a diagram to show the top of the cliff, T, the foot of the cliff, F, the climber, C, the boat, B, labelling all the angles and distances given above.

[2]

(b) Find the distance from the boat to the foot of the cliff.

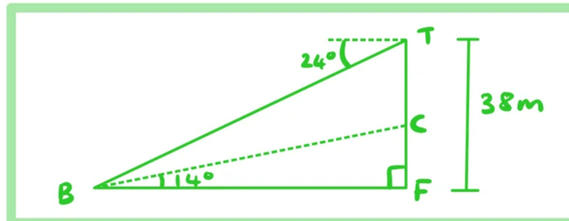
[2]

(c) Find how far the climber must climb to reach the top of the cliff.

[4]

(a) DEPRESSION = DOWN FROM HORIZONTAL

ELEVATION = UP FROM HORIZONTAL



A 38 m high cliff is perpendicular to the sea and the angle of depression from the cliff to a boat at sea is 24°. Climbing the cliff is a rock climber and the angle of elevation from the boat to the climber 14°.

(a) Draw and label a diagram to show the top of the cliff, T, the foot of the cliff, F, the climber, C, the boat, B, labelling all the angles and distances given above.

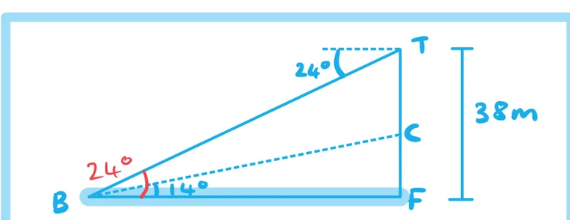
[2]

(b) Find the distance from the boat to the foot of the cliff.

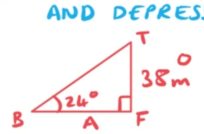
[2]

(c) Find how far the climber must climb to reach the top of the cliff.

[4]



(b) RIGHT ANGLED TRIG USING PARALLEL SEA AND DEPRESSION GIVES $T\hat{B}F = 24°$



$\tan\theta = \frac{O}{A}$ $\quad \tan T\hat{B}F = \frac{TF}{BF}$

$BF = \frac{TF}{\tan T\hat{B}F}$

$BF = \frac{38}{\tan 24} = 85.34939741$

$\boxed{BF = 85.3\text{ m}}$

A 38 m high cliff is perpendicular to the sea and the angle of depression from the cliff to a boat at sea is 24°. Climbing the cliff is a rock climber and the angle of elevation from the boat to the climber 14°.

(a) Draw and label a diagram to show the top of the cliff, T, the foot of the cliff, F, the climber, C, the boat, B, labelling all the angles and distances given above.

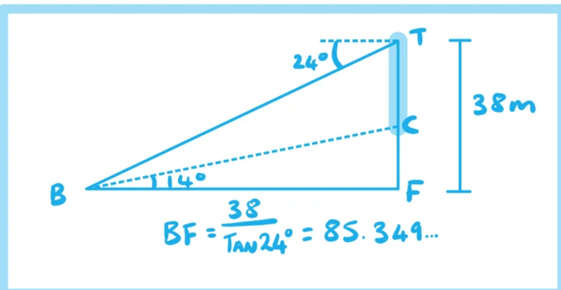
[2]

(b) Find the distance from the boat to the foot of the cliff.

[2]

(c) Find how far the climber must climb to reach the top of the cliff.

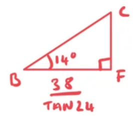
[4]



(c) TO FIND CLIMB DISTANCE CT

$CT = 38 - FC$

USE RIGHT ANGLED TRIG ON BFC



$FC = \tan 14° \times \dfrac{38}{\tan 24°}$

BF FROM (b) TO RETAIN ACCURACY

$FC = 21.2799948$

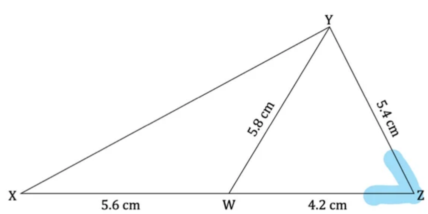
$CT = 38 - 21.2799948$

$CT = 16.7200052$

$\boxed{CT = 16.7 \text{ m} \ (3sf)}$

Question 7

The diagram below shows triangle XYZ with side length YZ = 5.4 cm. The point W is placed such that XW = 5.6 cm and WZ = 4.2 cm and YW = 5.8 cm.



(a) Find the angle $Y\hat{Z}W$.

[2]

(b) Find the area of triangle XYZ.

[2]

(c) Find the area of triangle XYW.

[3]

(a) THREE SIDES FINDING ANGLE = COSINE RULE

$\cos C = \dfrac{a^2 + b^2 - c^2}{2ab}$

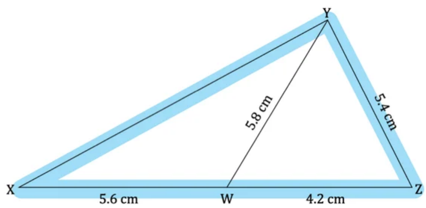
$\cos Y\hat{Z}W = \dfrac{WZ^2 + YZ^2 - WY^2}{2(WZ)(YZ)}$

$Y\hat{Z}W = \cos^{-1}\left(\dfrac{4.2^2 + 5.4^2 - 5.8^2}{2(4.2)(5.4)}\right)$

$Y\hat{Z}W = 73.13465266$

$\boxed{Y\hat{Z}W = 73.1° \ (3sf)}$

The diagram below shows triangle XYZ with side length YZ = 5.4 cm. The point W is placed such that XW = 5.6 cm and WZ = 4.2 cm and YW = 5.8 cm.



(a) Find the angle YẐW.

$\boxed{Y\hat{Z}W = 73.13465266}$ [2]

(b) Find the area of triangle XYZ.

[2]

(c) Find the area of triangle XYW.

[3]

(b) $\text{AREA} = \frac{1}{2} ab \sin C$

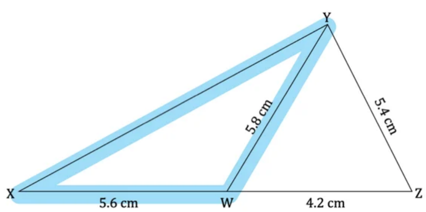
$= \frac{1}{2}(XW + WZ)(YZ) \sin(Y\hat{Z}W)$

$\text{AREA}_{XYZ} = \frac{1}{2}(5.6 + 4.2)(5.4) \sin(73.13465266)$

$\text{AREA}_{XYZ} = 25.32193494$

$\boxed{\text{AREA}_{XYZ} = 25.3 \text{ cm}^2 \text{ (3sf)}}$

The diagram below shows triangle XYZ with side length YZ = 5.4 cm. The point W is placed such that XW = 5.6 cm and WZ = 4.2 cm and YW = 5.8 cm.



(a) Find the angle YẐW.

$\boxed{Y\hat{Z}W = 73.13465266}$ [2]

(b) Find the area of triangle XYZ.

$\boxed{\text{AREA}_{XYZ} = 25.32193494}$ [2]

(c) Find the area of triangle XYW.

[3]

(c) $\text{AREA} = \frac{1}{2} ab \sin C$

$\text{AREA}_{XYW} = \text{AREA}_{XYZ} - \text{AREA}_{WYZ}$

$\text{AREA}_{XYW} = \text{AREA}_{XYZ} - \frac{1}{2}(4.2)(5.4) \sin(Y\hat{Z}W)$

USING VALUES FOR AREA_{XYZ} AND $Y\hat{Z}W$ FROM (a) AND (b) TO KEEP ACCURACY

$\text{AREA}_{XYW} = 25.321... - \frac{1}{2}(4.2)(5.4) \sin(73.134...)$

$\text{AREA}_{XYW} = 14.46967711$

$\boxed{\text{AREA}_{XYW} = 14.5 \text{ cm}^2 \text{ (3sf)}}$

Question 8

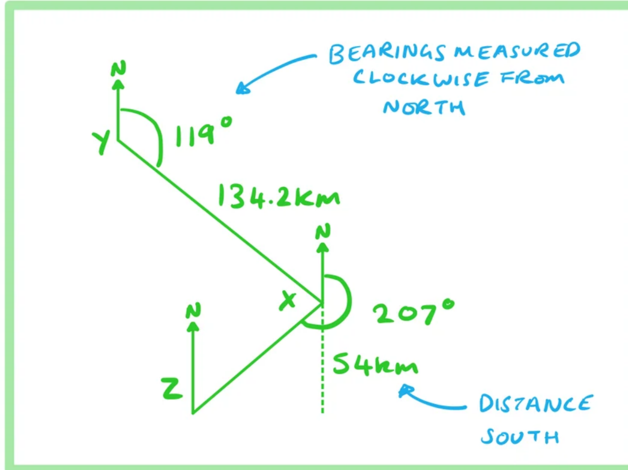
The distance between towns X and Y is 134.2 km. The bearing of town X from town Y is 119°. Town Z is 54 km south of town X. The bearing of town Z from town X is 207°.

(a) Draw and label a diagram to show towns X, Y and Z, clearly marking the bearings and distances given above. [2]

(b) Calculate the distance between towns X and Z. [2]

(c) Calculate the distance between towns Y and Z. [4]

(a)

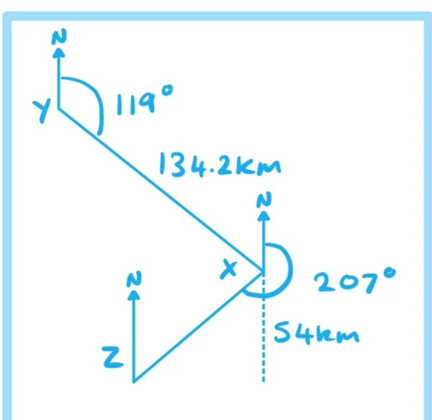


The distance between towns X and Y is 134.2 km. The bearing of town X from town Y is 119°. Town Z is 54 km south of town X. The bearing of town Z from town X is 207°.

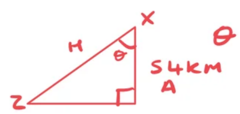
(a) Draw and label a diagram to show towns X, Y and Z, clearly marking the bearings and distances given above. [2]

(b) Calculate the distance between towns X and Z. [2]

(c) Calculate the distance between towns Y and Z. [4]



(b) DISTANCE XZ USES RIGHT ANGLED TRIG



$\theta = 207 - 180 = 27°$

$\cos\theta = \frac{A}{H}$ ← XZ

$XZ = \frac{54}{\cos 27°} = 60.60561683$

$\boxed{XZ = 60.6 \text{ km } (3sf)}$

The distance between towns X and Y is 134.2 km. The bearing of town X from town Y is 119°. Town Z is 54 km south of town X. The bearing of town Z from town X is 207°.

(a) Draw and label a diagram to show towns X, Y and Z, clearly marking the bearings and distances given above.

[2]

(b) Calculate the distance between towns X and Z.

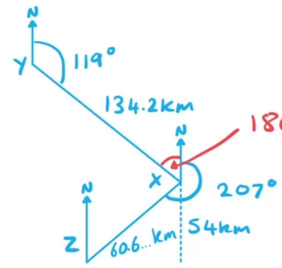
$$\boxed{XZ = \frac{54}{\cos 27^\circ} = 60.6\ldots \text{km}}$$

[2]

(c) Calculate the distance between towns Y and Z.

[4]

(c) USE BEARINGS TO FIND $Y\hat{X}Z$



$180 - 119 = 61°$

$Y\hat{X}Z = 360 - 207 - 61$
$= 92°$

USE COSINE RULE TO FIND YZ

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$YZ^2 = YX^2 + XZ^2 - 2(YX)(XZ)\cos(Y\hat{X}Z)$$

$$YZ^2 = (134.2)^2 + \left(\frac{54}{\cos 27}\right)^2 - 2(134.2)\left(\frac{54}{\cos 27}\right)\cos(92)$$

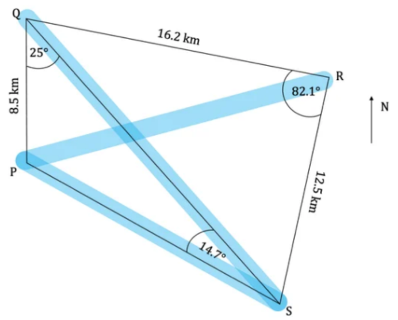
$$YZ = \sqrt{(134.2)^2 + \left(\frac{54}{\cos 27}\right)^2 - 2(134.2)\left(\frac{54}{\cos 27}\right)\cos(92)}$$

$$YZ = 149.1655963$$

$$\boxed{YZ = 149\text{ km } (3\text{ sf})}$$

Question 9

The diagram below shows four Islands P, Q, R and S. PQ = 8.5 km, QR = 16.2 km and RS = 12.5 km. Angle $P\hat{Q}S = 25°$, angle $Q\hat{S}P = 14.7°$ and angle $Q\hat{R}S = 82.1°$.
Island Q is due north from island P.

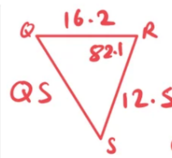


Mark is making deliveries around the Islands. He takes milk from Island Q to Island S, then takes wood from Island S to Island P, finally he delivers fruit from Island P to Island R.

Find the total distance Mark travels.

[8]

FOR EACH DISTANCE, FIRST FIND CORRESPONDING TRIANGLE TO USE

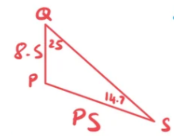


COSINE RULE $a^2 = b^2 + c^2 - 2bc\cos A$

$QS^2 = (16.2)^2 + (12.5)^2 - 2(16.2)(12.5)\cos(82.1)$

$QS = \sqrt{(16.2)^2 + (12.5)^2 - 2(16.2)(12.5)\cos(82.1)}$

QS = 19.05321387 km



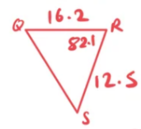
SINE RULE $\frac{a}{\sin A} = \frac{b}{\sin B}$

$\frac{PS}{\sin 25} = \frac{8.5}{\sin 14.7}$

$PS = \frac{8.5}{\sin 14.7} \times \sin 25$

PS = 14.15622762 km

TO FIND PR FIRST FIND $S\hat{Q}R$ USING SINE RULE

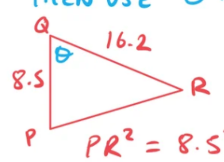


$\frac{\sin S\hat{Q}R}{12.5} = \frac{\sin 82.1}{QS}$

$S\hat{Q}R = \sin^{-1}\left(\frac{\sin 82.1}{19.05321387} \times 12.5\right)$

$S\hat{Q}R = 40.5288583\text{1}$

THEN USE $\theta = P\hat{Q}S + S\hat{Q}R$ TO FIND PR



$\theta = 25 + 40.52885831$
$= 65.52885831$

$PR^2 = 8.5^2 + 16.2^2 - 2(8.5)(16.2)\cos\theta$

$PR = \sqrt{8.5^2 + 16.2^2 - 2(8.5)(16.2)\cos(65.528\ldots)}$

PR = 14.85293632 km

TOTAL DISTANCE = QS + SP + PR

19.05321387 + 14.15622762 + 14.85293632

TOTAL = 48.06237781 km

$\boxed{\text{TOTAL} = 48.1 \text{ km (3sf)}}$

Question 10

Nathan (N) stands 10 m above the ground on the second-floor balcony of an apartment building and can see Melissa (M) in the car park. The angle of elevation from Melissa to Nathan is 21.6°.

(a) Calculate the distance from M to N.

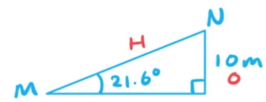
[2]

Louisa (L) is standing on the other side of the car park. The distance between Louisa and Nathan is 1.5 times the distance between Melissa and Nathan.

(b) Calculate the angle of depression from N to L.

[3]

(a) DRAW DIAGRAM



$\sin\theta = \frac{O}{H}$

$MN = \frac{10}{\sin 21.6}$

$MN = 27.16471892$

$\boxed{MN = 21.2 \text{ m } (3sf)}$

Nathan (N) stands 10 m above the ground on the second-floor balcony of an apartment building and can see Melissa (M) in the car park. The angle of elevation from Melissa to Nathan is 21.6°.

(a) Calculate the distance from M to N.

$\boxed{MN = \frac{10}{\sin 21.6} = 27.2\ldots}$

[2]

Louisa (L) is standing on the other side of the car park. The distance between Louisa and Nathan is 1.5 times the distance between Melissa and Nathan.

(b) Calculate the angle of depression from N to L.

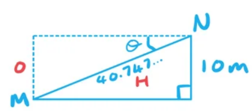
[3]

(b) $LN = 1.5 \times MN$

$LN = 1.5 \times 27.16471892$

$LN = 40.74707837$

DRAW DIAGRAM DEPRESSION IS DOWN FROM HORIZONTAL



$\sin\theta = \frac{O}{H}$ H ← LN

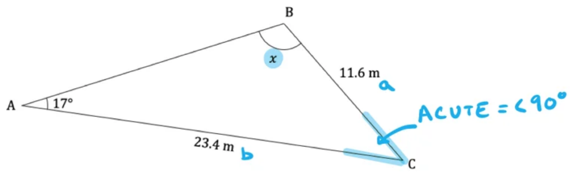
$\theta = \sin^{-1}\left(\frac{10}{LN}\right) = \sin^{-1}\left(\frac{10}{40.74707837}\right)$

$\theta = 14.20644114$

$\boxed{\theta = 14.2° \ (3sf)}$

Question 11

The diagram below shows a field ABC, with angle BÂC = 17°, BC = 11.6 m and AC = 23.4 m.



(a) Given that BĈA is acute, find the value of x.

[3]

(b) Calculate the perimeter of the field.

[3]

(a) USING SINE RULE $\frac{\sin B}{b} = \frac{\sin A}{a}$

$x = \sin^{-1}\left(\frac{\sin 17}{11.6} \times 23.4\right) = 36.1417\ldots$

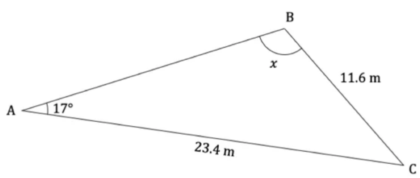
GIVEN BĈA IS ACUTE x MUST BE > 90°

∴ MUST BE AMBIGUOUS CASE OF SINE

$x = 180 - 36.1417\ldots = 143.858297$

$\boxed{x = 144° \quad 3sf}$

The diagram below shows a field ABC, with angle BÂC = 17°, BC = 11.6 m and AC = 23.4 m.



(a) Given that BĈA is acute, find the value of x.

$\boxed{x = 143.858297}$

[3]

(b) Calculate the perimeter of the field.

[3]

(b) USE EITHER SINE OR COSINE RULE TO CALCULATE LENGTH OF AB

FIRST FIND BĈA (COULD USE ORIGINAL $x = 36.14170303 - 17$)

$180 - 17 - 143.858297 = 19.14170303$

COSINE $a^2 = b^2 + c^2 - 2bc \cos A$

$AB^2 = AC^2 + BC^2 - 2\,AC \times BC \times \cos C$

$AB^2 = 23.4^2 + 11.6^2 - 2 \times 23.4 \times 11.6 \times \cos 19.14170303$

$= 169.2555667$

$AB = \sqrt{169.2555667} = 13.00982577$

FIND PERIMETER AB + BC + AC

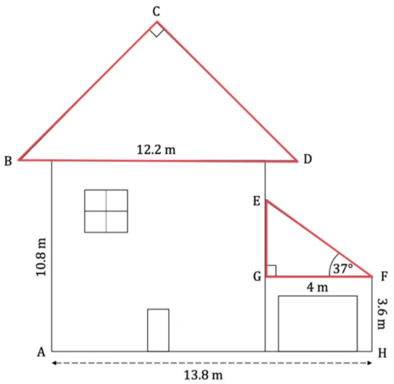
$P = 23.4 + 11.6 + 13.00982577$

$P = 48.00982577$

$\boxed{P = 48.0\text{ m} \quad 3sf}$

Question 12

The diagram below shows an architect's drawing of the front view of a house. The house is in the shape of a rectangle with a height of 10.8 m and has a roof in the shape of a right-angled isosceles triangle, BCD. BD = 12.2 m, angle $B\hat{C}D = 90°$. Next to the house is a garage in the shape of a rectangle measuring 4 m × 3.6 m with a roof in the shape of a right-angled triangle with a base, GF, of 4 m and angle $E\hat{F}G = 37°$.



(a) Find the length of

(i) EG

(ii) BC.

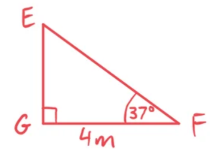
[2]

(b) Find the total area of the front view of the house.

[6]

a) Notice the right-angled triangles

i)

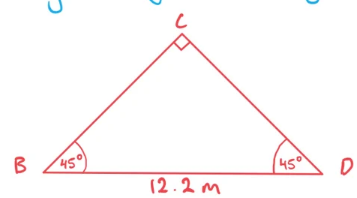


$\therefore \tan 37° = \frac{EG}{4}$

$EG = \tan 37° \times 4$

$\boxed{EG \approx 3.01\text{ m}}$

ii) Base angles of an isosceles right-angled triangle equal 45°.

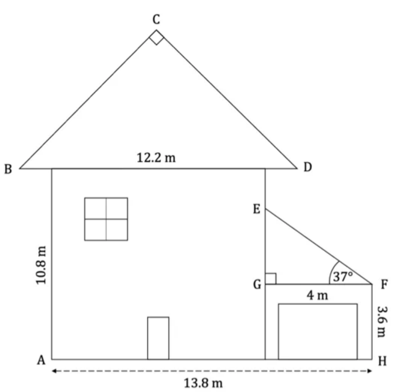


$\sin 45° = \frac{BC}{12.2}$

$BC = \sin 45° \times 12.2$

$\boxed{BC \approx 8.63\text{ m}}$

The diagram below shows an architect's drawing of the front view of a house. The house is in the shape of a rectangle with a height of 10.8 m and has a roof in the shape of a right-angled isosceles triangle, BCD. BD = 12.2 m, angle $B\hat{C}D = 90°$. Next to the house is a garage in the shape of a rectangle measuring 4 m × 3.6 m with a roof in the shape of a right-angled triangle with a base, GF, of 4 m and angle $E\hat{F}G = 37°$.



(a) Find the length of

(i) EG

(ii) BC.

$\boxed{BC \approx 8.63\text{ m}}$

[2]

(b) Find the total area of the front view of the house.

[6]

b) Total area (A) = House + Roof + Garage

House area = rectangle

= height × base

Height = 10.8 base = 13.8 − 4
= 9.8

Roof area = triangle

$= \frac{1}{2}(BC)(CD)$

$BC = CD \approx 8.63$

Garage area = trapezoid.

$= \frac{1}{2}(FH + (EG + FH)(GF)$

$FH = 3.6$ $EG \approx 3.01$ $GF = 4$

$\therefore A = (10.8)(9.8) + \frac{1}{2}(8.63)^2 + \frac{1}{2}(3.6 + (3.6 + 3.01))(4)$

$\boxed{A \approx 163.5\text{ m}^2}$