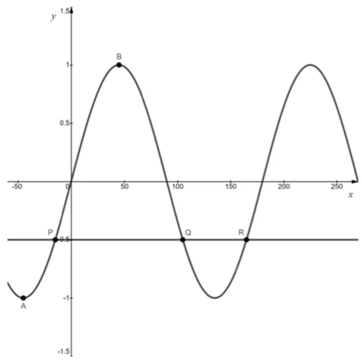
Question 1

The graph below shows the curve with equation $y = \sin 2x$ in the interval $-60° \leq x \leq 270°$.



(a) Point A has coordinates $(-45°, -1)$ and is the minimum point closest to the origin. Point B is the maximum point closest to the origin. State the coordinates of B.

[1]

(b) A straight line with equation $y = -\frac{1}{2}$ meets the graph of $y = \sin 2x$ at the three points P, Q and R, as shown in the diagram.

Given that point P has coordinates $(-15°, -\frac{1}{2})$, use graph symmetries to determine the coordinates of Q and R.

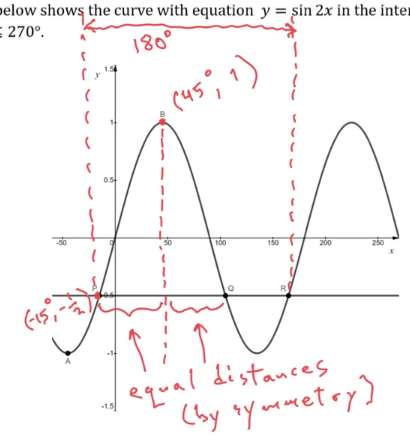
[2]

$\sin^{-1}(1) = 90°$ so $\sin(90°) = 1$

If $x = 45°$, $\sin(2x) = \sin(90°) = 1$ MAXIMUM

a) Point B has coordinates $(45°, 1)$

The graph below shows the curve with equation $y = \sin 2x$ in the interval $-60° \leq x \leq 270°$.



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[2]

$\sin 2x$ is a horizontal stretch of $\sin x$, with scale factor ½ (i.e., a 'squash'), around the y-axis. $\sin x$ repeats every 360°, so $\sin 2x$ repeats every 180°.

b) $45 - (-15) = 60$ } horizontal distance from point P to point B

$45 + 60 = 105$

Point Q has coordinates $(105°, -\frac{1}{2})$

$-15 + 180 = 165$

Point R has coordinates $(165°, -\frac{1}{2})$

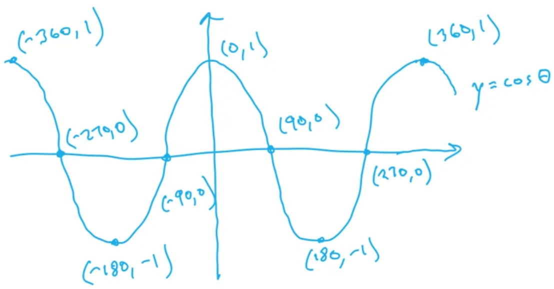
Note: Remember that you can use your GDC to check your answers on a question like this!

Question 2

(i) Sketch the graph of $y = \cos(\theta + 30°)$ in the interval $-180° \le \theta \le 360°$.

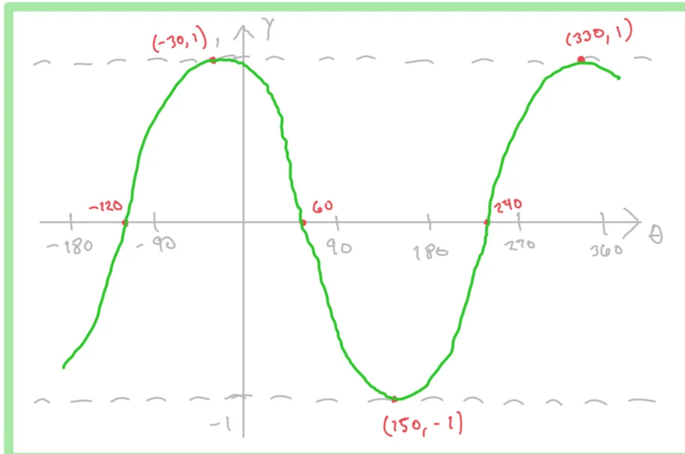
(ii) Write down all the values where $\cos(\theta + 30°) = 0$ in the given interval.

[4]



$y = \cos(\theta + 30°)$ is $y = \cos\theta$ translated 30° to the left

(i)



(ii) From the graph

$\boxed{\theta = -120°, 60°, 240°}$

Question 3

A dolphin is swimming such that it is diving in and out of the water at a constant speed. On each jump and dive the dolphin reaches a height of 2 m above sea level and a depth of 2 m below sea level.

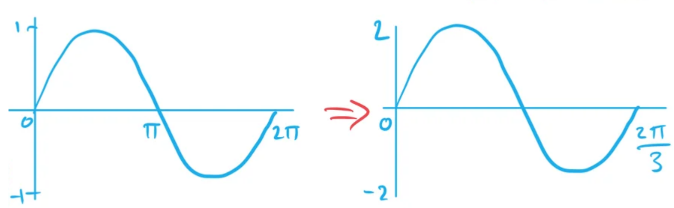
Starting at sea level, the dolphin takes $\frac{2\pi}{3}$ seconds to jump out of the water, dive back in and return to sea level.

Write down a model for the height, h m, of the dolphin, relative to sea level, at time t seconds, in the form $h = A\sin(Bt)$ where A and B are constants to be found.

[3]

SIN x
PERIOD $= 2\pi$
MAX/MIN $= \pm 1$

$A\sin(Bt)$
PERIOD $= \frac{2\pi}{3}$
MAX/MIN $= \pm 2$



A = VERTICAL STRETCH SF 2 = 2

B = HORIZONTAL STRETCH SF $\frac{1}{3}$ = 3

$\boxed{h = 2\sin(3t)}$

Question 4

A section of a new rollercoaster has a series of rises and falls. The vertical displacement of the rollercoaster carriage, y, measured in metres relative to a fixed reference height, can be modelled using the function $y = 30\cos(24t)°$, where t is the time in seconds.

stretches max. and min to +30 and −30

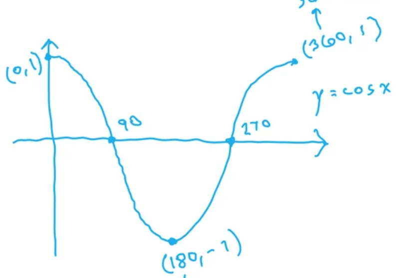
(i) Sketch the function for the interval $0 \le t \le 30$.

(ii) How many times will the rollercoaster carriage fall during the 30 seconds?

(iii) How long does the model suggest it will take for the rollercoaster carriage to reach the bottom of the first fall?

[6]

$360 \div 24 = 15$

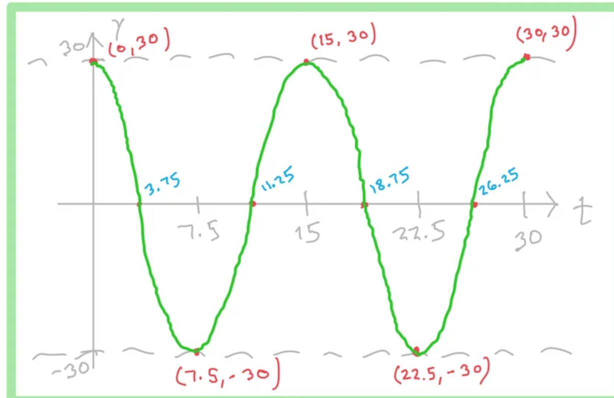


$180 \div 24 = 7.5$

Note: Remember that you can use your GDC to help you sketch a graph like this.

(i) $0 \le t \le 30$ means $0 \le 24t \le 720$

2 complete cycles of cos



(ii) $\boxed{\text{Twice}}$ from the graph

(iii) $24t = 180$

$t = 180 \div 24 = 7.5$

$\boxed{7.5 \text{ seconds}}$

Question 5

The height, h m, of water in a reservoir is modelled by the function

$$h(t) = A + B\sin\left(\frac{\pi}{6}t\right),\ t \ge 0,$$

where t is the time in hours after midnight. A and B are positive constants.

(a) In terms of A and B, write down the natural height of the water in the reservoir, as well as its maximum and minimum heights.

[3]

(b) The maximum level of water is 3 m higher than its natural level.
The level of water is three times higher at its maximum than at its minimum.

Find the maximum, minimum and natural water levels.

[3]

(c) (i) How many times per day does the water reach its maximum level?
(ii) Find the times of day when the water level is at its minimum?

[3]

a)

NATURAL HEIGHT = A m

MAX = (A + B) m

MIN = (A − B) m

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(ii) Find the times of day when the water level is at its minimum?

[3]

b) MAX = A + B MIN = A − B
NATURAL = A

MAX = A + 3 B = 3

MAX = 3(MIN)

$A + 3 = 3(A - 3)$

$A + 3 = 3A - 9$

$2A = 12$

$A = 6$

$\boxed{\text{NATURAL LEVEL} = 6\text{ m} \quad \text{MIN} = 3\text{ m} \quad \text{MAX} = 9\text{ m}}$

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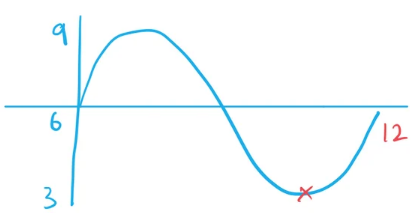

c) $h(t) = 6 + 3\sin\left(\frac{\pi}{6}t\right)$

i) $2\cancel{\pi} \times \frac{6}{\cancel{\pi}} = 12$ HOURS

$\boxed{\text{MAX 2 TIMES PER DAY}}$

ii) $6 + 3\sin\left(\frac{\pi}{6}t\right) = 3$ MIN = 3

CONSIDER RANGE
$0 \leq t \leq 24$

$\sin\left(\frac{\pi}{6}t\right) = -1$

$0 \leq \frac{\pi}{6}t \leq 4\pi$

$\sin^{-1}(-1) = -\frac{\pi}{2}$

+2π +4π

$\div\frac{\pi}{6}$

$\frac{\pi}{6}t = \frac{3}{2}\pi\ ,\ \frac{7}{2}\pi$

9 , 21

$\boxed{\text{9 AM AND 9 PM}}$

Question 6

A lifejacket falls over the side of a boat from a height of 3 m.
The height, h m, of the lifejacket above or below sea level ($h = 0$), at time t seconds after falling, is modelled by the equation $h = 3e^{-0.7t}\cos 4t$.

(a) The lifejacket reaches its furthest point below sea level after 0.742 seconds.
Find the total distance it has fallen, giving your answer to three significant figures.

[2]

(b) Write down the value of t for the first three times the lifejacket is at sea level.

[2]

(c) (i) Find the value of $3e^{-0.7t}$ when $t = 6.2$.

(ii) Hence justify why, from 6.2 seconds on, the lifejacket will always be within 4 centimetres of sea level.

[3]

a) $h = 3e^{-0.7 \times 0.742}\cos(4 \times 0.742)$

$= -1.75781\ldots$ BELOW SEA LEVEL

TOTAL DISTANCE

$3 + 1.7578\ldots = 4.7578\ldots$

$\boxed{4.76 \text{ m } (3sf)}$

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[3]

b) $h = y = 0$ CAN USE GDC TO SOLVE

AS $3e^{-0.7t} \neq 0$

$\cos(4t) = 0$

$\cos^{-1}(0) = \frac{\pi}{2}$

$+\pi \qquad +2\pi$

$4t = \frac{\pi}{2}, \frac{3}{2}\pi, \frac{5}{2}\pi$ $\div 4$

$\boxed{t = \frac{\pi}{8}, \frac{3}{8}\pi, \frac{5}{8}\pi \text{ SECONDS}}$

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[3]

c) i) $t = 6.2$

$3e^{-0.7 \times 6.2} = 0.0391095\ldots$

$\boxed{0.0391\ (3sf)}$

ii) AS t INCREASES $3e^{-0.7t}$ WILL DECREASE

SO FOR $t \geq 6.2$

$3e^{-0.7t} < 0.04$

GIVEN THAT $-1 \leq \cos(4t) \leq 1$

AT $t \geq 6.2$ THEN

$3e^{-0.7t} \cos(4t) \leq \pm 0.04$

SO LIFE JACKET WILL ALWAYS BE WITHIN 4cm OF SEA LEVEL AFTER 6.2s

Question 7

The number of daylight hours, h, in the UK, during a day d days after the spring equinox (the day in spring when the number of daylight hours is 12), is modelled using the function

$$h = 12 + \frac{9}{2}\sin\left(\frac{2\pi}{365}d\right)$$

(a) (i) Find the number of daylight hours during the day that is 100 days after the spring equinox.

(ii) Find the number of days after the spring equinox that the two days occur during which the number of daylight hours is closest to 9.

[5]

(b) For how many days of the year does the model suggest that the number of daylight hours exceeds 15 hours? Give your answer as a whole number of days.

[3]

a) i) $d = 100$

$h = 12 + \frac{9}{2}\sin\left(\frac{2\pi}{365}\cdot 100\right)$

$= 16.449\ldots$

$\boxed{16.4 \text{ HOURS}}$

ii) SET FUNCTION TO EQUAL 9 AND SOLVE

$12 + \frac{9}{2}\left(\sin \frac{2\pi}{365}d\right) = 9$

CAN USE GDC TO SOLVE

$\sin\left(\frac{2\pi}{365}d\right) = -3 \times \frac{2}{9} = -\frac{2}{3}$

$\sin^{-1}\left(-\frac{2}{3}\right) = -0.7297\ldots$

$\pi + 0.729\ldots \quad 2\pi - 0.729\ldots$

$3.871\ldots, \quad 5.553\ldots$

$\div \frac{2\pi}{365}$

$224.891\ldots, \quad 322.609\ldots$

CANNOT RELY JUST ON ROUNDING VALUES AS ONLY INTEGER VALUES OF d CAN BE USED

FOUR POSSIBLE DAYS = 224, 225, 322, 323

CHECK EACH VALUE IN FUNCTION TO FIND EXACT NUMBER OF HOURS

$h(224) = 9.052 \qquad h(225) = \underline{8.994}$

$h(322) = 8.965 \qquad h(323) = \underline{9.023}$

TWO DAYS CLOSEST TO 9 HOURS

$\boxed{225 \text{ AND } 323 \text{ DAYS}}$

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[3]

b) $12 + \frac{9}{2}\sin\left(\frac{2\pi}{365}d\right) = 15$

$\sin\left(\frac{2\pi}{365}d\right) = \frac{2}{3}$

CAN USE GDC TO SOLVE

$\sin^{-1}\left(\frac{2}{3}\right) = 0.7297...$

$\pi - 0.7297...$

$\frac{2\pi}{365}d = 0.7297..., \quad 2.4118...$

$\div \frac{2\pi}{365}$

$42.389... \quad 140.108...$

CANNOT RELY JUST ON ROUNDING VALUES AS ONLY INTEGER VALUES OF d CAN BE USED

SO DAYLIGHT WILL BE < 15 FOR 42
> 15 FOR 43

< 15 FOR 141
> 15 FOR 140

DAYLIGHT GREATER THAN 15 BETWEEN

43RD DAY AND 140TH DAY INCLUSIVE

$\boxed{98 \text{ DAYS}}$

BE CAREFUL NOT TO USE $140 - 43 = 97$ AS THIS IS NOT INCLUSIVE

Question 8

Felicity is a keen ice skater and has entered a competition that requires her to skate in a circular pathway in front of three judges. Her distance, d metres, away from the judges table, t seconds after commencing her routine can be modelled by the function

$$d = 12\cos\frac{\pi}{30}t + 15.$$

(a) (i) State the distance Felicity is away from the judges table at the start of her routine.

(ii) State the distance Felicity is away from the judges table after 15 seconds.

[3]

(b) Find, in terms of π, the circumference of Felicity's circular pathway on the ice rink.

[2]

(c) Find, in terms of π, Felicity's average speed for each lap on the ice rink.

[2]

Felicity's routine took three laps in total around the ice rink.

(d) Find the times during Felicity's routine where she was at a distance of 21 metres from the judges table.

[3]

a)

i) AT START OF ROUTINE $t = 0$

$d = 12\cos(0) + 15$

EXACT VALUE $\cos 0 = 1$

$d = 12 + 15 = 27$

$\boxed{d = 27\text{ m}}$

ii) $t = 15$

$d = 12\cos\left(\frac{15\pi}{30}\right) + 15$

$\frac{15\pi}{30} = \frac{\pi}{2}$

EXACT VALUE

$\cos\left(\frac{\pi}{2}\right) = 0$

$\boxed{d = 15\text{ m}}$

Felicity is a keen ice skater and has entered a competition that requires her to skate in a circular pathway in front of three judges. Her distance, d metres, away from the judges table, t seconds after commencing her routine can be modelled by the function

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b) LAP CIRCUMFERENCE $C = 2\pi r = \pi d$

CIRCLE DIAMETER $= d_{max} - d_{min}$

$-1 \leq \cos\frac{\pi}{30}t \leq 1$

$d_{max} = 12(1) + 15 = 27\ m$

$d_{min} = 12(-1) + 15 = 3\ m$

DIAMETER $= 27 - 3 = 24$

$C = 24\pi$

$\boxed{\text{CIRCUMFERENCE} = 24\pi\ m}$

Felicity is a keen ice skater and has entered a competition that requires her to skate in a circular pathway in front of three judges. Her distance, d metres, away from the judges table, t seconds after commencing her routine can be modelled by the function

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(b) Find, in terms of π, the circumference of Felicity's circular pathway on the ice rink.

$\boxed{\text{CIRCUMFERENCE} = 24\pi\ m}$

[2]

(c) Find, in terms of π, Felicity's average speed for each lap on the ice rink.

[2]

Felicity's routine took three laps in total around the ice rink.

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[3]

c) SPEED $= \dfrac{\text{DISTANCE}}{\text{TIME}}$

DISTANCE FROM (b) $= 24\pi$

TIME TAKEN FOR ONE LAP IS PERIOD OF FUNCTION

$2\pi = \frac{\pi}{30}t$ TIME = PERIOD $\Rightarrow \dfrac{2\pi}{|B|}$

$t = 2\cancel{\pi} \times \dfrac{30}{\cancel{\pi}} = 60$

SPEED $= \dfrac{24\pi}{60} = \dfrac{2\pi}{5}$

$\boxed{\text{SPEED} = \frac{2\pi}{5}\ m/s \text{ OR } ms^{-1}}$

Felicity is a keen ice skater and has entered a competition that requires her to skate in a circular pathway in front of three judges. Her distance, d metres, away from the judges table, t seconds after commencing her routine can be modelled by the function

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[3]

d) EACH LAP TAKES 60 SECONDS (PERIOD FROM c)

3 LAPS = 180 SECONDS

SOLVE FOR $d = 21$ $\quad 0 \leq t \leq 180$

$$12\cos\frac{\pi}{30}t + 15 = 21$$

$-15 \qquad -15$

$$12\cos\frac{\pi}{30}t = 6$$

$\div 12 \qquad \div 12$

$$\cos\frac{\pi}{30}t = \frac{1}{2}$$

EXACT VALUES $\cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$ OR $\cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$

$$\frac{\pi}{30}t = \frac{\pi}{3}, \frac{5\pi}{3}, \frac{7\pi}{3}, \frac{11\pi}{3}, \frac{13\pi}{3}, \frac{17\pi}{3}$$

$$t = \frac{\cancel{\pi}}{3} \times \frac{30}{\cancel{\pi}} = \frac{30}{3} = 10 \text{ SECONDS} \ldots$$

$$\boxed{t = 10s, 50s, 70s, 110s, 130s, 170s}$$