

# IB Maths: AA HL

## Transformations of Graphs

### Topic Questions

These practice questions can be used by students and teachers and is Suitable for IB Maths AA HL Topic Questions

Course	IB Maths
Section	2. Functions
Topic	2.6 Transformations of Graphs
Difficulty	Medium

**Level: IB Maths**

**Subject: IB Maths AA HL**

**Board: IB Maths**

**Topic: Transformations of Graphs**

### Question 1

The point  $P(-1, 4)$  lies on the curve with equation  $y = f(x)$ .

State the coordinates of the image of point  $P$  on the curves with the following equations:

- (i)  $y = f(x) + 3$
- (ii)  $y = f(x + 3)$
- (iii)  $y = 3f(x)$
- (iv)  $y = f(3x)$

[4 marks]

### Question 2

The point  $P(-3, -4)$  lies on the curve with equation  $y = f(x)$ .

State the coordinates of the image of point  $P$  on the curves with the following equations:

- (i)  $y = f(-x)$
- (ii)  $y = -f(x)$

[2 marks]

### Question 3

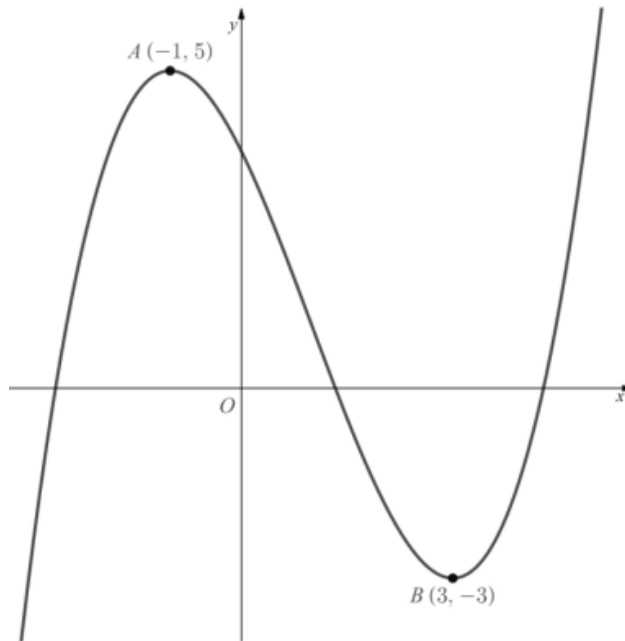
The point  $P(3, 2)$  lies on the curve with equation  $y = f(x)$ .

- (i) On the graph of  $y = f(x) + a$ , where  $a$  is a constant, the point  $P$  is mapped to the point  $(3, -5)$ . Determine the value of  $a$ .
- (ii) On the graph of  $y = f(x + b)$ , where  $b$  is a constant, the point  $P$  is mapped to the point  $(-1, 2)$ . Determine the value of  $b$ .
- (iii) On the graph of  $y = cf(x)$ , where  $c$  is a constant, the point  $P$  is mapped to the point  $(3, 1)$ . Determine the value of  $c$ .
- (iv) On the graph of  $y = f(dx)$ , where  $d$  is a constant, the point  $P$  is mapped to the point  $(1, 2)$ . Determine the value of  $d$ .

[4 marks]

### Question 4

The diagram below shows the graph of  $y = f(x)$ . The two marked points  $A(-1, 5)$  and  $B(3, -3)$  lie on the graph.



(a) In separate diagrams, sketch the curves with equation

(i)  $y = f(x - 1)$

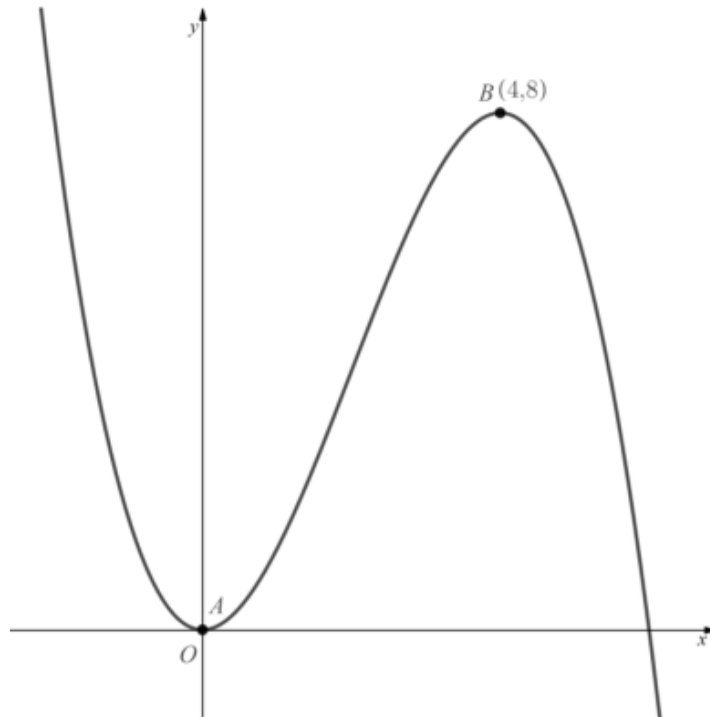
(ii)  $y = f(x) + 3$

On each diagram, give the coordinates of the images of points  $A$  and  $B$  under the given transformation.

[4 marks]

### Question 5

The diagram below shows the graph of  $y = f(x)$ . The marked point  $B(4, 8)$  lies on the graph, and the graph meets the origin at the marked point  $A$ .



(a) In separate diagrams, sketch the curves with equation

(i)  $y = -f(x)$

(ii)  $y = f(4x)$

On each diagram, give the coordinates of the images of points  $A$  and  $B$  under the given transformation.

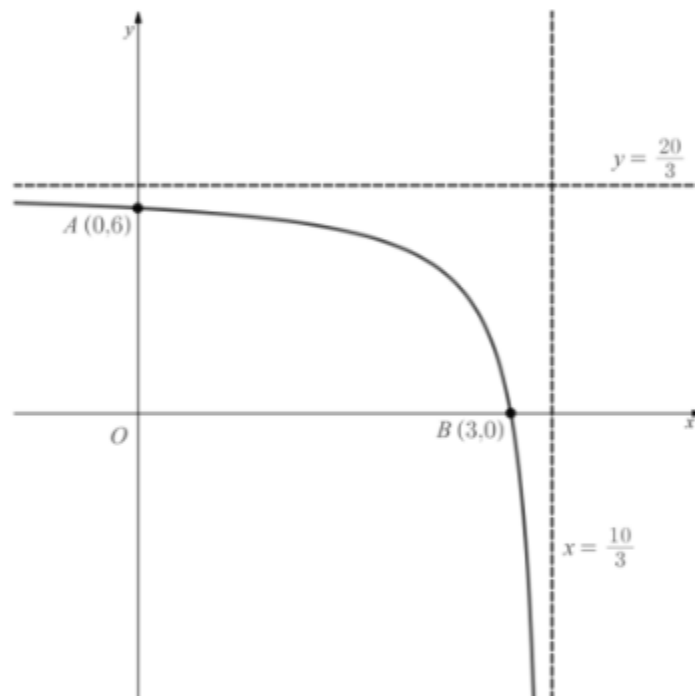
[4 marks]

(b) On the graph of  $y = af(x)$  the image of one of the two marked points has a  $y$  coordinate of 4. Find the value of  $a$ .

[2 marks]

### Question 6

The diagram below shows the graph of  $y = f(x)$ . The graph intersects the coordinate axes at the two marked points  $A(0, 6)$  and  $B(3, 0)$ . The graph has two asymptotes as shown, with equations  $y = \frac{20}{3}$  and  $x = \frac{10}{3}$ .



(a) In separate diagrams, sketch the curves with equation

(i)  $y = f(x) - 6$

(ii)  $y = f(-x)$

On each diagram, give the coordinates of the images of points  $A$  and  $B$  under the given transformation, as well as stating the equations of the transformed asymptotes.

[6 marks]

(b) The graph of  $y = f(x + a)$  has an asymptote at one of the coordinate axes. Find the value of  $a$ .

[2 marks]

### Question 7

Describe, in order, a sequence of transformations that maps the graph of  $y = f(x)$  onto the following graphs:

- (i)  $y = 3f(x + 2)$ ,
- (ii)  $y = f(-x) - 1$ .

[3 marks]

### Question 8

Given that  $f(x) = 3x^2 - 2x$  find an expression for  $g(x)$ , where  $g(x)$  is obtained by applying the following sequence of transformations to  $f(x)$ .

1. Translation by  $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$
2. Vertical stretch of scale factor 4
3. Translation by  $\begin{pmatrix} 0 \\ -3 \end{pmatrix}$

[4 marks]

### Question 9

- (a) (i) Sketch the graph of  $y = p(x)$ , where  $p(x) = 3x - 4$ .  
(ii) On the same set of axes, sketch the graph of  $y = p^{-1}(x)$ .  
Label the coordinates of the points where each graph crosses the coordinate axes.

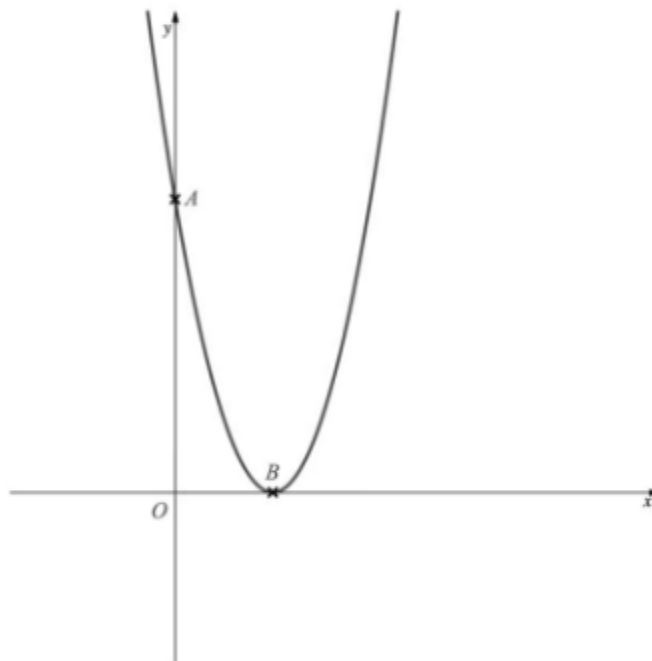
[4 marks]

- (b) (i) Find an expression for  $p^{-1}(x)$ .  
(ii) Find an expression for  $\frac{1}{9}[p(x) + 16]$ .  
(iii) What can you deduce about the sequence of transformations given by  $\frac{1}{9}[p(x) + 16]$ ?

[4 marks]

### Question 10

The equation  $y = f(x)$ , where  $f(x) = (x - a)^2$ , with  $a > 1$ , is shown below.



The points  $A$  and  $B$  are the points where the graph intercepts the coordinate axes.

- (a) Write down, in terms of  $a$ , the coordinates of  $A$  and  $B$ .

[2 marks]

(b) Sketch the graph of  $y = -f(-x)$ , labelling the images of the points  $A$  and  $B$  and stating their coordinates in terms of  $a$ .

[3 marks]

(c) Write down the value of  $a$  such that the point  $A$  is three times as far from the origin as the point  $B$ .

[1 mark]

### Question 11

The function  $f(x)$  is to be transformed by a sequence of functions, in the order detailed below:

1. A horizontal stretch by scale factor 2
2. A reflection in the  $x$ -axis
3. A translation by  $\begin{pmatrix} 0 \\ 2 \end{pmatrix}$

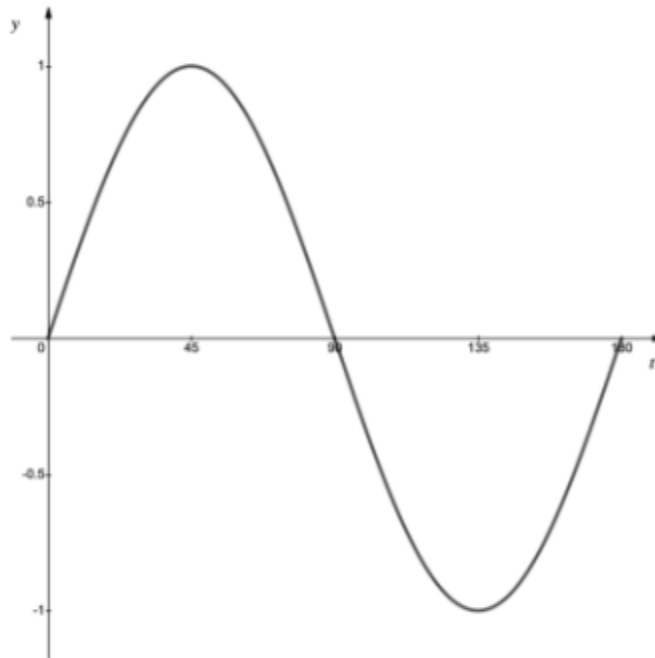
Write down an expression for the combined transformation in terms of  $f(x)$ .

[3 marks]



### Question 12

The diagram shows the graph of  $y = f(t)$ , where  $f(t) = \sin 2t$ ,  $0^\circ \leq x \leq 180^\circ$ .



- (a) (i) Write down the maximum value of  $y$  when  $y = 3f(t)$ .  
 (ii) Write down the first value of  $t$  for which this maximum occurs.

[2 marks]

- (b) (i) Write down the minimum value of  $y$  when  $y = 5f(t + 30^\circ)$ .  
 (ii) Write down the first value of  $t$  for which this minimum occurs.

[2 marks]

- (c) Find, in terms of  $f(t)$ , the combination of transformations that would map the graph of  $y = f(t)$  onto the graph of  $y = 2 + \sin t$ ,  $0^\circ \leq x \leq 180^\circ$ .

[2 marks]

### Question 13

Let  $f(x) = 3x^2 + 18x + 27$ .

(a) Write down the value of  $f(-3)$ .

[1 mark]

The function  $f$  can be written in the form of  $f(x) = a(x - h)^2 + k$ .

(b) Find the values of  $a$ ,  $h$  and  $k$ .

[3 marks]

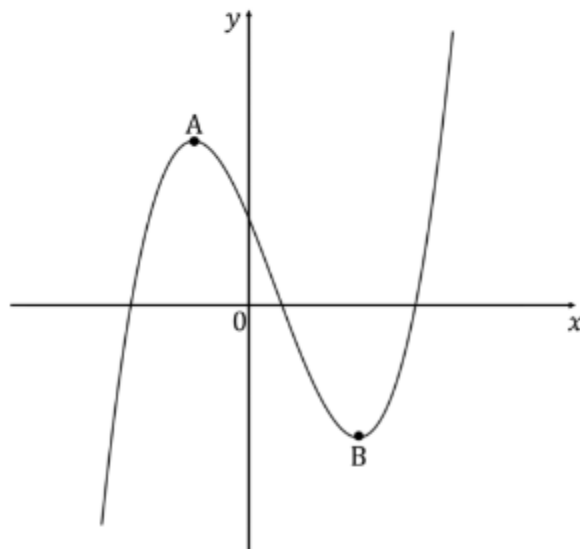
The graph of  $g$  is obtained from the graph of  $f$  by a reflection in the  $x$ -axis followed by a translation by the vector  $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ .

(c) Find  $g(x)$ , giving your answer in the form of  $g(x) = rx^2 + sx + t$ .

[4 marks]

### Question 14

The graph of  $f$  is shown below. The points  $A(-2, 10)$  and  $B(4, -10)$  lie on the curve.



Sketch the graph of:

(i)  $y = f(2x - 1)$ ,

(ii)  $y = f(4 - x)$ ,

Clearly indicate the new coordinates of the images of the points A and B.

[7 marks]

### Question 15

Describe a sequence of transformations that map the graph of  $y = \ln x$  onto the graph of  $y = 5 + \ln\left(\frac{1}{2}x + 4\right)$ .

[4 marks]

### Question 16

The function  $f$  is defined by

$$f(x) = \begin{cases} ax + 1 & \text{if } x \leq 7, \\ x^2 - 2x + 1 & \text{if } x > 7. \end{cases}$$

(a) Find the value of  $a$  such that the graph of  $f$  is continuous at  $x = 7$ .

[3 marks]

The graph of the function  $g$  is obtained by translating the graph of  $f$  by the vector  $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$ , followed by a reflection in the  $x$ -axis.

(b) Find  $g(x)$ .

[4 marks]

**Question 17**

Let  $f(x) = \frac{1}{x}$  and  $g(x) = \frac{x+1}{x-2}$ .

Explain fully the transformations of the graph of  $f$  to obtain the graph of  $g$ .

**[5 marks]**