Question 1

The point $P(-1, 4)$ lies on the curve with equation $y = \mathrm{f}(x)$.

State the coordinates of the image of point P on the curves with the following equations:

(i) $y = \mathrm{f}(x) + 3$
(ii) $y = \mathrm{f}(x + 3)$
(iii) $y = 3\mathrm{f}(x)$
(iv) $y = \mathrm{f}(3x)$

[4]

i) VERTICAL TRANSLATION y CHANGES +3

$\boxed{(-1, 7)}$

ii) HORIZONTAL TRANSLATION x CHANGES -3

$\boxed{(-4, 4)}$

iii) VERTICAL STRETCH y CHANGES $\times 3$

$\boxed{(-1, 12)}$

iv) HORIZONTAL STRETCH x CHANGES $\times \frac{1}{3}$ $\div 3$

$\boxed{(-\frac{1}{3}, 4)}$

Question 2

The point $P(-3, -4)$ lies on the curve with equation $y = \mathrm{f}(x)$.

State the coordinates of the image of point P on the curves with the following equations:

(i) $y = \mathrm{f}(-x)$
(ii) $y = -\mathrm{f}(x)$

[2]

i) HORIZONTAL REFLECTION y AXIS x CHANGES

$\boxed{(3, -4)}$

ii) VERTICAL REFLECTION x AXIS y CHANGES

$\boxed{(-3, 4)}$

Question 3

The point $P(3, 2)$ lies on the curve with equation $y = \mathrm{f}(x)$.

(i) On the graph of $y = \mathrm{f}(x) + a$, where a is a constant, the point P is mapped to the point $(3, -5)$. Determine the value of a.

(ii) On the graph of $y = \mathrm{f}(x + b)$, where b is a constant, the point P is mapped to the point $(-1, 2)$. Determine the value of b.

(iii) On the graph of $y = c\mathrm{f}(x)$, where c is a constant, the point P is mapped to the point $(3, 1)$. Determine the value of c.

(iv) On the graph of $y = \mathrm{f}(dx)$, where d is a constant, the point P is mapped to the point $(1, 2)$. Determine the value of d.

[4]

i) VERTICAL TRANSLATION y CHANGED $+a$

$\underbrace{(3,2) \Rightarrow (3,-5)}_{+a}$ $\boxed{a = -7}$

ii) HORIZONTAL TRANSLATION x CHANGED $-b$

$\underbrace{(3,2) \Rightarrow (-1,2)}_{-b}$ $\boxed{b = 4}$

iii) VERTICAL STRETCH y CHANGED $\times c$

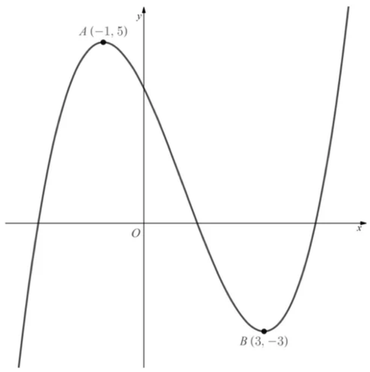
$\underbrace{(3,2) \Rightarrow (3,1)}_{\times c}$ $\boxed{c = \frac{1}{2}}$

iv) HORIZONTAL STRETCH x CHANGED $\times \frac{1}{d}$

$\underbrace{(3,2) \Rightarrow (1,2)}_{\times \frac{1}{d}}$ $\boxed{d = 3}$

Question 4

The diagram below shows the graph of $y = \mathrm{f}(x)$. The two marked points $A(-1, 5)$ and $B(3, -3)$ lie on the graph.



(a) In separate diagrams, sketch the curves with equation TRANSLATIONS

(i) $y = \mathrm{f}(x - 1)$ HORIZONTAL x CHANGES +1

(ii) $y = \mathrm{f}(x) + 3$ VERTICAL y CHANGES +3

On each diagram, give the coordinates of the images of points A and B under the given transformation.

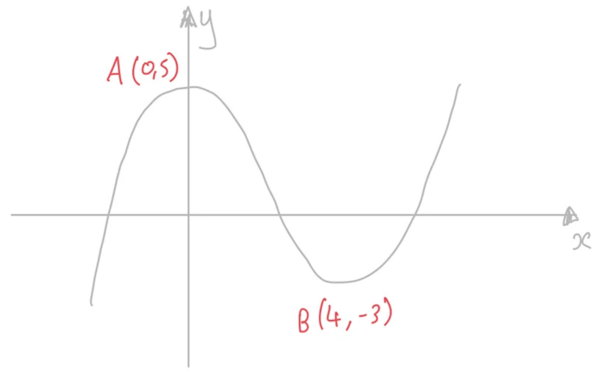
[4]

(b) On the graph of $y = \mathrm{f}(x + a)$ the image of one of the two marked points has an x coordinate of 2. Find the two possible values of a.

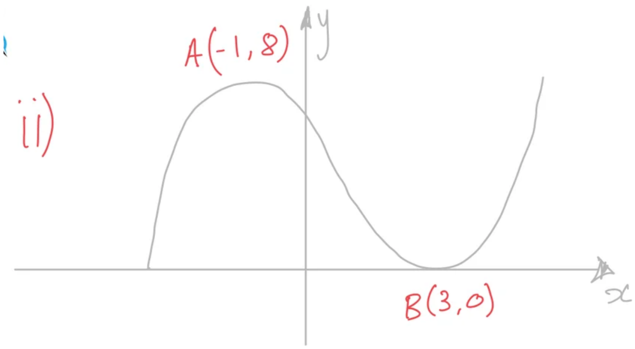
[2]

a)

i)

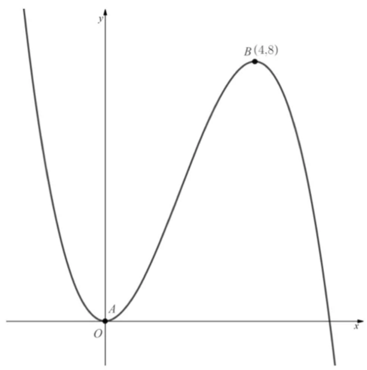


ii)



Question 5

The diagram below shows the graph of $y = \mathrm{f}(x)$. The marked point $B(4, 8)$ lies on the graph, and the graph meets the origin at the marked point A.



(a) In separate diagrams, sketch the curves with equation

(i) $y = -\mathrm{f}(x)$ VERTICAL REFLECTION y CHANGES (x AXIS)

(ii) $y = \mathrm{f}(4x)$ HORIZONTAL STRETCH x CHANGES $\times \frac{1}{4}$

On each diagram, give the coordinates of the images of points A and B under the given transformation.

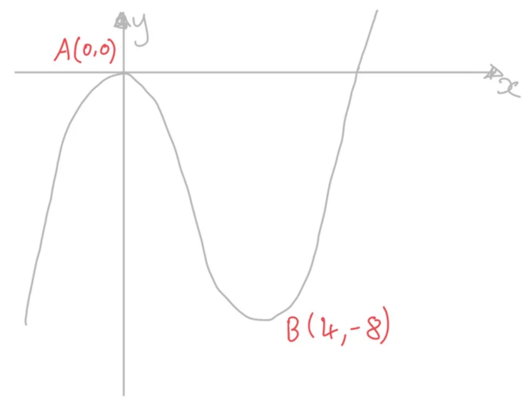
[4]

(b) On the graph of $y = a\mathrm{f}(x)$ the image of one of the two marked points has a y coordinate of 4. Find the value of a.

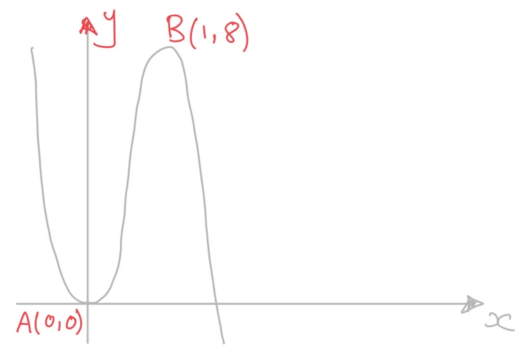
[2]

a)

i)



ii)



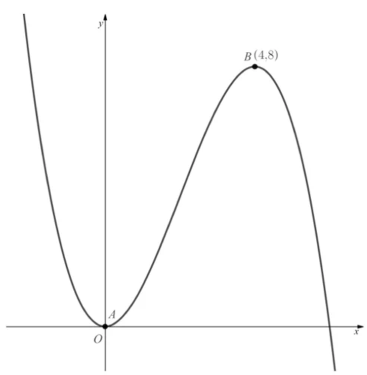
b) VERTICAL STRETCH y CHANGES $\times a$

$A = (0, 0)$ WONT CHANGE

$B = (4, 8) \xrightarrow{\times a} (4, 4)$

$\boxed{a = \frac{1}{2}}$

The diagram below shows the graph of $y = \mathrm{f}(x)$. The marked point $B(4, 8)$ lies on the graph, and the graph meets the origin at the marked point A.



(a) In separate diagrams, sketch the curves with equation

(i) $y = -\mathrm{f}(x)$

(ii) $y = \mathrm{f}(4x)$

On each diagram, give the coordinates of the images of points A and B under the given transformation.

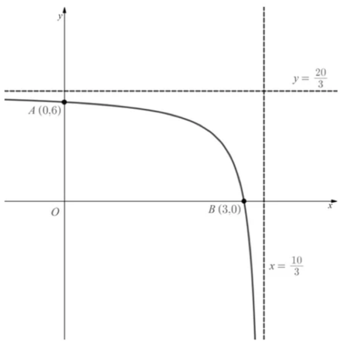
[4]

(b) On the graph of $y = a\mathrm{f}(x)$ the image of one of the two marked points has a y coordinate of 4. Find the value of a.

[2]

Question 6

The diagram below shows the graph of $y = \mathrm{f}(x)$. The graph intersects the coordinate axes at the two marked points $A(0, 6)$ and $B(3, 0)$. The graph has two asymptotes as shown, with equations $y = \frac{20}{3}$ and $x = \frac{10}{3}$.



(a) In separate diagrams, sketch the curves with equation

(i) $y = \mathrm{f}(x) - 6$ VERTICAL TRANSLATION y CHANGES − 6

(ii) $y = \mathrm{f}(-x)$ HORIZONTAL REFLECTION x CHANGES (y AXIS)

On each diagram, give the coordinates of the images of points A and B under the given transformation, as well as stating the equations of the transformed asymptotes.

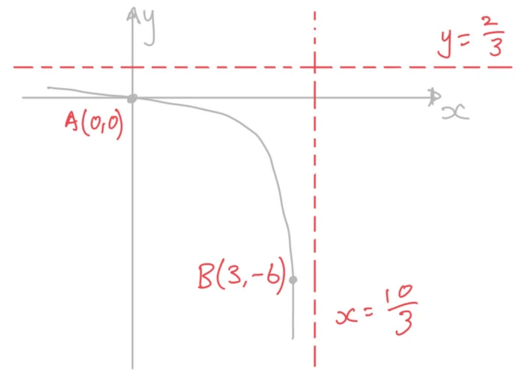
[6]

(b) The graph of $y = \mathrm{f}(x + a)$ has an asymptote at one of the coordinate axes. Find the value of a.

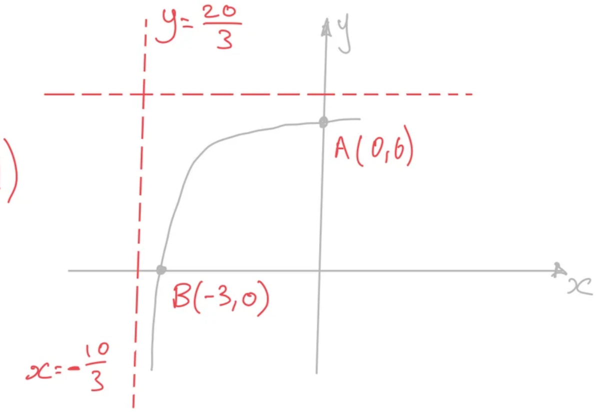
[2]

a)

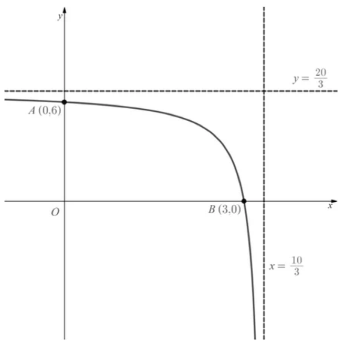
i)



ii)



The diagram below shows the graph of $y = \mathrm{f}(x)$. The graph intersects the coordinate axes at the two marked points $A(0, 6)$ and $B(3, 0)$. The graph has two asymptotes as shown, with equations $y = \frac{20}{3}$ and $x = \frac{10}{3}$.



(a) In separate diagrams, sketch the curves with equation

(i) $y = \mathrm{f}(x) - 6$

(ii) $y = \mathrm{f}(-x)$

On each diagram, give the coordinates of the images of points A and B under the given transformation, as well as stating the equations of the transformed asymptotes.

[6]

(b) The graph of $y = \mathrm{f}(x + a)$ has an asymptote at one of the coordinate axes. Find the value of a.

[2]

b) HORIZONTAL TRANSLATION CHANGES $-a$

$x = \frac{10}{3} \xrightarrow{-\frac{10}{3}} x = 0$

$\boxed{a = \frac{10}{3}}$

Question 7

Describe, in order, a sequence of transformations that maps the graph of $y = \mathrm{f}(x)$ onto the following graphs:

(i) $y = 3\mathrm{f}(x + 2)$,

(ii) $y = \mathrm{f}(-x) - 1$.

[3]

DEAL WITH INSIDE BRACKETS FIRST

INSIDE = HORIZONTAL

OUTSIDE = VERTICAL

i) TRANSLATION BY $\begin{pmatrix} -2 \\ 0 \end{pmatrix}$

VERTICAL STRETCH SF. 3

ii) REFLECTION IN y AXIS (OR $x = 0$)

TRANSLATION BY $\begin{pmatrix} 0 \\ -1 \end{pmatrix}$

Question 8

Given that $f(x) = 3x^2 - 2x$ find an expression for g(x), where g(x) is obtained by applying the following sequence of transformations to f(x).

1. Translation by $\binom{2}{0}$
2. Vertical stretch of scale factor 4
3. Translation by $\binom{0}{-3}$

[4]

1. $f(x) \Rightarrow f(x-2)$

$3x^2 - 2x \Rightarrow 3(x-2)^2 - 2(x-2)$

$3(x^2-4x+4) - 2x + 4$

$3x^2 - 14x + 16$

2. $f(x-2) \Rightarrow 4f(x-2)$

$4(3x^2 - 14x + 16)$

$12x^2 - 56x + 64$

3. $4f(x-2) \Rightarrow 4f(x-2) - 3$

$12x^2 - 56x + 64 - 3$

$\boxed{g(x) = 12x^2 - 56x + 61}$

Question 9

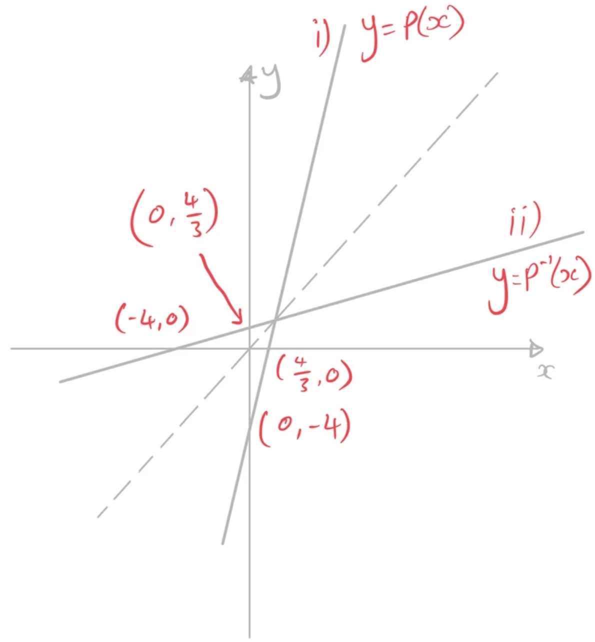
(a) (i) Sketch the graph of $y = \mathrm{p}(x)$, where $\mathrm{p}(x) = 3x - 4$.
(ii) On the same set of axes, sketch the graph of $y = p^{-1}(x)$.
Label the coordinates of the points where each graph crosses the coordinate axes.

[4]

(b) (i) Find an expression for $\mathrm{p}^{-1}(x)$.
(ii) Find an expression for $\frac{1}{9}[\mathrm{p}(x) + 16]$.
(iii) What can you deduce about the sequence of transformations given by $\frac{1}{9}[\mathrm{p}(x) + 16]$?

[4]

a)



INVERSE = REFLECTION IN $y = x$

(a) (i) Sketch the graph of $y = \mathrm{p}(x)$, where $\mathrm{p}(x) = 3x - 4$.
(ii) On the same set of axes, sketch the graph of $y = p^{-1}(x)$.
Label the coordinates of the points where each graph crosses the coordinate axes.

[4]

(b) (i) Find an expression for $\mathrm{p}^{-1}(x)$.
(ii) Find an expression for $\frac{1}{9}[\mathrm{p}(x) + 16]$.
(iii) What can you deduce about the sequence of transformations given by $\frac{1}{9}[\mathrm{p}(x) + 16]$?

[4]

b) i) LET $y = 3x - 4$ REARRANGE

$y + 4 = 3x$

$\frac{y+4}{3} = x$

$\boxed{p^{-1}(x) = \frac{x+4}{3} \text{ OR } p^{-1}(x) = \frac{1}{3}(x+4)}$

ii)

$\frac{1}{9}\left[(3x-4) + 16\right]$

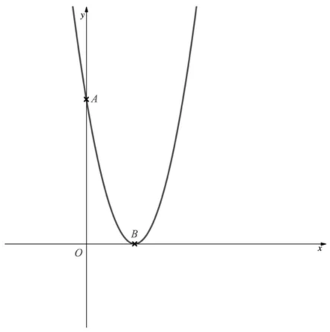
$\frac{1}{9}(3x + 12)$

$\frac{1}{3}x + \frac{4}{3} = \boxed{\frac{1}{3}(x+4)}$

iii) $\boxed{\frac{1}{9}\left[p(x) + 16\right] = p^{-1}(x)}$

Question 10

The equation $y = \mathrm{f}(x)$, where $\mathrm{f}(x) = (x - a)^2$, with $a > 1$, is shown below.



The points A and B are the points where the graph intercepts the coordinate axes.

(a) Write down, in terms of a, the coordinates of A and B.

[2]

(b) Sketch the graph of $y = -\mathrm{f}(-x)$, labelling the images of the points A and B and stating their coordinates in terms of a.

[3]

(c) Write down the value of a such that the point A is three times as far from the origin as the point B.

[1]

a) BOTH POINTS ON AXES SO

$y = 0$ OR $x = 0$

A $x = 0$ $\quad f(x) = (0 - a)^2$

$(-a)^2$

$y = a^2$

$\boxed{A = (0, a^2)}$

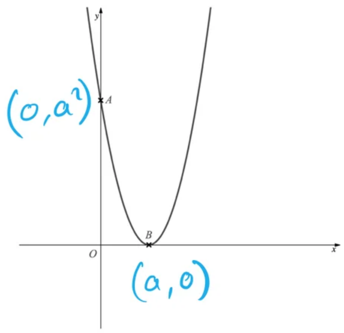
B $y = 0$ $\quad 0 = (x - a)^2$

$x - a = 0$

$x = a$

$\boxed{B = (a, 0)}$

The equation $y = \mathrm{f}(x)$, where $\mathrm{f}(x) = (x - a)^2$, with $a > 1$, is shown below.



The points A and B are the points where the graph intercepts the coordinate axes.

(a) Write down, in terms of a, the coordinates of A and B.

[2]

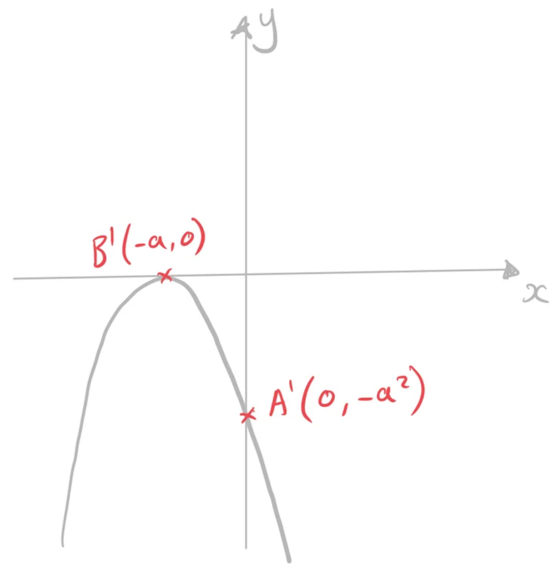
(b) Sketch the graph of $y = -\mathrm{f}(-x)$, labelling the images of the points A and B and stating their coordinates in terms of a.

[3]

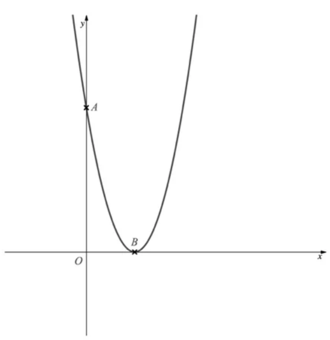
(c) Write down the value of a such that the point A is three times as far from the origin as the point B.

[1]

b) $f(-x)$ REFLECTING y $\quad -f(x)$ REFLECT IN x



The equation $y = \mathrm{f}(x)$, where $\mathrm{f}(x) = (x - a)^2$, with $a > 1$, is shown below.



The points A and B are the points where the graph intercepts the coordinate axes.

(a) Write down, in terms of a, the coordinates of A and B.

[2]

(b) Sketch the graph of $y = -\mathrm{f}(-x)$, labelling the images of the points A and B and stating their coordinates in terms of a.

[3]

(c) Write down the value of a such that the point A is three times as far from the origin as the point B.

[1]

c) BOTH ON AXES SO HORIZONTAL AND VERTICAL DISTANCES FROM ORIGIN

$$A = 3B$$

$$(0, a^2) = (a, 0)$$

$$a^2 = 3a$$

$$\boxed{a = 3}$$

Question 11

The function $f(x)$ is to be transformed by a sequence of functions, in the order detailed below:

1. A horizontal stretch by scale factor 2
2. A reflection in the x-axis
3. A translation by $\binom{0}{2}$

Write down an expression for the combined transformation in terms of $f(x)$.

[3]

1. HORIZONTAL = INSIDE BRACKETS
(SCALE FACTOR = $\frac{1}{a}$)

STRETCH = MULTIPLY

$$f\left(\frac{1}{2}x\right)$$

2. VERTICAL REFLECTION = OUTSIDE
(NEGATIVE)

$$-f\left(\frac{1}{2}x\right)$$

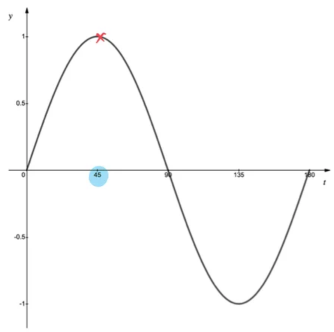
3. VERTICAL TRANSLATION = OUTSIDE

$$-f\left(\frac{1}{2}x\right)+2$$

$$\boxed{2-f\left(\frac{1}{2}x\right)}$$

Question 12

The diagram shows the graph of $y = \mathrm{f}(t)$, where $\mathrm{f}(t) = \sin 2t$, $0° \leq x \leq 180°$.



(a) (i) Write down the maximum value of y when $y = 3\mathrm{f}(t)$.
(ii) Write down the first value of t for which this maximum occurs.

[2]

(b) (i) Write down the minimum value of y when $y = 5\mathrm{f}(t + 30°)$.
(ii) Write down the first value of t for which this minimum occurs.

[2]

(c) Find, in terms of $\mathrm{f}(t)$, the combination of transformations that would map the graph of $y = \mathrm{f}(t)$ onto the graph of $y = 2 + \sin t$, $0° \leq x \leq 180°$.

[2]

a)

i) VERTICAL STRETCH SF 3

MAX OF $\sin 2t = 1$

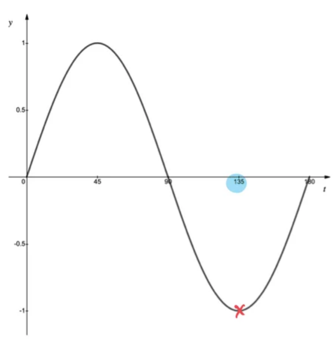
MAX $3(\sin 2t) = 3$

$\boxed{y = 3}$

ii) VERTICAL STRETCH WON'T AFFECT HORIZONTAL VALUE

$\boxed{t = 45°}$

The diagram shows the graph of $y = \mathrm{f}(t)$, where $\mathrm{f}(t) = \sin 2t$, $0° \leq x \leq 180°$.



(a) (i) Write down the maximum value of y when $y = 3\mathrm{f}(t)$.
(ii) Write down the first value of t for which this maximum occurs.

[2]

(b) (i) Write down the minimum value of y when $y = 5\mathrm{f}(t + 30°)$.
(ii) Write down the first value of t for which this minimum occurs.

[2]

(c) Find, in terms of $\mathrm{f}(t)$, the combination of transformations that would map the graph of $y = \mathrm{f}(t)$ onto the graph of $y = 2 + \sin t$, $0° \leq x \leq 180°$.

[2]

b) TRANSLATION $\begin{pmatrix} -30 \\ 0 \end{pmatrix}$ ONLY AFFECTS t

VERTICAL STRETCH SF5 ONLY AFFECTS y

i) MIN $\sin 2t = -1$

MIN $5(\sin 2t + 30) = -5$

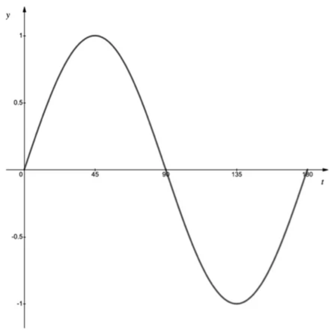
$\boxed{y = -5}$

ii) TRANSLATION $\begin{pmatrix} -30 \\ 0 \end{pmatrix}$

$t = 135 - 30$

$\boxed{t = 105°}$

The diagram shows the graph of $y = \mathrm{f}(t)$, where $\mathrm{f}(t) = \sin 2t$, $0° \leq x \leq 180°$.



(a) (i) Write down the maximum value of y when $y = 3\mathrm{f}(t)$.
(ii) Write down the first value of t for which this maximum occurs.

[2]

(b) (i) Write down the minimum value of y when $y = 5\mathrm{f}(t + 30°)$.
(ii) Write down the first value of t for which this minimum occurs.

[2]

(c) Find, in terms of $\mathrm{f}(t)$, the combination of transformations that would map the graph of $y = \mathrm{f}(t)$ onto the graph of $y = 2 + \sin t$, $0° \leq x \leq 180°$.

[2]

c) $f(t) = \sin 2t$

$f(\frac{1}{2}t) = \sin t$

$f(\frac{1}{2}t) + 2 = 2 + \sin t$

$\boxed{f(\frac{1}{2}t) + 2}$

Question 13

Let $f(x) = 3x^2 + 18x + 27$.

(a) Write down the value of $f(-3)$.

[1]

a) SUB IN $x = -3$

$3(-3)^2 + 18(-3) + 27$

$27 - 54 + 27 = 0$

$\boxed{f(-3) = 0}$

The function f can be written in the form of $f(x) = a(x - h)^2 + k$.

(b) Find the values of a, h and k.

[3]

The graph of g is obtained from the graph of f by a reflection in the x-axis followed by a translation by the vector $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$.

(c) Find $g(x)$, giving your answer in the form of $g(x) = rx^2 + sx + t$.

[4]

Let $f(x) = 3x^2 + 18x + 27$.

(a) Write down the value of $f(-3)$. [1]

The function f can be written in the form of $f(x) = a(x-h)^2 + k$.

(b) Find the values of a, h and k. [3]

The graph of g is obtained from the graph of f by a reflection in the x-axis followed by a translation by the vector $\binom{0}{1}$.

(c) Find $g(x)$, giving your answer in the form of $g(x) = rx^2 + sx + t$. [4]

b) FACTORISE 3 FROM x TERMS

$f(x) = 3(x^2 + 6x) + 27$

COMPLETE THE SQUARE ON $x^2 + 6x$

$f(x) = 3[(x+3)^2 - 3^2] + 27$

$f(x) = 3[(x+3)^2 - 9] + 27$

EXPAND AND SIMPLIFY INTO FORM $a(x-h)^2 + k$

$f(x) = 3(x+3)^2 \cancel{-27} \cancel{+27}$

$f(x) = 3(x+3)^2$

$\boxed{a = 3 \quad h = -3 \quad k = 0}$

ALTERNATE METHOD EXPAND $a(x-h)^2 + k$

$a(x-h)^2 + k = ax^2 - 2ahx + ah^2 + k$

COMPARE COEFFICIENTS TO $3x^2 + 18x + 27$

$ax^2 = 3x^2 \Rightarrow$ $\boxed{a = 3}$

$-2ahx = 18x \Rightarrow -2(3)h = 18 \Rightarrow$ $\boxed{h = -3}$

$ah^2 + k = 27 \Rightarrow 3(-3)^2 + k = 27 \Rightarrow$ $\boxed{k = 0}$

Let $f(x) = 3x^2 + 18x + 27$.

(a) Write down the value of $f(-3)$. [1]

The function f can be written in the form of $f(x) = a(x-h)^2 + k$.

(b) Find the values of a, h and k. [3]

The graph of g is obtained from the graph of f by a reflection in the x-axis followed by a translation by the vector $\binom{0}{1}$.

(c) Find $g(x)$, giving your answer in the form of $g(x) = rx^2 + sx + t$. [4]

c) REFLECTION OF $f(x)$ IN x-AXIS : $-f(x)$

TRANSLATION BY $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$: $-f(x) + 1$

$g(x) = -f(x) + 1$

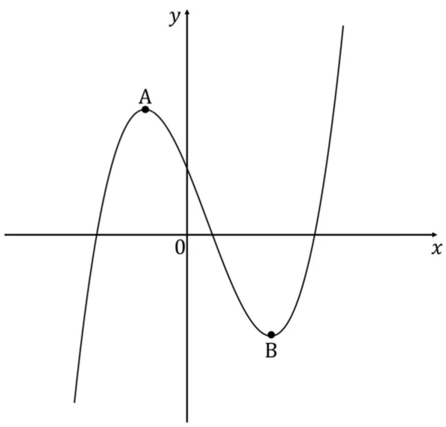
$g(x) = -[3x^2 + 18x + 27) + 1$

$= -3x^2 - 18x - 27 + 1$

$\boxed{g(x) = -3x^2 - 18x - 26}$

Question 14

The graph of f is shown below. The points $A(-2, 10)$ and $B(4, -10)$ lie on the curve.



(a) Sketch the graph of:

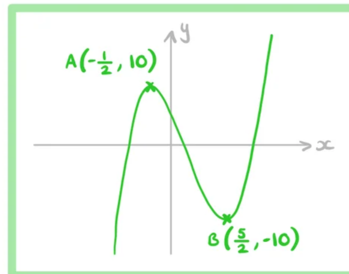
(i) $y = f(2x - 1)$,

(ii) $y = f(4 - x)$,

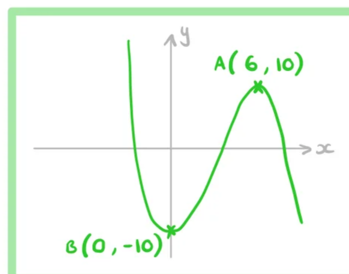
Clearly indicate the new coordinates of the images of the points A and B.

[7]

(a) (i) Translation $(1, 0)$ followed by horizontal stretch of factor ½



(ii) Translation of $(-3, 0)$ followed by reflection in the y-axis



Question 15

Describe a sequence of transformations that map the graph of $y = \ln x$ onto the graph of $y = 5 + \ln\left(\frac{1}{2}x + 4\right)$.

[4]

$\ln x \rightarrow 5 + \ln\left(\frac{1}{2}x + 4\right)$

$f(x) \rightarrow f\left(\frac{1}{2}x + 4\right) + 5$

Translation of $\begin{pmatrix} -4 \\ 5 \end{pmatrix}$ followed by horizontal stretch of factor 2

OR

Horizontal stretch of factor 2 followed by translation $\begin{pmatrix} -8 \\ 5 \end{pmatrix}$

Question 16

The function f is defined by

$$f(x)=\begin{cases} ax+1 & \text{if } x\le 7,\\ x^2-2x+1 & \text{if } x>7.\end{cases}$$

(a) Find the value of a such that the graph of f is continuous at $x=7$.

[3]

(a) $f(7)$ must have the same value for both parts of the piecewise function if it is to be continuous at $x=7$

$$a(7)+1=(7)^2-2(7)+1$$

$$7a=35$$

$$\boxed{a=5}$$

The graph of the function g is obtained by translating the graph of f by the vector $\binom{1}{-1}$, followed by a reflection in the x-axis.

(b) Find $g(x)$.

[4]

The function f is defined by

$$f(x)=\begin{cases} ax+1 & \text{if } x\le 7,\\ x^2-2x+1 & \text{if } x>7.\end{cases}$$

(a) Find the value of a such that the graph of f is continuous at $x=7$.

$$\boxed{a=5}$$

[3]

The graph of the function g is obtained by translating the graph of f by the vector $\binom{1}{-1}$, followed by a reflection in the x-axis.

(b) Find $g(x)$.

[4]

(b) Translation of $f(x)$ by $\binom{1}{-1}$

$$f(x)\to f(x-1)-1$$

Now 8 because of the translation

$$f(x-1)-1=\begin{cases}5(x-1)+1-1 & x\le 8\\ (x-1)^2-2(x-1)+1-1 & x>8\end{cases}$$

$$f(x-1)-1=\begin{cases}5x-5 & x\le 8\\ x^2-4x+3 & x>8\end{cases}$$

Reflection in the x-axis

$$f(x-1)-1\to-(f(x-1)-1)$$

$$g(x)=\begin{cases}-(5x-5) & x\le 8\\ -(x^2-4x+3) & x>8\end{cases}$$

$$\boxed{g(x)=\begin{cases}5-5x & x\le 8\\ -x^2+4x-3 & x>8\end{cases}}$$

Question 17

Let $f(x) = \frac{1}{x}$ and $g(x) = \frac{x+1}{x-2}$.

Explain fully, the transformations of the graph of f to obtain the graph of g.

[5]

Rewrite $g(x)$ so that x appears just in the denominator

$x + 1 = (x - 2) + 3$

$$\Rightarrow \frac{x+1}{x-2} = \frac{(x-2)+3}{x-2}$$

$$= \cancel{\frac{x-2}{x-2}} + \frac{3}{x-2}$$

$$= 1 + \frac{3}{x-2}$$

$$\Rightarrow f(x) \to g(x)$$

$$\frac{1}{x} \to 1 + \frac{3}{x-2}$$

Translation of $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$, then a stretch with SF=3 in y direction followed by a translation $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$