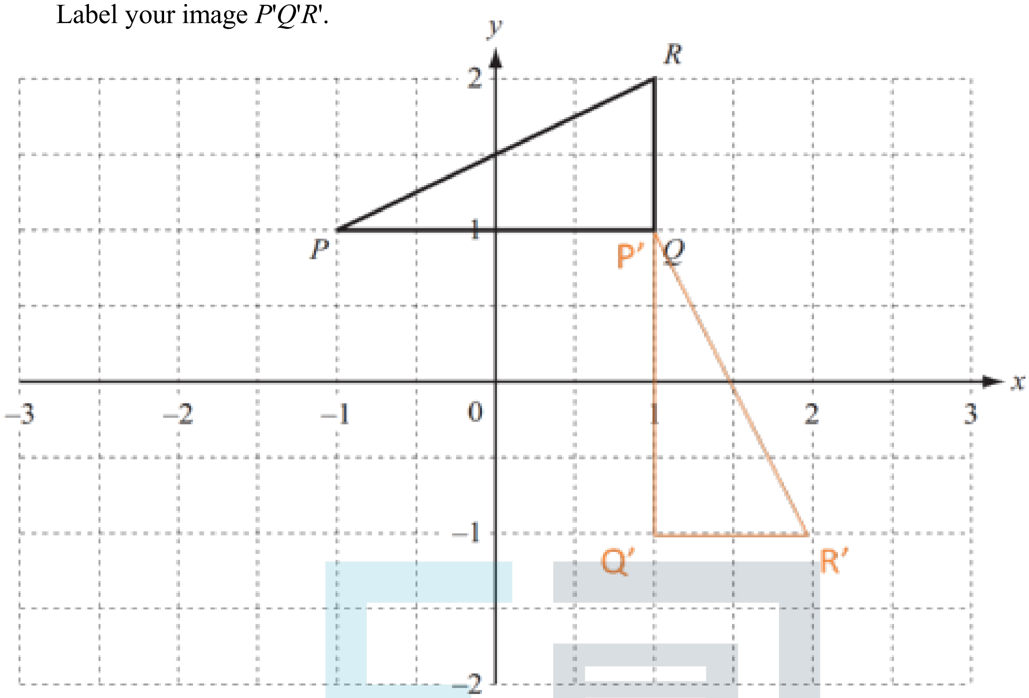
EXAM PAPERS PRACTICE

Transformations

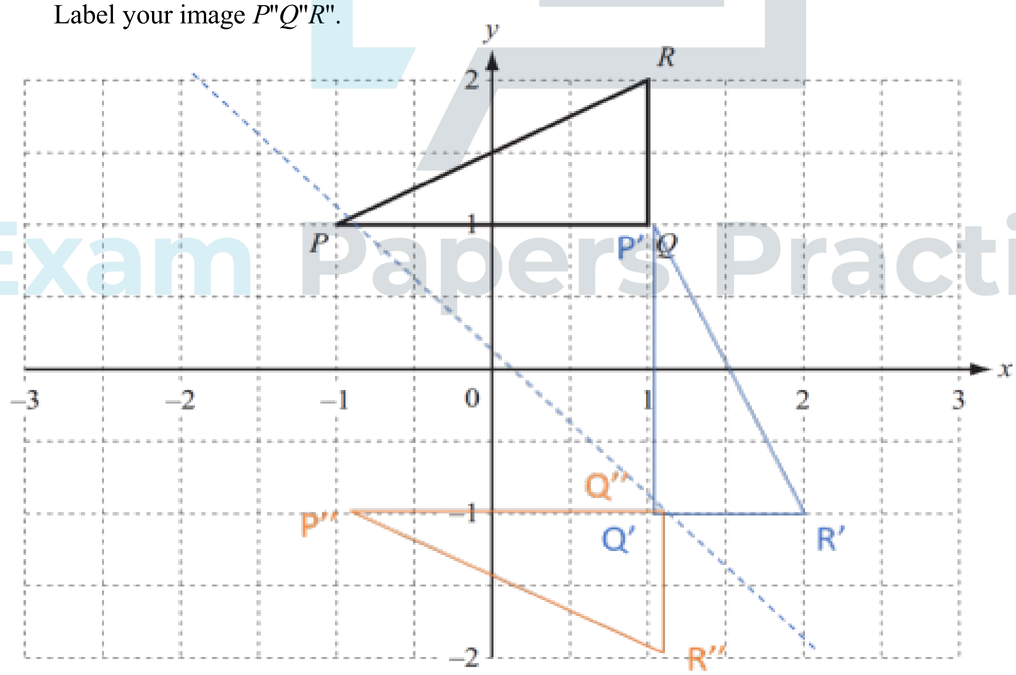
Model Answer

The triangle *PQR* has co-ordinates *P*(-1, 1), *Q*(1, 1) and *R*(1, 2).

(a) Rotate triangle *PQR* by 90° clockwise about (0, 0).
Label your image *P'Q'R'*. [2]

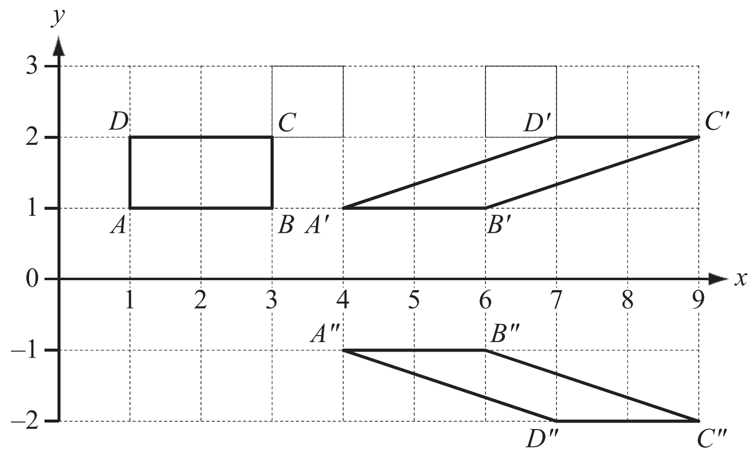


(b) Reflect your triangle *P'Q'R'* in the line $y = -x$.
Label your image *P''Q''R''*. [2]



(c) Describe fully the single transformation which maps triangle *PQR* onto triangle *P''Q''R''*. [2]

A reflection in the line $y = 0$ (the x-axis)



(a) Describe the single transformation which maps *ABCD* onto *A′ B′ C′ D′*. [3]

It is a shear of scale factor **3** with **x**-axis invariant.

(b) A single transformation maps *A′ B′ C′ D′* onto *A″ B″ C″ D″*.
Find the matrix which represents this transformation.

It is a reflection in the x-axis, i.e.

$$\begin{pmatrix} x \\ y \end{pmatrix} \to \begin{pmatrix} x \\ -y \end{pmatrix}$$

This is represented by

$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

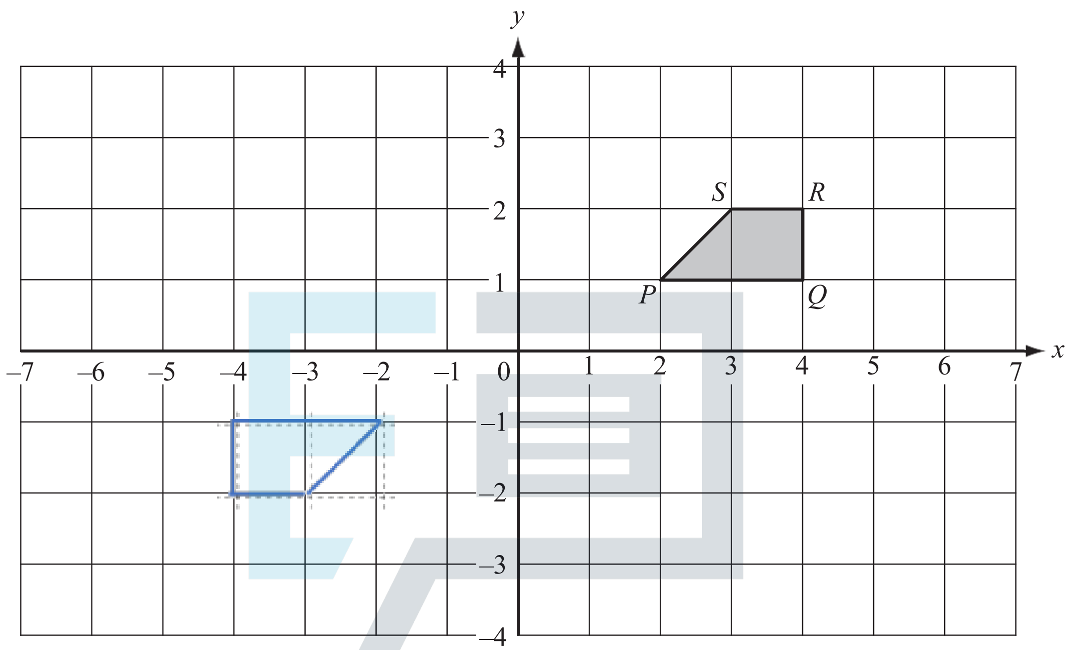
[2]

We can see this by

$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ -y \end{pmatrix}$$

$$\mathbf{A} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \mathbf{B} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$$

On the grid on the next page, draw the image of *PQRS* after the transformation represented by **BA**.



[5]

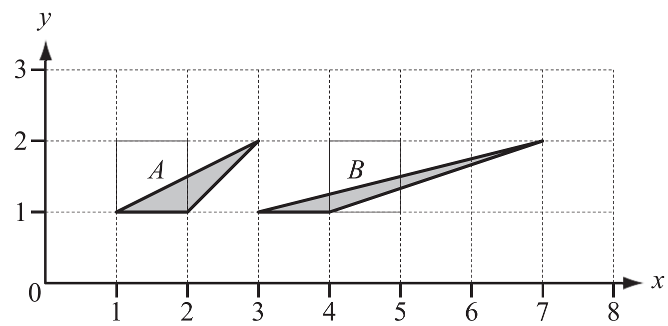
We apply A first, so we have

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} y \\ x \end{pmatrix}$$

A reflection in the line $y = x$. Now apply B

$$\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} y \\ x \end{pmatrix} = \begin{pmatrix} -x \\ -y \end{pmatrix}$$

From the original shape this is a reflection in x-axis and a reflection in the y-axis (drawn in blue).



(a) Describe fully the single transformation that maps triangle *A* onto triangle *B*.

Shear, scale factor 2, x axis invariant. [3]

(b) Find the 2 × 2 matrix which represents this transformation. [2]

We require

$$\begin{pmatrix} x \\ y \end{pmatrix} \to \begin{pmatrix} x + 2y \\ y \end{pmatrix}$$

This is done using the matrix

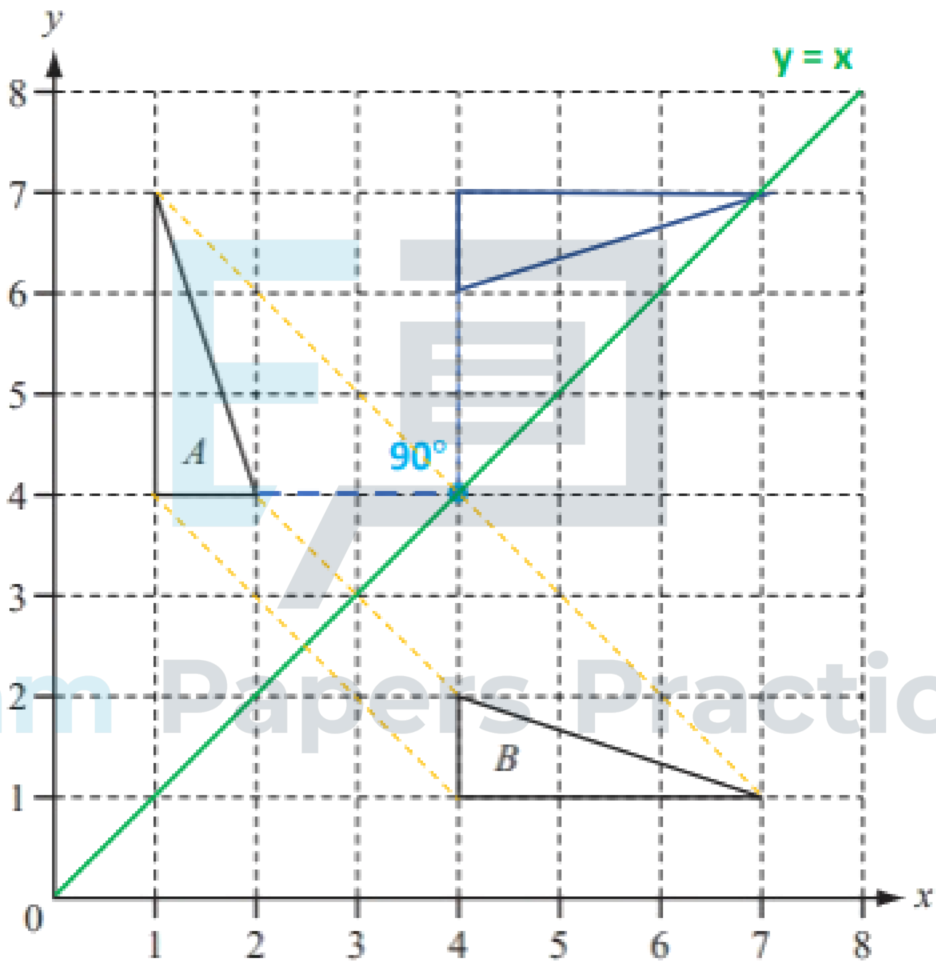
$$\begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$$

(a) Describe fully the **single** transformation which maps triangle *A* onto triangle *B*.

The single transformation which maps triangle A onto triangle B is a reflection with the mirror line the line of equation y = x, represented on the figure below in green. The distance from each point of the object and corresponding point of the image to the mirror line is equal. The distances are also perpendicular on the mirror line.

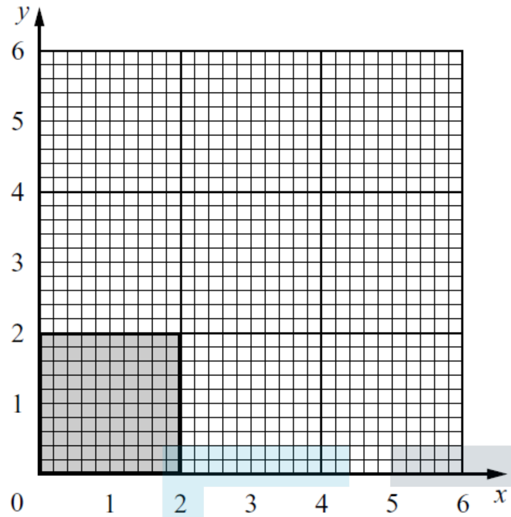
(b) On the grid, draw the image of triangle *A* after rotation by 90° clockwise about the point (4, 4).

[2]



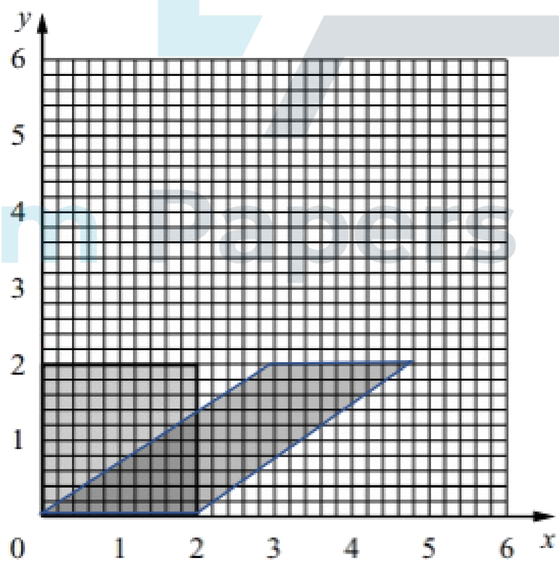
[2]

(a)



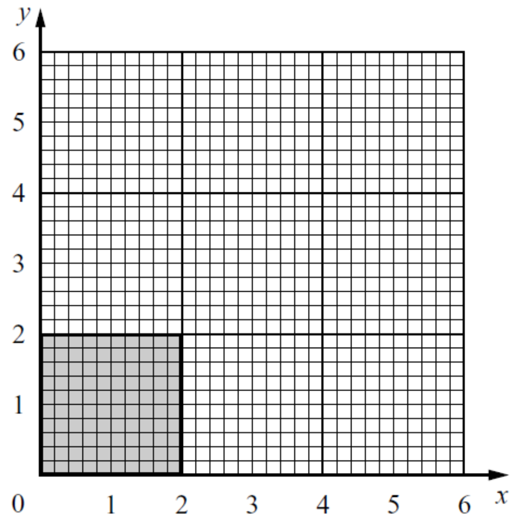
Draw the shear of the shaded square with the x-axis invariant and the point (0, 2) mapping onto the point (3, 2).

[2]



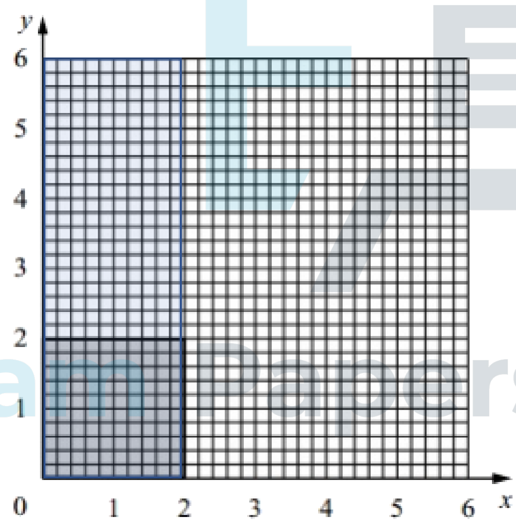
x-axis invariant - the shear happens parallel to the x-axis

(b)



(i) Draw the one-way stretch of the shaded square with the x-axis invariant and the point (0, 2) mapping onto the point (0, 6).

[2]



x-axis invariant – so the x coordinates of each point remains the same while the y coordinates change, parallel to the y-axis. [1]
Point (0, 2) is mapped onto point (0, 6), therefore, the sides are stretch in the vertical direction by 3.

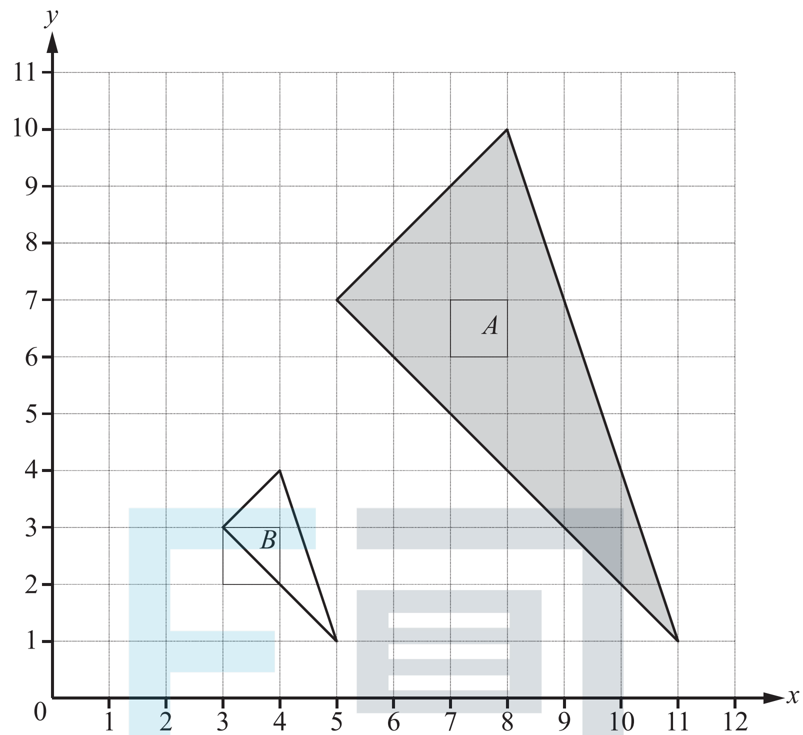
(ii) Write down the matrix of this stretch.

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} 0 & 2 & 2 & 0 \\ 0 & 0 & 2 & 2 \end{pmatrix} = \begin{pmatrix} 0 & 2 & 2 & 0 \\ 0 & 0 & 6 & 6 \end{pmatrix}$$

The matrix for this stretch multiplied by the matrix representing the object results in the matrix representing the image.

The matrix giving this stretch is:

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 3 \end{pmatrix}$$



Describe fully the **single** transformation that maps triangle *A* onto triangle *B*. [3]

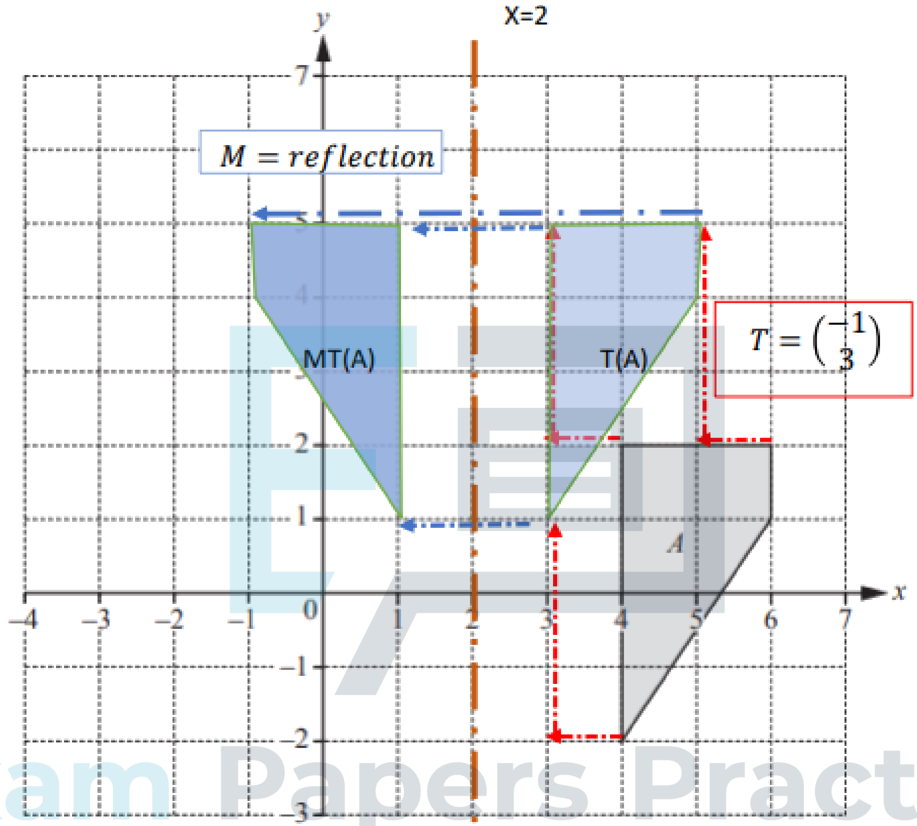
Enlargement, scale factor $1/3$, centre $(2, 1)$.

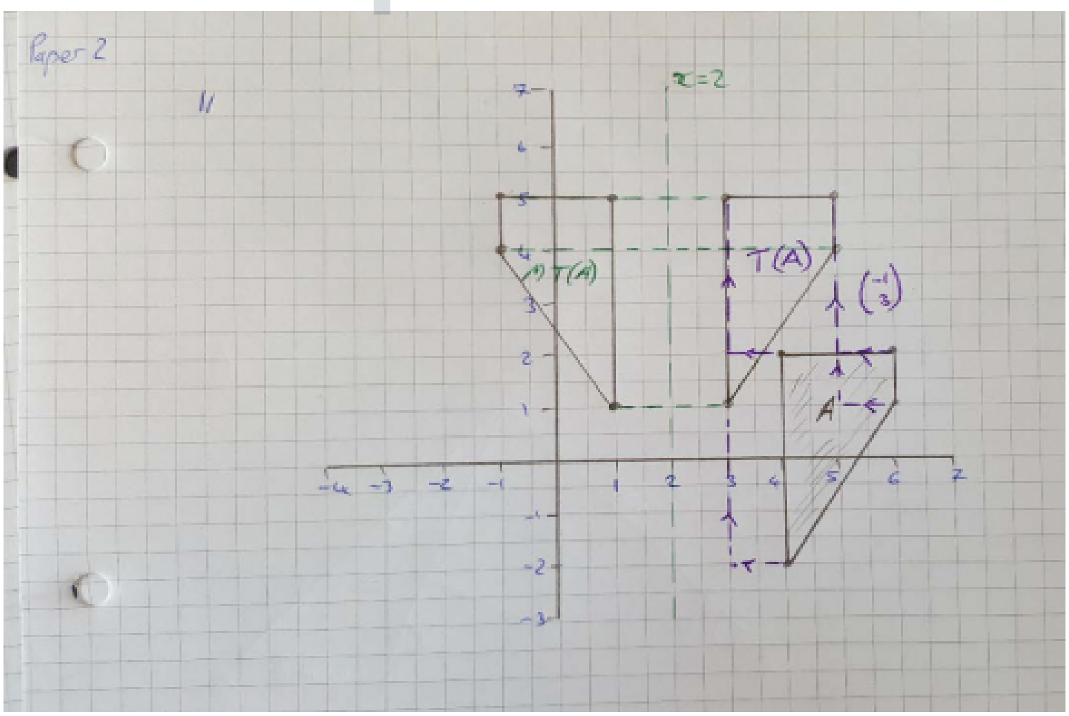
T(X) is the image of the shape X after translation by the vector $\begin{pmatrix} -1 \\ 3 \end{pmatrix}$.

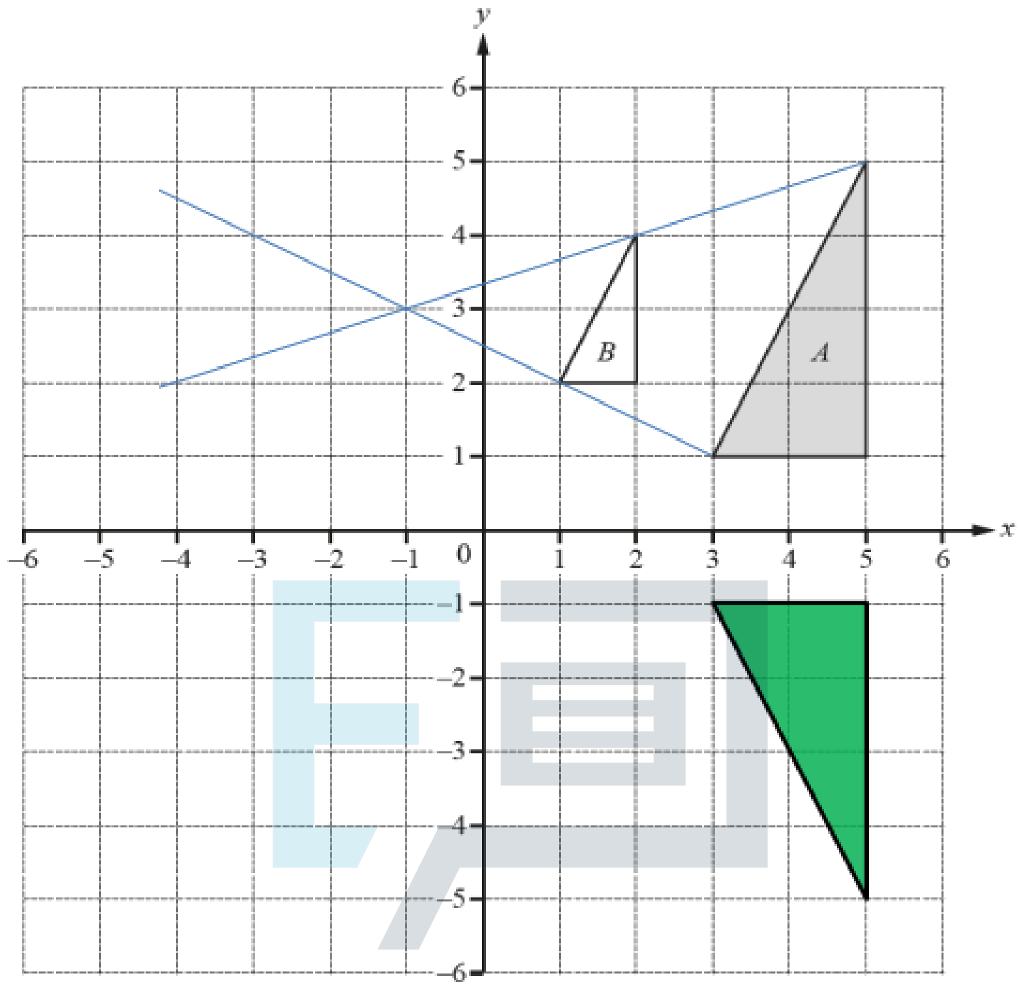
M(Y) is the image of the shape Y after reflection in the line $x = 2$.

On the grid, draw MT(A), the image of shape A after the transformation MT. [3]

In this question we need to first translate, and then reflect the shape







[3]

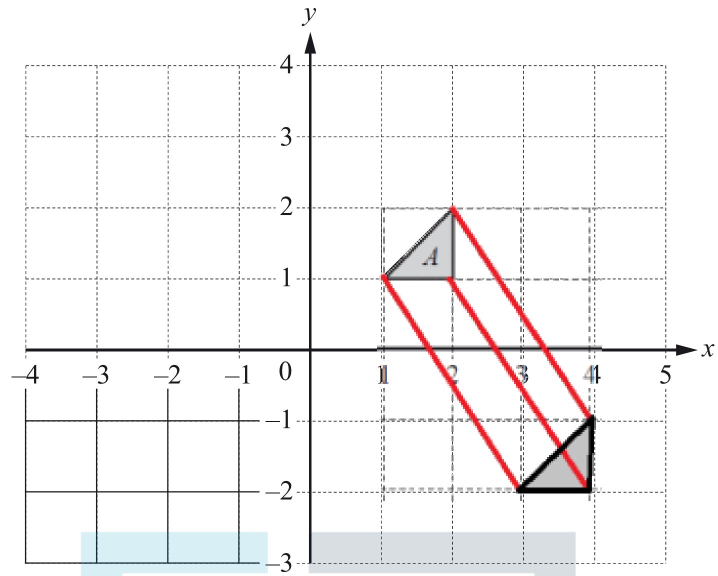
(a) Describe fully the **single** transformation that maps triangle A onto triangle B.

Change in size so it's an Enlargement - need to find Centre and Scale Factor.

Draw lines through equivalent points on A and B - where they cross is Centre: $(-1, 3)$

Use equivalent sides to find Scale Factor $= \frac{\text{Side of } B}{\text{Side of } A} = \frac{4}{2} = 2$

Enlargement, Centre (-1,3), Scale Factor 2

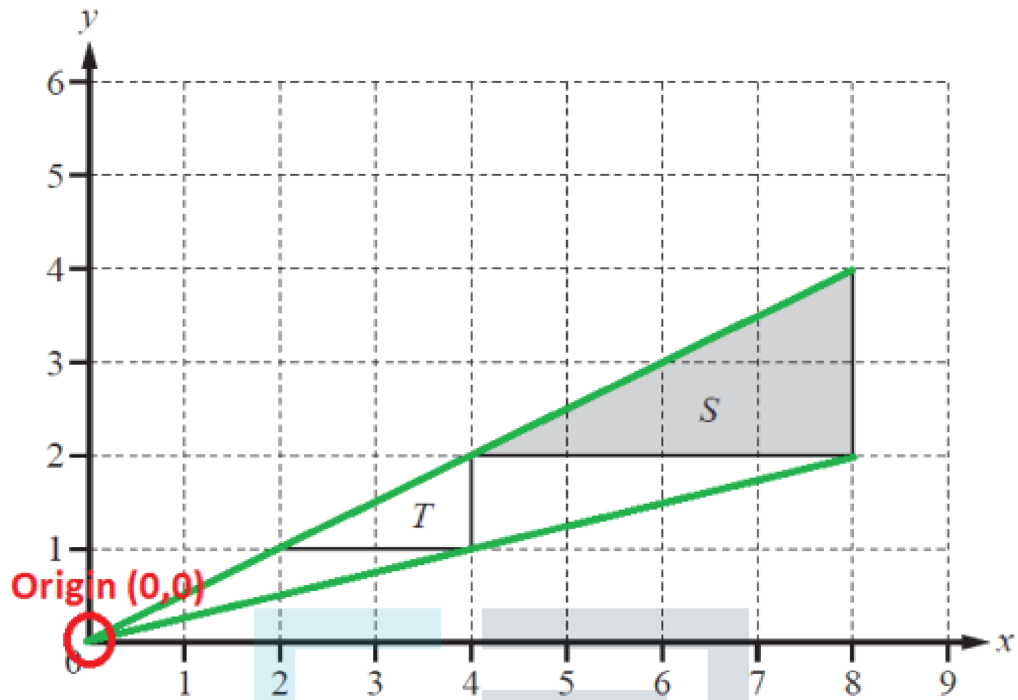


Draw the image of shape A after a translation by the vector $\begin{pmatrix} 2 \\ -3 \end{pmatrix}$. [2]

This translation represents a shift by 2 units in the positive x direction and by 3 units in the negative y direction

The new triangle has vertices $(\mathbf{3}, -\mathbf{2}), (\mathbf{4}, -\mathbf{2})$ and $(\mathbf{4}, -\mathbf{1})$.

Question 11



[3]

The transformation is an enlargement with centre (0, 0) and the scale factor 1/2.

(a) Describe fully the **single** transformation that maps triangle *S* onto triangle *T*.

When we join the corresponding vertices of triangles S and T, the lines meet at the origin (0,0).
The distance from (0,0) to a vertex of triangle T is half the distance from (0,0) to a corresponding vertex of triangle S.
This suggests that the scale factor of the enlargement is 1/2.

(b) Find the matrix which represents the transformation that maps triangle *S* onto triangle *T*.

The value of every coordinate has to be halved.

This can be achieved by a matrix $\begin{pmatrix} 1/2 & 0 \\ 0 & 1/2 \end{pmatrix}$

Example: $\begin{pmatrix} 1/2 & 0 \\ 0 & 1/2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \frac{1}{2}x \\ \frac{1}{2}y \end{pmatrix}$

[2]

Find the 2 × 2 matrix that represents a rotation through 90° clockwise about (0, 0). [2]

A general matrix for rotation looks like $\begin{pmatrix} \cos x & -\sin x \\ \sin x & \cos x \end{pmatrix}$ where x is an angle of anticlockwise rotation.

This matrix becomes $\begin{pmatrix} \mathbf{0} & \mathbf{1} \\ \mathbf{-1} & \mathbf{0} \end{pmatrix}$ for $x = -90°$ (minus sign because we want a clockwise rotation).

This is a matrix that represents a rotation through 90° clockwise about $(0, 0)$

(p, q) is the image of the point (x, y) under this combined transformation.

$$\begin{pmatrix} p \\ q \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} 3 \\ 2 \end{pmatrix}$$

(a) Draw the image of the triangle under the combined transformation. [3]

$$\begin{pmatrix} p \\ q \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} 3 \\ 2 \end{pmatrix}$$

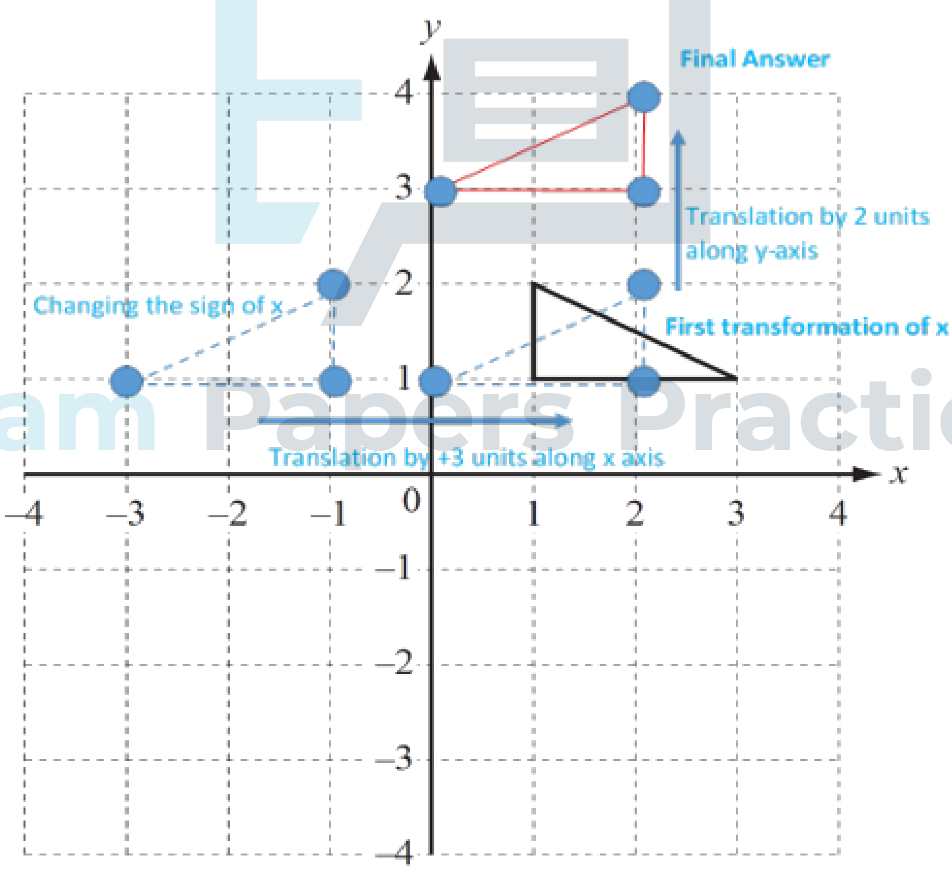
Multiply the matrix with x and y,

$$\begin{pmatrix} p \\ q \end{pmatrix} = \begin{pmatrix} -x \\ y \end{pmatrix} + \begin{pmatrix} 3 \\ 2 \end{pmatrix}$$

$$= \begin{pmatrix} -x+3 \\ y+2 \end{pmatrix}$$

Therefore the transformations are:

1. Changing the sign of x and translating it by 3 units along the x axis
2. Translating 2 units along the y axis



(b) Describe fully the **single** transformation represented by $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$. [2]

It is a reflection on the y-axis.