2002

XVIII

1583

Time allowed

Score

/

Percentage

%

Maths

**AQA
AS & A LEVEL**

Topic Questions

3.9 H: Integration

3 **(a)** **(i)** Given that $f(x) = x^4 + 2x$, find $f'(x)$. *(1 mark)*

(ii) Hence, or otherwise, find $\displaystyle\int \frac{2x^3 + 1}{x^4 + 2x}\, dx$. *(2 marks)*
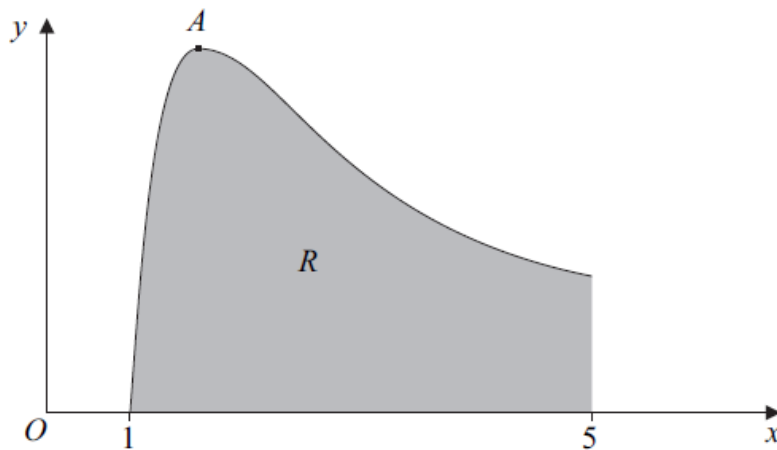
(b) **(i)** Use the substitution $u = 2x + 1$ to show that

$$\int x\sqrt{2x + 1}\, dx = \frac{1}{4}\int \left(u^{\frac{3}{2}} - u^{\frac{1}{2}}\right) du$$ *(3 marks)*

(ii) Hence show that $\displaystyle\int_0^4 x\sqrt{2x + 1}\, dx = 19.9$ correct to three significant figures. *(4 marks)*

(b) Using integration by parts, find $\displaystyle\int x^{-2}\ln x\, dx$. *(4 marks)*

(c) The sketch shows the graph of $y = x^{-2}\ln x$.
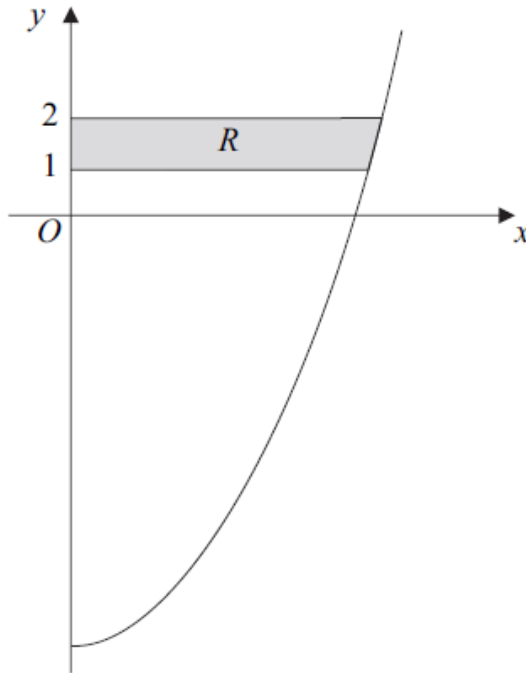

(ii) The region R is bounded by the curve, the x-axis and the line $x = 5$. Using your answer to part (b), show that the area of R is

$$\frac{1}{5}(4 - \ln 5)$$ *(3 marks)*

(b) Use the substitution $u = 2x + 1$ to find $\displaystyle\int x(2x + 1)^8\, dx$, giving your answer in terms of x. *(4 marks)*

4   (a)   Use integration by parts to find $\int x\sin x\,dx$.   *(4 marks)*

     (b)   Using the substitution $u = x^2 + 5$, or otherwise, find $\int x\sqrt{x^2 + 5}\,dx$.   *(4 marks)*

     (c)   The diagram shows the curve $y = x^2 - 9$ for $x \geqslant 0$.

The shaded region R is bounded by the curve, the lines $y = 1$ and $y = 2$, and the y-axis.

Find the exact value of the volume of the solid generated when the region R is rotated through 360° **about the y-axis**.   *(4 marks)*

_____

6   (a)   Use integration by parts to find $\int xe^{5x}\,dx$.   *(4 marks)*

     (b)   (i)   Use the substitution $u = \sqrt{x}$ to show that

$$\int \frac{1}{\sqrt{x}(1 + \sqrt{x})}\,dx = \int \frac{2}{1 + u}\,du$$

                         (2 marks)

         (ii)   Find the exact value of $\displaystyle\int_1^9 \frac{1}{\sqrt{x}(1 + \sqrt{x})}\,dx$.   *(3 marks)*

_____