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Practice questions created by actual examiners and assessment experts

Detailed mark scheme

Suitable for all boards

Designed to test your ability and thoroughly prepare you

2002

XVIII

1583

Time allowed

Score

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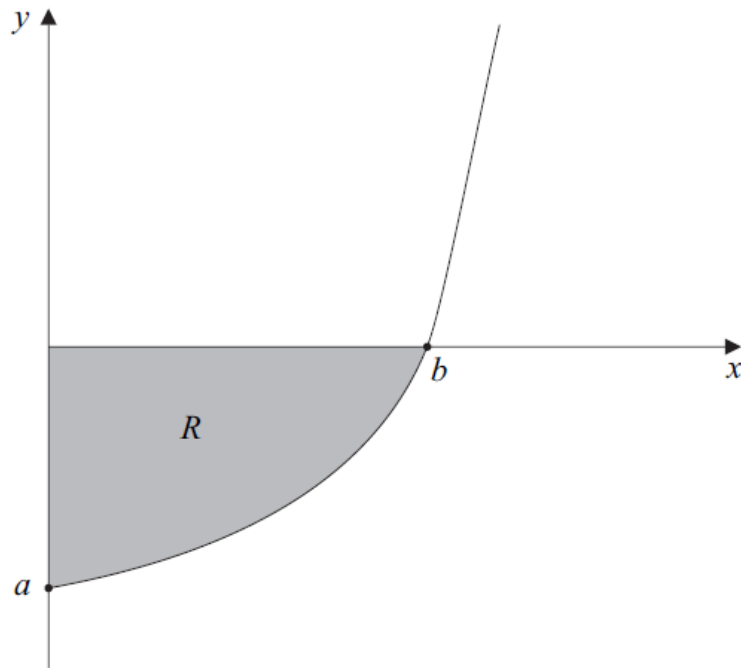
Percentage

%

Maths

AQA
AS & A LEVEL

Topic Questions

3.7 F: Exponentials and logarithms

5   The diagram shows part of the graph of $y = e^{2x} - 9$. The graph cuts the coordinate axes at $(0, a)$ and $(b, 0)$.


(a)   State the value of a, and show that $b = \ln 3$.   (3 marks)

(b)   Show that $y^2 = e^{4x} - 18e^{2x} + 81$.   (1 mark)

(c)   The shaded region R is rotated through $360°$ about the x-axis. Find the volume of the solid formed, giving your answer in the form $\pi(p \ln 3 + q)$, where p and q are integers.   (6 marks)

(d)   Sketch the curve with equation $y = |e^{2x} - 9|$ for $x \geqslant 0$.   (2 marks)

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9   (a)   Given that $y = x^{-2} \ln x$, show that $\dfrac{dy}{dx} = \dfrac{1 - 2\ln x}{x^3}$.   (4 marks)

(c)   The sketch shows the graph of $y = x^{-2} \ln x$.
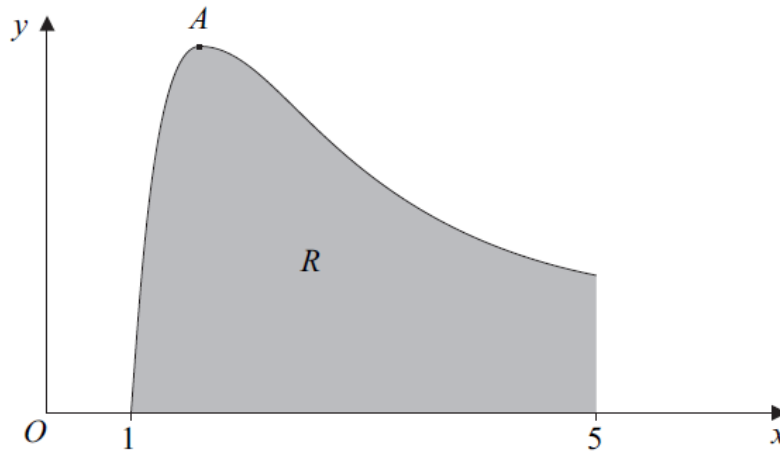
   (i)   Using the answer to part (a), find, in terms of e, the x-coordinate of the
         stationary point A.   *(2 marks)*

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5   (a)   A curve has equation $y = e^{2x} - 10e^x + 12x$.

   (i)   Find $\dfrac{dy}{dx}$.   *(2 marks)*

   (ii)   Find $\dfrac{d^2y}{dx^2}$.   *(1 mark)*

(b)   The points P and Q are the stationary points of the curve.

   (i)   Show that the x-coordinates of P and Q are given by the solutions of the equation

   $$e^{2x} - 5e^x + 6 = 0$$ *(1 mark)*

   (ii)   By using the substitution $z = e^x$, or otherwise, show that the x-coordinates of
          P and Q are $\ln 2$ and $\ln 3$.   *(3 marks)*

   (iii)   Find the y-coordinates of P and Q, giving each of your answers in the form
           $m + 12 \ln n$, where m and n are integers.   *(3 marks)*

   (iv)   Using the answer to part (a)(ii), determine the nature of each stationary point.

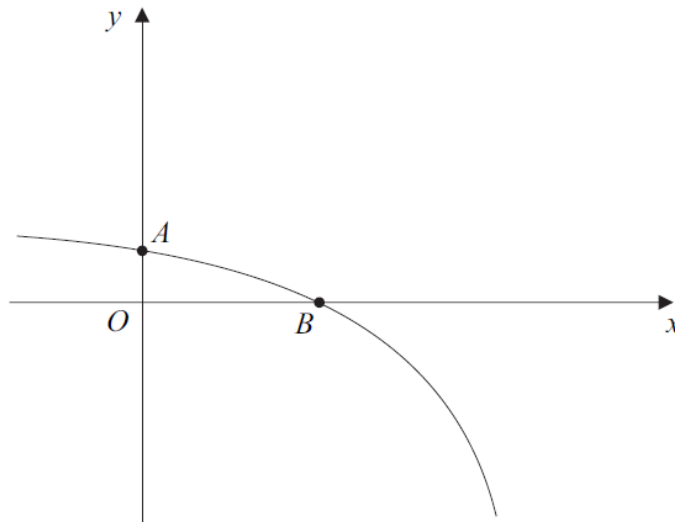(b)   (i)   Given that $y = x \ln x$, find $\dfrac{dy}{dx}$.   *(2 marks)*

       (ii)   Hence, or otherwise, find $\displaystyle\int \ln x \, dx$.   *(2 marks)*

       (iii) Find the exact value of $\displaystyle\int_1^5 \ln x \, dx$.   *(2 marks)*

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(b)   (i)   Find $\dfrac{dx}{dy}$ when $x = 2y^3 + \ln y$.   *(1 mark)*

       (ii)   Hence find an equation of the tangent to the curve $x = 2y^3 + \ln y$ at the point (2,1).   *(3 marks)*

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9   The sketch shows the graph of $y = 4 - e^{2x}$. The curve crosses the y-axis at the point A and the x-axis at the point B.

(a) (i) Find $\int (4 - e^{2x})dx$. (2 marks)

(ii) Hence show that $\int_0^{\ln 2} (4 - e^{2x})dx = 4\ln 2 - \frac{3}{2}$. (2 marks)

(b) (i) Write down the y-coordinate of A. (1 mark)

(ii) Show that $x = \ln 2$ at B. (2 marks)

(c) Find the equation of the normal to the curve $y = 4 - e^{2x}$ at the point B. (4 marks)

(d) Find the area of the region enclosed by the curve $y = 4 - e^{2x}$, the normal to the curve at B and the y-axis. (3 marks)

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1 (a) Differentiate $\ln x$ with respect to x. (1 mark)

(b) Given that $y = (x + 1)\ln x$, find $\frac{dy}{dx}$. (2 marks)

(c) Find an equation of the normal to the curve $y = (x + 1)\ln x$ at the point where $x = 1$.

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7 (a) A curve has equation $y = (x^2 - 3)e^x$.

(i) Find $\frac{dy}{dx}$. (2 marks)

(ii) Find $\frac{d^2y}{dx^2}$. (2 marks)

(b) (i) Find the x-coordinate of each of the stationary points of the curve. (4 marks)

(ii) Using your answer to part (a)(ii), determine the nature of each of the stationary points. (2 marks)