Question 1

Prove that $(4x-1)(2x+3)-(2x+1)^2 = 2(2x-1)(x+2)$.

[3]

EXPAND BRACKETS ON LHS

$(4x-1)(2x+3) - (2x+1)(2x+1)$ FOIL

$(8x^2 - 2x + 12x - 3) - (4x^2 + 2x + 2x + 1)$

$(8x^2 + 10x - 3) - (4x^2 + 4x + 1)$

SIMPLIFY, TAKE CARE WITH NEGATIVES

$8x^2 + 10x - 3 - 4x^2 - 4x - 1 = 4x^2 + 6x - 4$

FACTOR OF 2

$2(2x^2 + 3x - 2)$

FACTORISE REMAINING QUADRATIC

$\therefore$ $2(2x-1)(x+2)$ = RHS AS REQUIRED

$$\boxed{(4x-1)(2x+3) - (2x+1) = 2(2x-1)(x+2)}$$

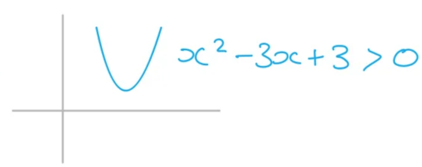
Question 2

Prove that $x^2 - 3x + 3$ is positive for all values of x.

[3]

PROVE QUADRATIC IS ALWAYS POSITIVE USING DISCRIMINANT $b^2 - 4ac < 0$

$b^2 - 4ac < 0$



$a = 1 \quad b = -3 \quad c = 3$

$(-3)^2 - 4(1)(3) < 0$

$9 - 12 < 0$

$-3 < 0$ ✓✓

$\therefore$ $x^2 - 3x + 3 > 0$ FOR ALL VALUES OF x

Question 3

Prove that $(a-b)^2 - (a+b)^2 = -4ab$.

[3]

EXPAND BRACKETS ON LHS

$(a-b)(a-b) - (a+b)(a+b)$

$(a^2 - 2ab + b^2) - (a^2 + 2ab + b^2)$

SIMPLIFY $\cancel{a^2} - 2ab + \cancel{b^2} - \cancel{a^2} - 2ab - \cancel{b^2}$

$-4ab$ = RHS AS REQUIRED

$\therefore$

$\boxed{(a-b)^2 - (a+b)^2 = -4ab}$

Question 4

Prove that the sum of any three consecutive integers is a multiple of 3.

+

[3]

LET THREE CONSECUTIVE INTEGERS BE

$n-1, n, n+1$

THEN

$n-1 + n + n+1$

$\equiv 3n$

WHICH IS A MULTIPLE OF 3

$\therefore$

THE SUM OF 3 CONSECUTIVE INTEGERS IS A MULTIPLE OF 3

Question 5

Prove that $x^2 + 2 \geq 2$ for all values of x.

[2]

GIVEN THAT

$$x^2 + 2 \geqslant 2$$

THEN

$$x^2 \geqslant 0$$

ALL SQUARE NUMBERS ARE ALWAYS POSITIVE

$\therefore$ $\boxed{x^2 + 2 \geqslant 2 \text{ FOR ALL VALUES OF } x}$

Question 6

Prove that the square of an even number is a multiple of 4.

[3]

LET AN EVEN NUMBER BE 2N

THEN

$$(2n)^2 = 4n^2$$

$$\equiv 4(n^2)$$

WHICH IS A MULTIPLE OF 4

$\therefore$ $\boxed{\text{THE SQUARE OF AN EVEN NUMBER IS ALWAYS A MULTIPLE OF 4}}$

Question 7

(a) Factorise $n^2 + 3n + 2$. [1]

a) $\boxed{(n+1)(n+2)}$

(b) Hence show that $n^3 + 3n^2 + 2n = n(n+1)(n+2)$. [1]

(c) Given that n is even, write down whether $(n+1)$ *and* $(n+2)$ are odd or even. [2]

(d) Hence deduce whether $n^3 + 3n^2 + 2n$ is odd or even. Justify your answer. [2]

(a) Factorise $n^2 + 3n + 2$. [1]

(b) Hence show that $n^3 + 3n^2 + 2n = n(n+1)(n+2)$. [1]

b) $n^3 + 3n^2 + 2n$

$n(n^2 + 3n + 2)$

$\boxed{n(n+1)(n+2)}$

(c) Given that n is even, write down whether $(n+1)$ *and* $(n+2)$ are odd or even. [2]

(d) Hence deduce whether $n^3 + 3n^2 + 2n$ is odd or even. Justify your answer. [2]

(a) Factorise $n^2 + 3n + 2$.

[1]

(b) Hence show that $n^3 + 3n^2 + 2n = n(n+1)(n+2)$.

[1]

(c) Given that n is even, write down whether $(n+1)$ *and* $(n+2)$ are odd or even.

[2]

(d) Hence deduce whether $n^3 + 3n^2 + 2n$ is odd or even. Justify your answer.

[2]

c)

GIVEN THAT n IS EVEN

CONSECUTIVE INTEGERS ALTERNATE BETWEEN ODD AND EVEN

$n+1$ MUST BE ODD

$n+2$ MUST BE EVEN

(a) Factorise $n^2 + 3n + 2$.

[1]

(b) Hence show that $n^3 + 3n^2 + 2n = n(n+1)(n+2)$.

[1]

(c) Given that n is even, write down whether $(n+1)$ *and* $(n+2)$ are odd or even.

[2]

(d) Hence deduce whether $n^3 + 3n^2 + 2n$ is odd or even. Justify your answer.

[2]

d)

GIVEN THAT

$$n^3+3n^2+2n \equiv n(n+1)(n+2)$$

THEN

IF n IS ODD

$n+1$ = EVEN

$n+2$ = ODD

IF n IS EVEN

$n+1$ = ODD

$n+2$ = EVEN

AT LEAST ONE TERM OF $n(n+1)(n+2)$ IS ALWAYS EVEN, THEREFORE

n^3+3n^2+2n MUST ALWAYS BE EVEN

Question 8

(a) Show that $(3n+2)^2 - (n+2)^2 = 8n^2 + 8n$, where $n \in \mathbb{Z}$. SET OF INTEGERS [2]

(b) Hence, or otherwise, prove that $(3n+2)^2 - (n+2)^2$ is a multiple of 8. [2]

a) EXPAND BRACKETS ON LHS

$(3n+2)(3n+2) - (n+2)(n+2)$

$(9n^2 + 12n + 4) - (n^2 + 4n + 4)$

SIMPLIFY $9n^2 + 12n + \cancel{4} - n^2 - 4n - \cancel{4}$

$8n^2 + 8n$ = RHS AS REQUIRED

$\therefore$ $\boxed{(3n+2)^2 - (n+2)^2 = 8n^2 + 8n}$

(a) Show that $(3n+2)^2 - (n+2)^2 = 8n^2 + 8n$, where $n \in \mathbb{Z}$. [2]

(b) Hence, or otherwise, prove that $(3n+2)^2 - (n+2)^2$ is a multiple of 8. [2]

b) USING $(3n+2)^2 - (n+2)^2 = 8n^2 + 8n$ FROM (a)

FACTORISE

$8n^2 + 8n = 8(n^2 + n)$

$8(n^2+n)$ IS A MULTIPLE OF 8

$\therefore$ $\boxed{(3n+2)^2 - (n+2)^2 \text{ IS A MULTIPLE OF } 8}$