Exam Papers Practice

Simple Harmonic Motion

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Course	HL IB Physics
Section	Wave Behaviour
Topic	Simple Harmonic Motion
Difficulty	Medium

To be used by all students preparing for HL IB Physics
Students of other boards may also find this useful

1

The correct answer is **B** because:

- Acceleration is at a maximum at the point of **maximum** displacement from the equilibrium
- Velocity is also zero at the point of maximum displacement from the equilibrium position
    - The point of maximum displacement from the equilibrium position is Y
- Therefore, the only answer that fits these criteria is **B**

Position Y is the negative maximum displacement of string S. Acceleration acts in the opposite direction to displacement, so position Y is the point of positive maximum acceleration. Velocity is a maximum at the equilibrium (position Z) and is zero at the points of maximum displacement (position Y) (both in the negative and positive direction).

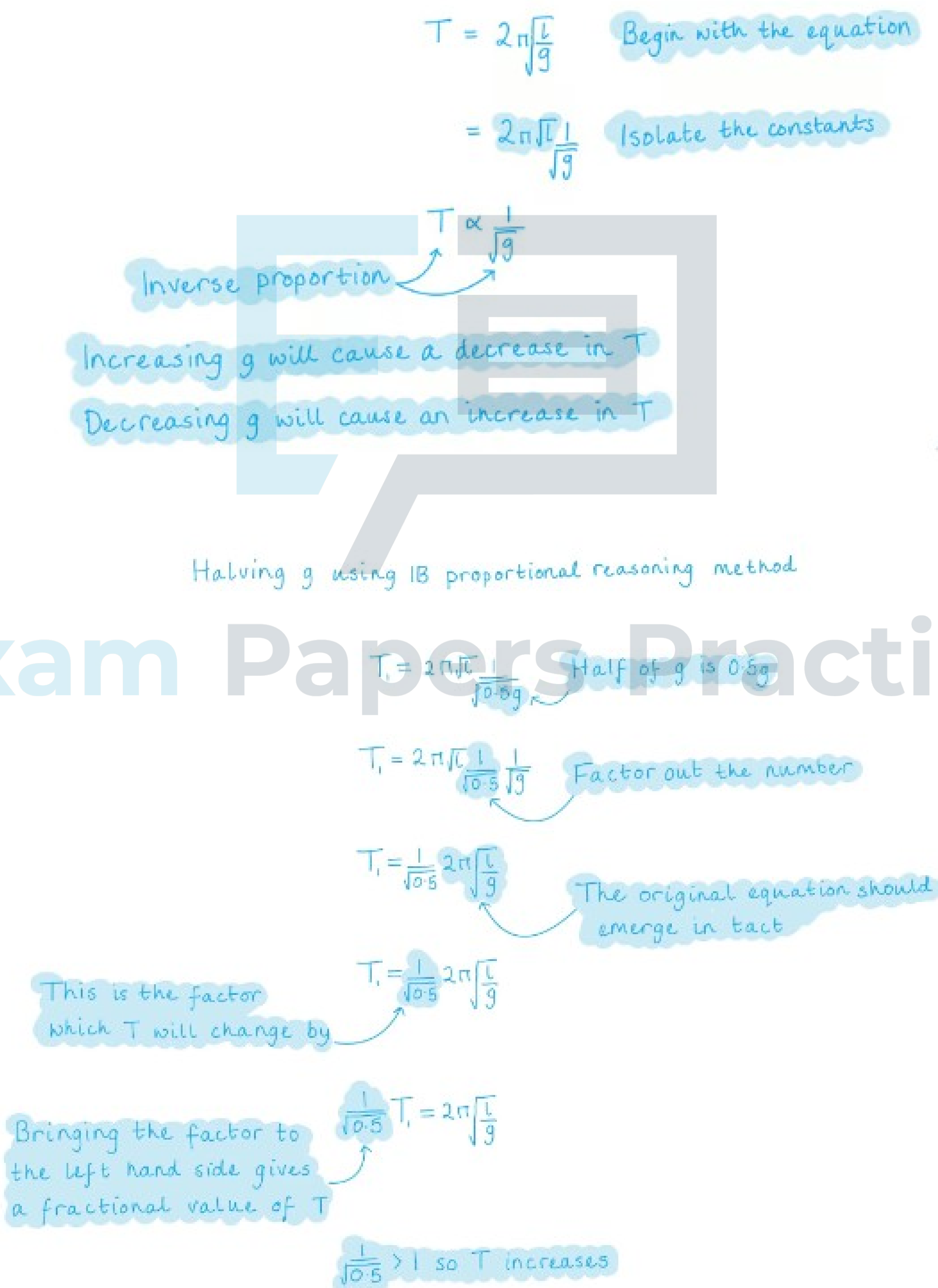
2

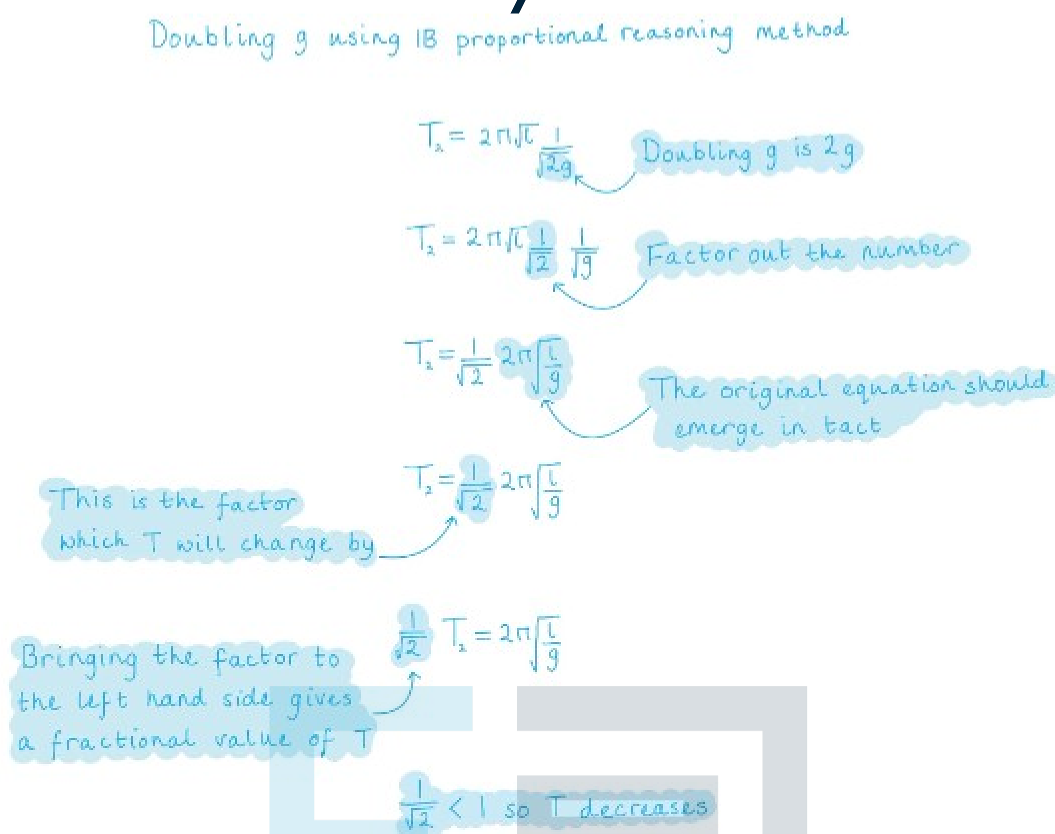
The correct answer is **D** because:

- The equation relating the period *T* to the mass-spring system is $T = 2\pi\sqrt{\frac{m}{k}}$
    - Therefore, the only factors that affect the period of a mass-spring system are the mass *m* and the spring constant *k*
    - The change in acceleration of free fall has no effect on the period of the mass-spring system
    - Hence, the period of the mass-spring system will be the same on Mars as it was on Earth
    - This eliminates options **A** and **C**
- The equation relating the period *T* to the simple pendulum is $T = 2\pi\sqrt{\frac{l}{g}}$
    - The factors that affect the period of a simple pendulum are the length *l* and the acceleration of free fall *g*
    - Therefore, the period of the simple pendulum will be different on Mars than it is on Earth

- This eliminates option **B**
- Therefore, the only answer that fits the criteria is **D**

In order to answer this question, you do not need to know by what factor the period of the simple pendulum would change, as the only options given are that it stays the same or that it increases. But if you would like a reminder to show you how to work it out, here is one below.





3

The correct answer is **B** because:

- The period of oscillation of an object in SHM is constant
  - This is one of the conditions for SHM
  - The oscillation is isochronous
- The frequency of the oscillation is the inverse of the period
  - Therefore, if period is constant, frequency is also constant
- Option **B** is the only answer that meets these criteria

A is incorrect as if the period is constant, then the frequency must also be constant

C is incorrect as if the period is constant, then the frequency must also be constant. The spring constant defines how stiff a spring is, it is not a condition of SHM

D is incorrect as the spring constant defines how stiff a spring is, it is not a condition of SHM. The acceleration of freefall describes the gravitational field strength of the environment, but it is not a condition of SHM Isochronous means occurring at the same time.

4

The correct answer is **A** because:

- The total energy equation can be written as $E = \frac{1}{2}m\omega^2 A^2$
- The mass m increases by half becoming $1.5m$
- Amplitude A increases to $4A$
- Therefore:
  - $E = \frac{1}{2}(1.5m)\,\omega^2(4A)^2$
  - $E = \frac{1}{2}1.5m\omega^2 16A^2$
- Using proportional reasoning and separating out the factors gives:
  - $E = (1.5 \times 16)\frac{1}{2}m\,\omega^2 A^2$
  - $E = 24\frac{1}{2}m\,\omega^2 A^2$
  - Hence, $24E = \frac{1}{2}m\,\omega^2 A^2$
- Increasing the mass m by half and increasing the amplitude to $4A$ would cause the energy to increase by a factor of 24

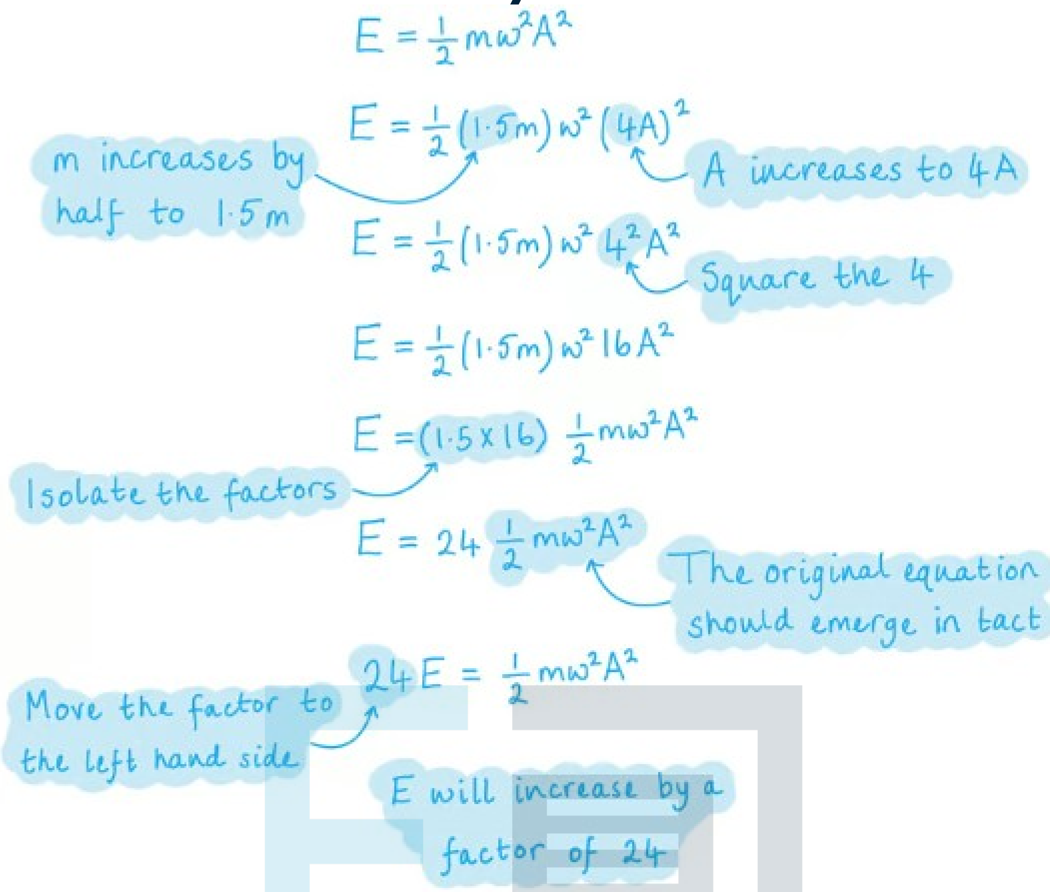
B is incorrect as factor 24 was multiplied by the $\frac{1}{2}$ from $\frac{1}{2}m\omega^2 A^2$.

Remember that when using the proportional reasoning technique, the equation needs to emerge intact once you have removed the factors.

C is incorrect as the question has been misinterpreted. This calculation was performed with $0.5m$. The question reads that m has **increased** by half, not that m has been halved

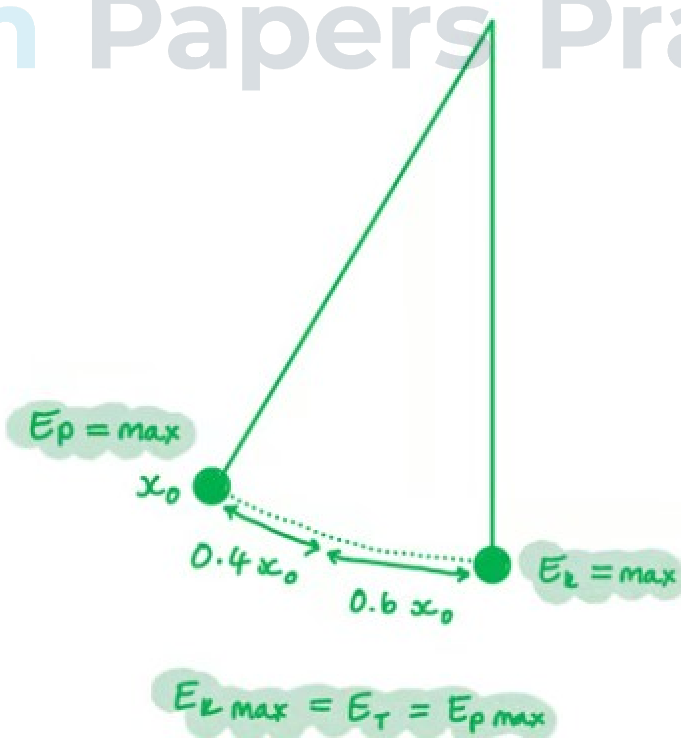
D is incorrect as the 4 from $4A$ has not been squared

If you need a reminder on how to use proportional reasoning to find the factor:



5

The correct answer is **A** because:



- At $0.4x_0$ from the maximum displacement, this is 0.6 of the amplitude x_0
- Maximum kinetic energy is the same as total energy at equilibrium
- The equation for total energy is $E_T = \frac{1}{2} m\omega^2 {x_0}^2$
    - Adding in the value 0.6 *A*, it becomes $E_T = \frac{1}{2} m\omega^2 (0.6x_0)^2 = \frac{1}{2} m\omega^2 0.36{x_0}^2$
    - Using the proportional reasoning method to isolate the factor, $E_T = 0.36 \frac{1}{2} m\omega^2 {x_0}^2$
    - So, $0.36E_T = \frac{1}{2} \mathrm{m}\omega^2 {x_0}^2$
- The potential energy of the system at a distance of $0.4x_0$ from maximum displacement will be 0.36 of the maximum kinetic energy, 0.36 E_k

Drawing a diagram can be crucial to understanding which values to use in the calculation. If you are at all unsure of the scenario being presented, sketch a quick diagram to visualise what's going on.

The wording of the question is difficult because it uses the term maximum kinetic energy to mean total energy at equilibrium which gives you the opportunity to use the wrong equation. It also gives the distance from the amplitude as a fraction of the amplitude inviting you to use the wrong value. **Make sure you don't fall into these traps**! Sketching a diagram is the best way to avoid them.

If you need a reminder of how to find the factor using the proportional reasoning method:

$E_T = \frac{1}{2}m\omega^2 x_0^2$

$E_T = \frac{1}{2}m\omega^2 (0.6x_0)^2$ ← 0.6 of amplitude

$E_T = \frac{1}{2}m\omega^2 0.6^2 x_0^2$ ← Square the 0.6

$E_T = \frac{1}{2}m\omega^2 0.36 x_0^2$

$E_T = 0.36 \frac{1}{2}m\omega^2 x_0^2$ ← The original equation should emerge in tact

$0.36 E_T = \frac{1}{2}m\omega^2 x_0^2$ ← Moving the factor to the left hand side gives a fractional value of E_T

6

The correct answer is **D** because:

- The defining equation for SHM is $a = -\omega^2 x$
    - Maximum displacement means that $x = x_0$ and acceleration is at a maximum at amplitude x_0
    - $a_{max} = -\omega^2 x_0$
- Rearranging to solve for x_0 gives:
    - $x_0 = -\frac{a_{max}}{\omega^2}$
- Angular velocity is $\omega = \frac{2\pi}{T}$
- Combining these equations gives:
    - $x_0 = -\frac{a_{max}}{(\frac{2\pi}{T})^2}$
    - $x_0 = -\frac{a_{max}T^2}{4\pi^2}$

ω can also be expressed as $2\pi f$, since $T = \frac{1}{f}$. The correct arrangement using frequency would be $x_0 = -\frac{a_{max}}{4\pi^2 f^2}$, which is close to, but not exactly options **A** or **B**.

There are lots of equations to keep track of in SHM. If you familiarise yourself with the interchangeable expressions as part of your revision, this will greatly reduce the amount of time you have to spend looking up equations and rearranging equations in the exam!

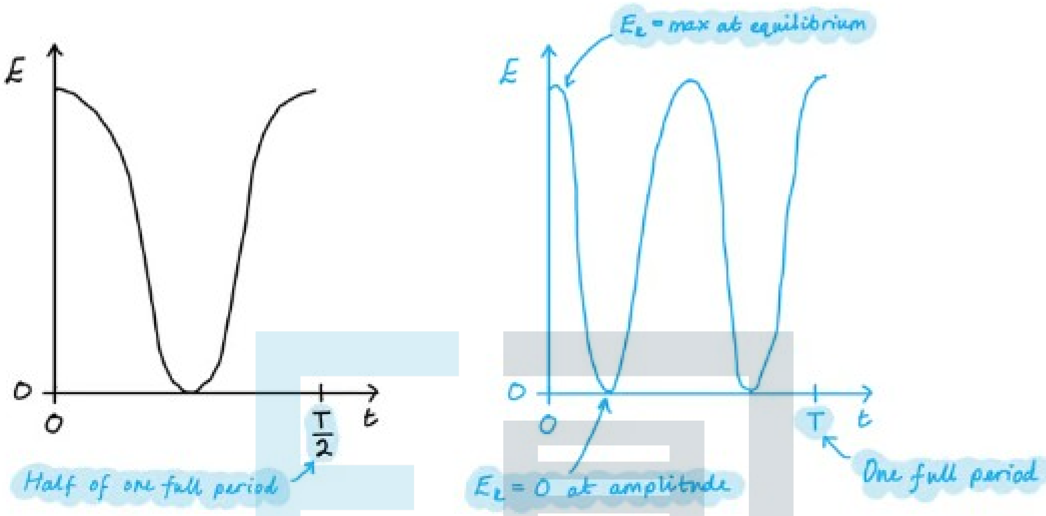
7

The correct answer is **B** because:

- The value for energy is always positive
  - This eliminates option **A**
- Kinetic energy is at a maximum at the point of equilibrium
  - $t = 0$ at the point of equilibrium
  - At $t = 0$, kinetic energy is at a maximum
  - This eliminates option **D**
- The time axis shows that the graph is for half on one period
  - The ion begins at equilibrium, passes equilibrium at $\frac{T}{2}$, and returns to equilibrium in one full period
  - The graph for half of one period would begin at equilibrium and end at equilibrium at $\frac{T}{2}$
  - This eliminates option **C**
- The only answer that fits these criteria is option **B**

This question is only concerned with half a period, so the kinetic energy will go from maximum at equilibrium, to zero at amplitude, and back to maximum as it passes equilibrium again.

For a full period, the kinetic energy would go from maximum at equilibrium, to zero at positive amplitude, to maximum as it passes through equilibrium, to zero at negative amplitude, and back to maximum as it approaches the equilibrium again.



8

The correct answer is **B** because :

- The equation describing the period of a pendulum is $T = 2\pi\sqrt{\frac{l}{g}}$
  - Changing the mass of the pendulum bob will have no effect on the period
- $T \propto \sqrt{l}$
  - Decreasing the length will therefore decrease the period
  - This eliminates option **A**
- The length of the pendulum is halved so $T = 2\pi\frac{\sqrt{0.5l}}{\sqrt{g}}$
  - T will decrease by a factor of $\sqrt{0.5}$
  - Using an approximation, $\sqrt{0.5} \approx 0.7$
    - 0.50 is close to 0.49, therefore $\sqrt{0.5} \approx \sqrt{0.49}$
    - $7^2 = 49$, therefore $\sqrt{0.49} = 0.7$
  - This eliminates options **C** and **D**
- The only answer that fits these criteria is option **B**

Make sure you practice approximations for proportionality questions as part of your revision. These type of questions come up a lot and you only have a very limited time to perform the calculations in the exam. Approximations can be made using the nearest square numbers and will be accurate enough to lead you to the correct answer. Remember that approximations are not entirely accurate, so the answer you get may not be exactly the same as the answer options given. In these instances, you are looking for the closest answer option to your approximation.

If you need a more detailed look at how to get to the answer:

$$T = 2\pi\sqrt{\frac{l}{g}}$$

$$T = 2\pi\sqrt{\frac{0.5l}{g}}$$

Half the length is $0.5l$

Isolate the factor: $T = 2\pi\sqrt{0.5}\sqrt{\frac{l}{g}}$

$$T = \sqrt{0.5}\,2\pi\sqrt{\frac{l}{g}}$$

The equation should emerge in tact

Move the factor to the left hand side: $\frac{}{}$$\sqrt{0.5}\,T = 2\pi\sqrt{\frac{l}{g}}$

$\sqrt{0.5} < 1$ so T will decrease

To approximate $\sqrt{0.5}$ use square numbers close to 50

$$\sqrt{0.5} = \sqrt{0.50}$$

$$\sqrt{0.5} \approx \sqrt{0.49}$$

So, $\sqrt{0.5} \approx 0.7$

x	x^2
6	36
7	49
8	64

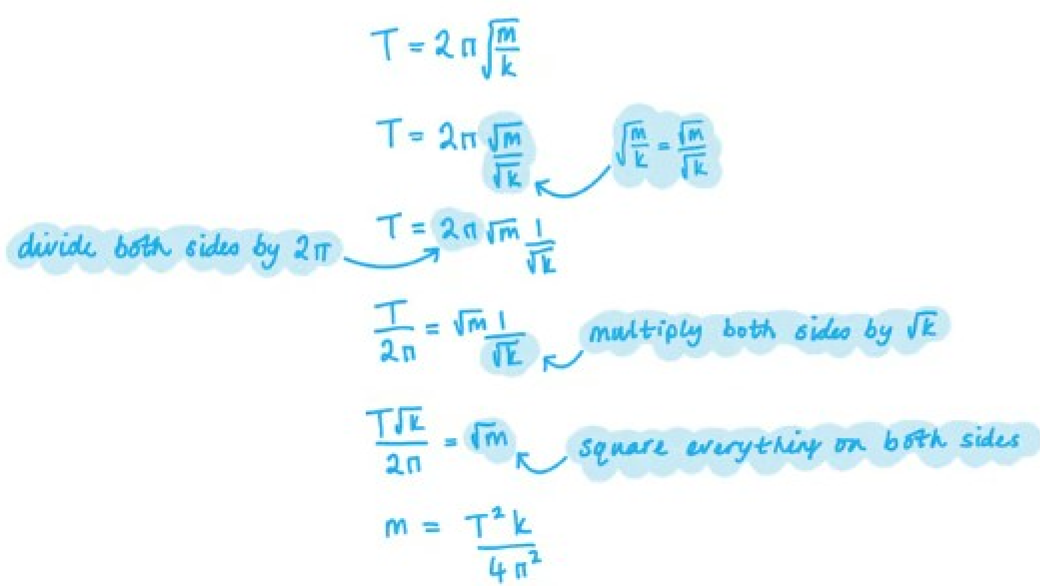
Therefore, $\sqrt{0.5}\,T \approx 0.7T$

9

The correct answer is D because:

- The equation linking the period T and mass m of a mass-spring system is $T = 2\pi\sqrt{\frac{m}{k}}$
- Rearranging the equation to solve for mass gives:
    - $m = \frac{T^2 k}{4\pi^2}$
- Doubling the period gives:
    - $m = \frac{(2T)^2 k}{4\pi^2}$
    - $m = \frac{4T^2 k}{4\pi^2}$
    - $m = 4\frac{T^2 k}{4\pi^2}$
- The mass would need to be **4** times larger to double the period
    - $4m = \frac{T^2 k}{4\pi^2}$

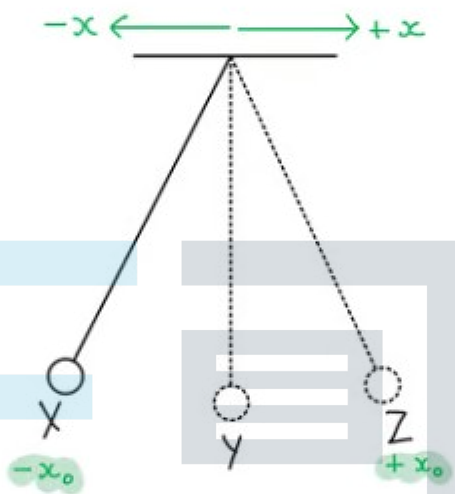
If you need a reminder on how to rearrange the original equation to solve for m:



10

The correct answer is **B** because:

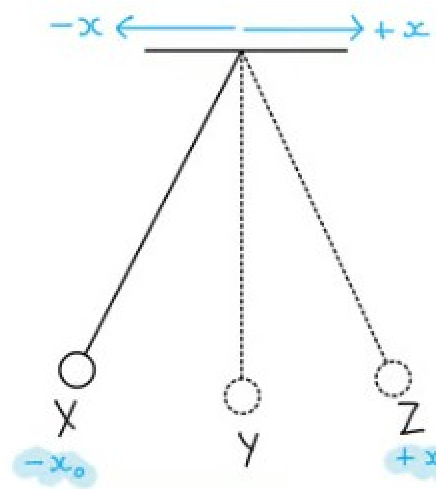
- Since the diagram shows the pendulum being released from position X, it can be assumed that the forward direction is positive therefore:



- Acceleration is zero at the point of equilibrium, position Y
  - This discounts options **A** and **C**
- Displacement is at a maximum at amplitude, so the maximum negative displacement will be at $-x_0$
  - $-x_0$ is position X
  - Both options **B** and **D** still apply
- Velocity is at a maximum at equilibrium, Position Y
  - This discounts option **D**
- The only answer that fits these criteria is option **B**

The best way to tackle a question like this is by the process of elimination. Take one variable at a time and determine what it must be, then discount the answer options that do not apply. Working methodically in this way will save time and avoid confusion.

If you need a reminder of when the displacement, velocity, acceleration and Force are at their maximums:



Position	x	v	a	F
X	− max	zero	+ max	+ max
Y	zero	± max	zero	zero
Z	+ max	zero	− max	− max

11

The correct answer is **D** because:

- The total energy, E_T is equal to the sum of the kinetic and potential energy of the oscillating system
  - This means that when the potential energy = 0 then the kinetic energy is at a maximum
- This means $E_T = \frac{1}{2}mv^2$
- From circular motion it is stated that $v = \omega r$
  - In this case, the radius, r is the displacement x
  - Since this is the **maximum** kinetic energy, $x = x_0$ (the amplitude)
- Therefore, the total energy can be written as:
  - $E_T = \frac{1}{2}m\omega^2 x_o^2$ where m = mass, ω^2 = the *square* of the angular frequency, and x_0^2 is the square of the amplitude
- Since mass, m is constant, this means $E_T \propto \omega^2$ and $E_T \propto x_0^2$

12

The correct answer is **A** because:

- The defining equation of SHM states that acceleration $a = -\omega^2 x$
- The negative sign shows that the vectors on each side of the equation, acceleration *a*, and displacement x, are pointing in opposite directions

13

The correct answer is **B** because:

- A phase difference of $\frac{\pi}{4}$ is the same as 45° of $\frac{1}{16}$ of a cycle out of phase
  - This is shown by graph **B**

A is incorrect as this is a phase difference of $\frac{\pi}{2}$ or 90° or $\frac{1}{4}$ cycle out of phase

C is incorrect as this is a phase difference of π or 180° or $\frac{1}{2}$ cycle out of a phase

D is incorrect as this is a phase difference of 0 (or 2π)

Try not to get mixed up with $\frac{\pi}{4}$ (45°) and $\frac{1}{4}$ cycle out of a phase (90°). 1 full cycle is 360°, or 2π.

14

The correct answer is **C** because:

- In SHM, the restoring force is always towards the equilibrium position
  - Therefore, acceleration is always towards the equilibrium position

- Since the equilibrium position is at Y, the direction is towards Z (as it will pass through Y then continue to Z)
- The particle will be at maximum velocity (and therefore, kinetic energy) at point Y
  - So, a particle moving from maximum displacement to equilibrium must be accelerating
- It then **decelerates** again from Y to Z, but at this point the restoring force would be pointing towards Y

A is incorrect as maximum kinetic energy occurs at equilibrium, and maximum potential energy at maximum displacement, so both options are wrong

B is incorrect as total energy is equal to the maximum kinetic energy, at point Y, the equilibrium, instead of point X

D is incorrect as the restoring force is always towards equilibrium, point Y in this diagram. We do not have enough information to say whether the particle is accelerating or decelerating

15

The correct answer is **C** because:

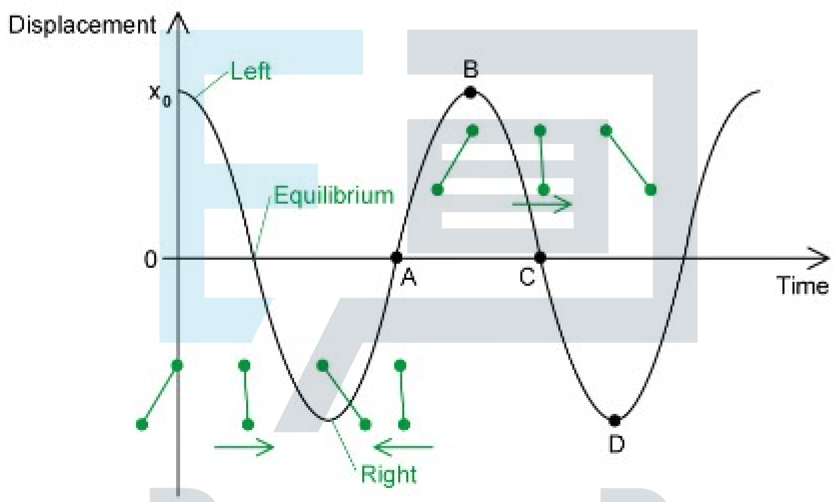
- The angular frequency $\omega = \frac{2\pi}{T}$ where T is the time period
- Therefore, ω is **inversely** proportional to T:
  - $\omega \propto \frac{1}{T}$
- Therefore, if T is multiplied by 3, then ω must be **divided** by 3

Remember to look at the circular motion equations in the data booklet to help with simple harmonic motion too. The equations are relevant because both circular motion and simple harmonic motion are **periodic**.

16

The correct answer is **C** because:

- When time = 0, the bob starts at the left as stated in the question
- The bob then passes through equilibrium, reaches maximum displacement on the right and then returns to equilibrium at point **A**
- At point **B**, the bob has returned to the maximum displacement on the left and its velocity is 0
- At point **C**, the bob is in equilibrium but heading towards the right
  - This is also the direction of its velocity
  - Therefore, **C** is correct
- At point **D,** the bob is at maximum displacement on the right and its velocity is 0



17

The correct answer is **D** because:

- The total energy, E_T is equal to the sum of the kinetic and potential energy of the oscillating system
  - This means that when the potential energy = 0, then the kinetic energy is at a **maximum**
- This means total energy is $E_T = \frac{1}{2}mv^2$

- Therefore:
  - Energy during first oscillation, $E_1 = \frac{1}{2}mv^2$
  - Energy during second oscillation, $E_2 = \frac{1}{2}m(2v)^2$
- Comparing E_2 to E_1:
  - $E_2 = 4 \times \left(\frac{1}{2}mv^2\right)$
  - Therefore, $E_2 = 4E_1$
- The period of the oscillation is **independent** of the energy
  - Hence, period T does not change
- Hence, option **D** is correct

Remember to always check the relation between two variables with equations where you can. Especially look out for when variables are **squared**.

Remember that all oscillations in SHM are **isochronous**, meaning that the period and the amplitude are independent of one another, so changing either one won't affect the other. In fact, it is the initial displacement, or amplitude, that affects the total energy in the oscillating system and the energy determines the speed of the oscillation

18

The correct answer is **B** because:

- For an object in SHM, when its displacement, x is 0 (at equilibrium), the object travels at its faster speed v
- When its displacement is at a maximum (at its amplitude), the object has a speed $v = 0$
  - The only graph that matches this pattern is graph **B**

A is incorrect as in this graph, when x is 0 then v is also 0. This is the opposite way around

C & **D are** incorrect as the question mentions that v is the speed, which is a scalar quantity and cannot be negative

19

The correct answer is **A** because:

- The defining equation of SHM states that $a = -\omega^2 x$
  - The negative sign shows that the vectors on each side of the equation are pointing in opposite directions
- Angular frequency is constant for a given set of oscillations
- We are looking for a graph which shows **direct** proportionality, which means a straight line through the origin, and has negative gradient

B & **D** are incorrect as they are not directly proportional (not straight lines through the origin)

C is incorrect as the line has a positive gradient instead of negative

20

The correct answer is **C** because:

- The time period, *T* is given by the equation:
  - $T = \frac{1}{f}$ where *f* is the frequency
- *T* is the time for 1 complete oscillation
- From the graph, this is around 7.5 s
- Therefore, the frequency is:
  - $f = \frac{1}{7.5}$ Hz

A is incorrect as this is only considered half the oscillation, instead of one full oscillation

B is incorrect as this is the time period, not the frequency

D is incorrect as this is the time period of only half of an oscillation, not the frequency