EXAM PAPERS PRACTICE

Similarity

Model Answer

Question 1

The length of a backpack of capacity 30 litres is 53 cm.
Calculate the length of a mathematically similar backpack of capacity 20 litres.

[3]

$\frac{l_1}{l_2} = \frac{c_1}{c_2}$

Plugging in the given values, we get:

$\frac{53\text{ cm}}{l_2} = \frac{30\text{ liters}}{20\text{ liters}}$

To solve for l_2, we can cross-multiply and then divide both sides by $\frac{30\text{ liters}}{20\text{ liters}}$

$l_2 = \frac{53\text{ cm} \times 20\text{ liters}}{30\text{ liters}}$

$l_2 = \frac{1060}{30}\text{ cm}$

$l_2 = 35.33\text{ cm}$

Therefore, the length of the second backpack is 35.33 centimeters.

Question 2

The two barrels in the diagram are mathematically similar.

NOT TO SCALE

The smaller barrel has a height of h cm and a capacity of 100 litres.
The larger barrel has a height of 90 cm and a capacity of 160 litres.

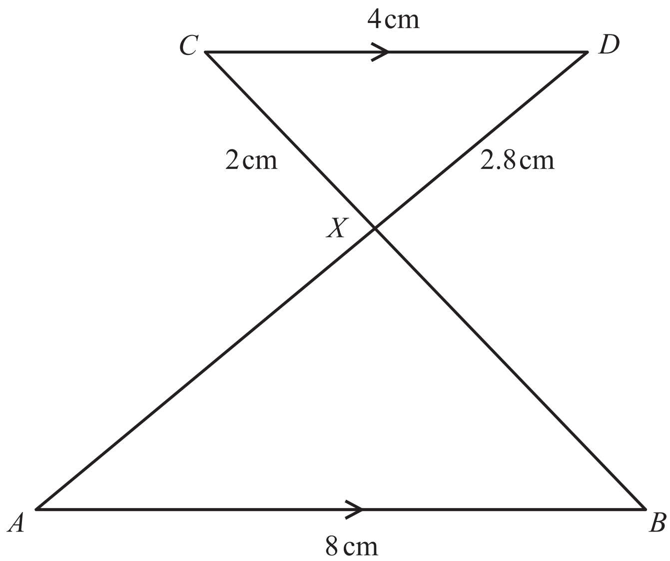
Work out the value of h. [3]

$\frac{100}{h} = \frac{160}{90}$

Now, solve for h :

$h = \frac{100 \times 90}{160}$

$h = 56.25\text{ cm}$



NOT TO SCALE

In the diagram, AB and CD are parallel.
AD and BC intersect at X.
$AB = 8$ cm, $CD = 4$ cm, $CX = 2$ cm and $DX = 2.8$ cm.

(a) Complete this mathematical statement. [1]

Triangle ABX is **similar** to triangle DCX.

(b) Calculate AX. [2]

Answer: $AX = 5.6$ cm

(c) The area of triangle ABX is y cm^2.

Find the area of triangle DCX in terms of y. [1]

DCX is $\frac{16}{9}y = \frac{16y}{9}$ cm^2.

Question 4

Two bottles and their labels are mathematically similar.
The smaller bottle contains 0.512 litres of water and has a label with area $96\,cm^2$.
The larger bottle contains 1 litre of water.

Calculate the area of the larger label. [3]

Answer:

area of larger lebal is equal to 96/0.512 which is 187.5 cm2

Question 5

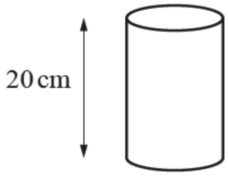
Two cups are mathematically similar.
The larger cup has capacity 0.5 litres and height 8 cm.
The smaller cup has capacity 0.25 litres.

Find the height of the smaller cup. [3]

Answer:

The smaller cup has a height of 4 cm

(a)

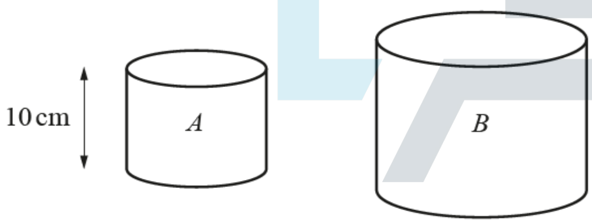


A cylinder has height 20cm.
The area of the circular cross section is 74cm^2.

Work out the volume of this cylinder. [1]

The given cylinder has a radius of $r = \sqrt{74\text{ cm}^2/\pi} = 4.9$ cm. Therefore, its volume is $V = \pi r^2 h = \pi(4.9\text{ cm})^2(20\text{ cm}) = 1480\text{ cm}^3$.

(b) Cylinder A is mathematically similar to cylinder B.



The height of cylinder A is 10 cm and its surface area is 440 cm^2.
The surface area of cylinder B is 3960 cm^2.

Calculate the height of cylinder B. [3]

$\frac{A_A}{A_B} = \left(\frac{h_A}{h_B}\right)^2$

Given that $A_A = 440\text{ cm}^2$ and $A_B = 3960\text{ cm}^2$, and $h_A = 10$ cm, you can use this information to find h_B :

$\frac{440}{3960} = \left(\frac{10}{h_B}\right)^2$

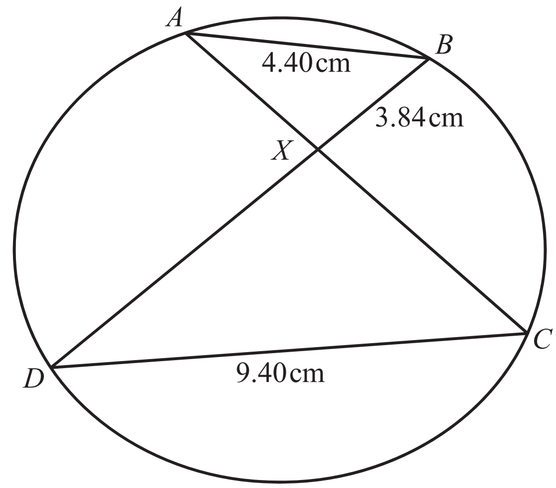
Solve for h_B :

$\frac{1}{9} = \frac{100}{h_B^2}$

$h_B^2 = 900$

$h_B = 30$ cm

Therefore, the height of cylinder B is 30 cm.



NOT TO SCALE

A, *B*, *C* and *D* lie on a circle.
AC and *BD* intersect at *X*.

(a) Give a reason why angle *BAX* is equal to angle *CDX*. [1]

Angle *BAX* is equal to angle CDX because they are opposite angles of a cyclic quadrilateral.

Therefore, angle BAX is congruent to angle CDX.

(b) *AB* = 4.40 cm, *CD* = 9.40 cm and *BX* = 3.84 cm.

(i) Calculate the length of *CX*. [2]

CX/BX = CD/AB
⇒ CX/3.84 = 9.40/4.40
⇒ CX = (9.40*3.84)/4.40
⇒ CX = 8.2
Thus, length of CX is 8.2 cm

(ii) The area of triangle *ABX* is 5.41 cm^2.

Calculate the area of triangle *CDX*. [2]

Since $\angle ABX = \angle CDX$, we can form a proportion of the corresponding sides:

$$\frac{BX}{CX} = \frac{AB}{CD}$$

Substituting in the given values, we get:

$$\frac{3.84}{CX} = \frac{4.40}{9.40}$$

Solving for CX, we get:

$$CX = \frac{3.84 \cdot 9.40}{4.40} = 8.32 \text{ cm}$$

Now, we can calculate the area of triangle CDX using the fact that it is similar to triangleABX :

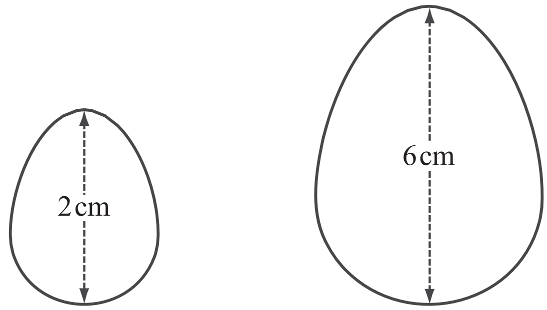
$$\frac{CDX}{ABX} = \left(\frac{CD}{AB}\right)^2$$

Substituting in the given values, we get:

$$\frac{CDX}{5.41} = \left(\frac{9.40}{4.40}\right)^2$$

Solving for CDX, we get:

$$CDX = 5.41 \cdot \left(\frac{9.40}{4.40}\right)^2 = 25.54 \text{ cm}^2$$



A company makes solid chocolate eggs and their shapes are mathematically similar.
The diagram shows eggs of height 2 cm and 6 cm.
The mass of the small egg is 4 g.

Calculate the mass of the large egg. [2]

$$\frac{\text{Volume of Small Egg}}{\text{Volume of Large Egg}} = \left(\frac{\text{Height of Small Egg}}{\text{Height of Large Egg}}\right)^3$$

Given that the height of the small egg is 2 cm and the height of the large egg is 6 cm, the ratio is $\frac{2}{6} = \frac{1}{3}$.

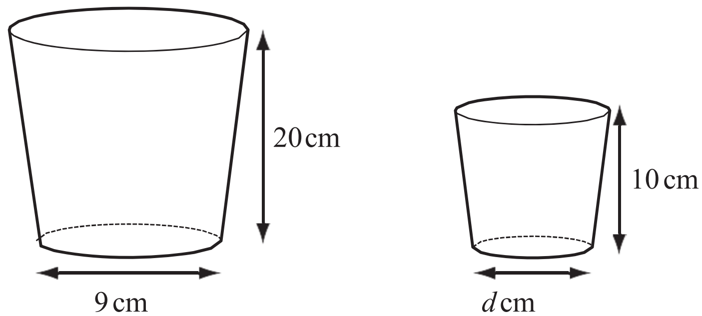
$$\frac{\text{Volume of Small Egg}}{\text{Volume of Large Egg}} = \left(\frac{1}{3}\right)^3 = \frac{1}{27}$$

Since the mass is directly proportional to the volume for a given material, the mass ratio is also $\frac{1}{27}$.

Given that the mass of the small egg is 4 g, the mass of the large egg is:

$$\text{Mass of Large Egg} = 4 \times 27 = 108 \text{ g}$$

Therefore, the mass of the large egg is 108 g.



The diagrams show two mathematically similar containers.
The larger container has a base with diameter 9 cm and a height 20 cm.
The smaller container has a base with diameter d cm and a height 10 cm.

(a) Find the value of d. [1]

$\frac{d}{9} = \frac{10}{20}$

Now, solve for d :

$d = \frac{10}{20} \times 9$

$d = \frac{1}{2} \times 9$

$d = 4.5$

Therefore, the value of d is 4.5 cm.

(b) The larger container has a capacity of 1600 ml. [2]

Calculate the capacity of the smaller container.

The ratio of volumes for similar shapes is the cube of the ratio of corresponding lengths. In this case, the corresponding length is the height.

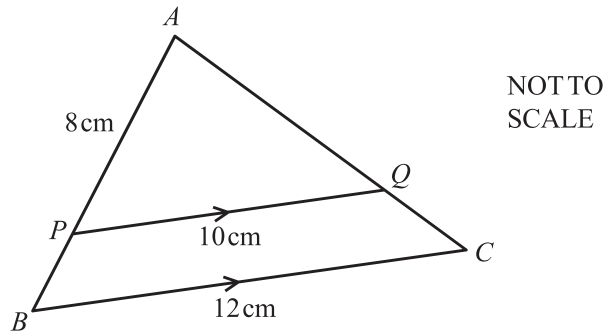
$$\text{Volume ratio } = \left(\frac{\text{height of larger container}}{\text{height of smaller container}}\right)^3$$

$$\text{Volume ratio } = \left(\frac{20}{10}\right)^3 = 2^3 = 8$$

Since the larger container has a capacity of 1600ml, the capacity of the smaller container is:

$$\text{Capacity of smaller container } = \frac{\text{Capacity of larger container}}{\text{Volume ratio}}$$

$$\text{Capacity of smaller container } = \frac{1600}{8} = 200\text{ml}$$



APB and *AQC* are straight lines. *PQ* is parallel to *BC*.
AP = 8 cm, *PQ* = 10 cm and *BC* = 12 cm. [2]
Calculate the length of *AB*.

$AP/AB = PQ/BC$
$=> 8/AB = 10/12$
$=> AB = 8 \times 12/10$
$=> AB = 9.6$ cm
length of $AB = 9.6$ cm

Question 11

A cylindrical glass has a radius of 3 centimetres and a height of 7 centimetres.
A large cylindrical jar full of water is a similar shape to the glass.
The glass can be filled with water from the jar exactly 216 times.
Work out the radius and height of the jar. [3]

Step 1: Find the volume of the glass

$V \approx 63\pi \mathrm{cm}^3$

Step 2: Find the volume of the jar

V_jar $= 216V_{\text{glass}} = 138,24\pi \mathrm{cm}^3$

Step 3: Find the radius and height of the jar

$r\_jar \approx 18$ cm

h_jar ≈ 42 cm

Therefore, the radius of the jar is 18 cm and the height of the jar is 42 cm.

Question 12

A car manufacturer sells a similar, scale model of one of its real cars.

(a) The fuel tank of the real car has a volume of 64 litres and the fuel tank of the model has a volume of 0.125 litres.
Show that the length of the real car is 8 times the length of the model car. [2]

The length of the real car is 8 times the length of the model car because the ratio of their fuel tank volumes is 512, which is the cube of 8.

(b) The area of the front window of the model is 0.0175 m .
Find the area of the front window of the real car.

Let r represent the scale factor.

Area ratio $= r^2$

Given that the area of the front window of the model is $0.0175 \mathrm{~m}^2$, we can set up the equation:

$0.0175 = r^2 \times$ Area of front window of real car [2]

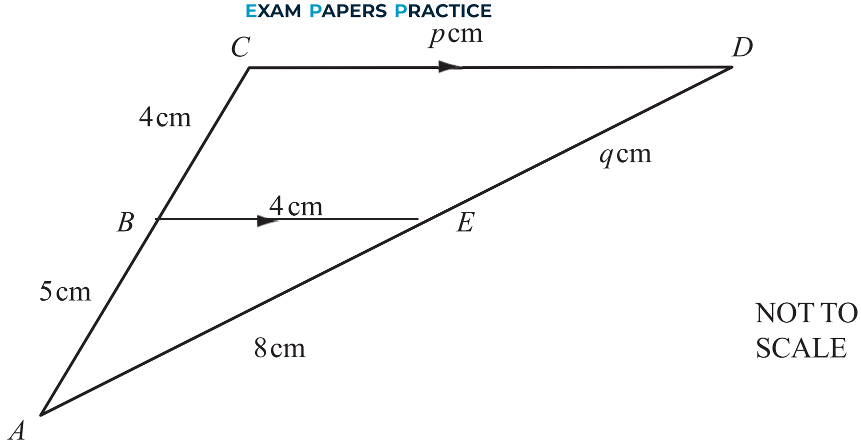
Solve for the area of the front window of the real car:

Area of front window of real car $= \frac{0.0175}{r^2}$

Without information about the specific scale factor r, we cannot provide a numerical answer.

Question 13

(a)



In the diagram triangles *ABE* and *ACD* are similar.
BE is parallel to *CD*.
AB = 5 cm, *BC* = 4 cm, *BE* = 4 cm, *AE* = 8 cm, *CD* = *p* cm and *DE* = *q* cm.
Work out the values of *p* and *q*.

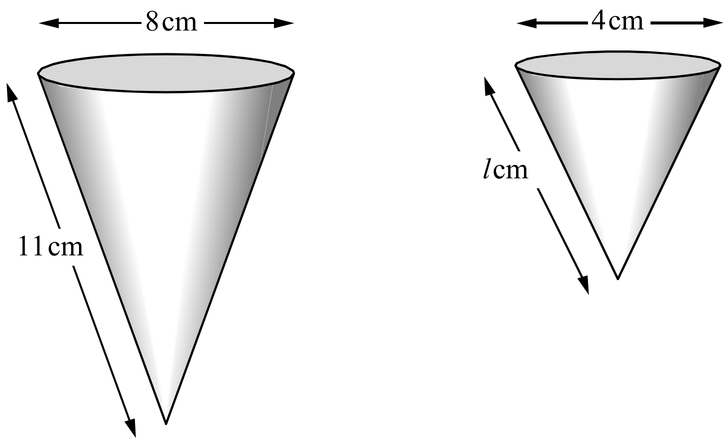
Answer:
$p = 7.2$
$q = 6.4$
Step-by-step explanation:
if the triangles are similar then the side lengths must be proportional :
$\mathrm{AC/BA = CD/BE}$
$9/5 = p/4$ cross multiply expressions
$5p = 36$ divide both sides by 5
$p = 7.2$
same goes for q
$9/5 = (q+8)/8$ cross multiply expressions
$72 = 5(q+8)$ divide both sides by 5
$14.4 = \mathrm{q} + 8$ subtract 8 from both sides
$6.4 = q$

[4]

(b) A spherical balloon of radius 3 metres has a volume of 36π cubic metres.
It is further inflated until its radius is 12 m.
Calculate its new volume, leaving your answer in terms of π.

The volume V of a sphere is given by the formula:
$V = \frac{4}{3}\pi r^3$
Given that the initial radius r of the spherical balloon is 3 m and the initial volume is $36\pi \mathrm{m}^3$, we can set up an equation:
$36\pi = \frac{4}{3}\pi\left(3^3\right)$
Now, solve for the scale factor (k) :
$k^3 = \frac{36\pi \times 3}{4\pi}$
$k^3 = 27$
$k = \sqrt[3]{27}$
$k = 3$
Now, the new radius (r') is 12 m, so the new volume (V') is:
$V' = \frac{4}{3}\pi\left(12^3\right)$
$V' = \frac{4}{3}\pi(1728)$
$V' = 2304\pi$
Therefore, the new volume of the spherical balloon, when inflated to a radius of 12 m, is$2304\pi \mathrm{m}^3$.

[2]



NOT TO SCALE

The two cones are similar.

(a) Write down the value of l. [1]

Since both cones, as shown in the figure attached below, are similar, 11/8 = 1/4

Cross multiply to find I

$4^{*11} = 8* |$

$44 = 8 |$ [2]

$|= 44/8$

$|= 5.5$ cm

(b) When full, the larger cone contains 172 cm^3 of water.
How much water does the smaller cone contain when it is full?

b. Given the slant height and diameter of cone, volume of cone can be calculated using the formula $1/3\pi r^{2*}\sqrt{\left(|^2 - r^2\right)}$

Where, $r =$ diameter $\div 2 = 4/2 = 2$ cm

I = 5.5 cm

Volume of smaller cone $= 1/3^*3.142^*2^{2*} \cdot \sqrt{\left(5.5^2 - 2^2\right)}$

$= 13^*3.142^*4^*\sqrt{30.25 - 4})$

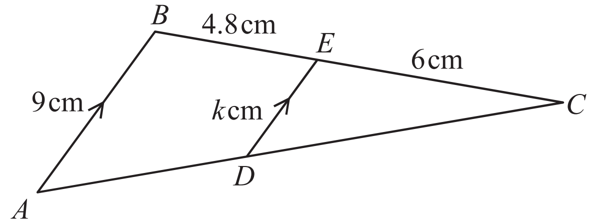
$= 13^*12.568^*\sqrt{26.25}$

$= 13^*12.568^*5.124$

$= 64.40/3$

Volume of small cone $= 21.5$ cm^3

(a)



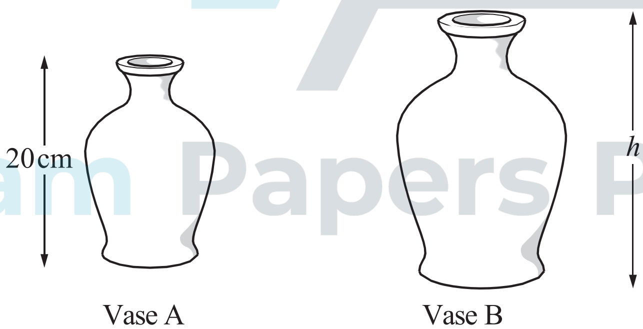
NOT TO SCALE

Triangles *CBA* and *CED* are similar.
AB is parallel to *DE*.
AB = 9 cm, *BE* = 4.8 cm, *EC* = 6 cm and *ED* = *k* cm.

Work out the value of *k*. [2]

k = 5 cm

(b)



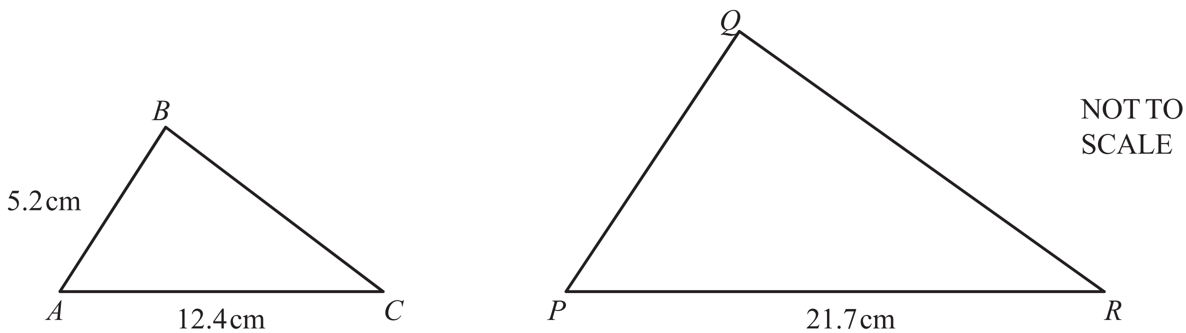
NOT TO SCALE

The diagram shows two mathematically similar vases.
Vase A has height 20 cm and volume 1500 cm^3.
Vase B has volume 2592 cm^3.

Calculate *h*, the height of vase B. [3]

the height of vase b is 34.56 cm.

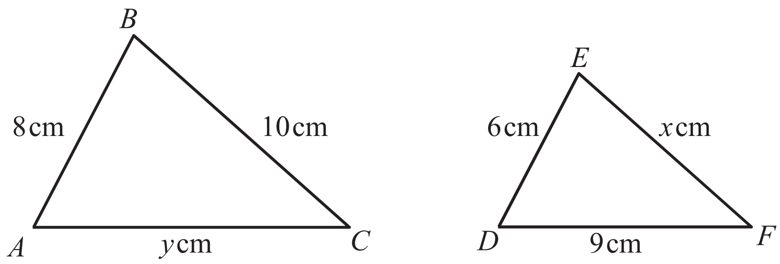
Triangle *ABC* is similar to triangle *PQR*.



Find *PQ*.

Answer: the two triangles are similar triangles..
So from the given information
ab/pq = ac/pr
5.2/pq = 12.4/21.7
52/10pq = 124/217
13/10pq = 31/217
13/10pq = 1/7
Pq = 9.1 cm
So side pq = 9.1 cm

[2]



Triangle ABC is similar to triangle DEF.

Calculate the value of

(a) x,

Given $\triangle ABC \sim \triangle DEF$

By property of similar triangles,

$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$

$\therefore \frac{8}{6} = \frac{10}{x} = \frac{y}{9}$

Equating first and second ratios, we get,

$\frac{8}{6} = \frac{10}{x}$

$\therefore 8x = 60$

$\therefore x = 7.5$

[2]

(b) y.

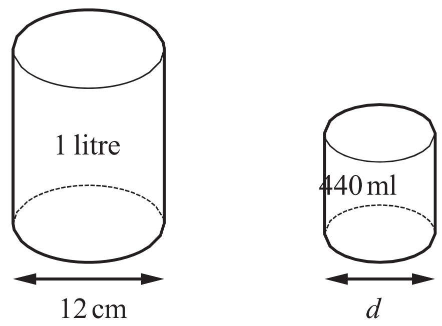
Equating first and third ratios, we get,

$\frac{8}{6} = \frac{y}{9}$

$\therefore 6y = 72$

$\therefore y = 12$

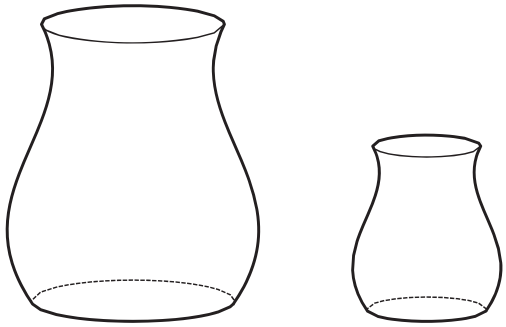
[2]



Two cylindrical cans are mathematically similar.
The larger can has a capacity of 1 litre and the smaller can has a capacity of 440ml.

Calculate the diameter, d, of the 440ml can. [3]

The diameter of the smaller can is $\cong 9.14$ cm



The two containers are mathematically similar in shape.
The larger container has a volume of $3456\,cm^3$ and a surface area of $1024\,cm^2$.
The smaller container has a volume of $1458\,cm^3$.

Calculate the surface area of the smaller container. [4]

$\frac{V_l}{V_s} = k^3$ and $\frac{A_l}{A_s} = k^2$

Substituting in the given values, we can solve for k and then the surface area of the smaller container:

$k = \sqrt[3]{\frac{3456}{1458}} = \frac{12}{9}$

$A_s = \frac{1024}{\left(\frac{12}{9}\right)^2} = 576\ \text{cm}^2$

The volumes of two similar cones are 36π cm^3 and 288π cm^3.
The base radius of the smaller cone is 3 cm.

Calculate the base radius of the larger cone. [3]

$\left(\frac{r_{\text{larger}}}{3}\right)^3 = \frac{288\pi}{36\pi}$

Solving for r_{larger} :

$r_{\text{larger}} = 2 \times 3 = 6$ cm

So, the base radius of the larger cone is 6 cm.

A company sells cereals in boxes which measure 10 cm by 25 cm by 35 cm.

They make a special edition box which is mathematically similar to the original box.

The volume of the special edition box is 15 120 cm^3.

[3]

Work out the dimensions of this box.

The dimensions of the special edition box, which is mathematically similar to the original (10 cm by 25 cm by 35 cm) and has a volume of 15120 cm^3, are approximately **18 cm by 45 cm by 63 cm**.