GCSE OCR Math J560

Similarity - Area & Volume

Answers
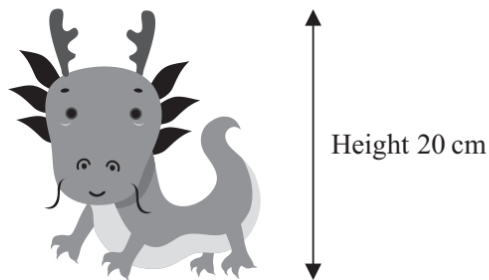
"We will help you to achieve A Star "

Answer 1

A company makes monsters.

The company makes small monsters with a height of 20 cm.

Height 20 cm

A small monster has a surface area of 300 cm^2.

The company also makes large monsters with a height of 120 cm.

A small monster and a large monster are mathematically similar.

Work out the surface area of a large monster.

SCALE FACTOR = $\dfrac{120}{20}$ = 6

SIMILAR SHAPES

AREA FACTOR
= (SCALE FACTOR)^2

VOLUME FACTOR
= (SCALE FACTOR)^3

AREA FACTOR = 6^2 = 36

LARGE SA = 300 × 36

= 10800 cm^2

$\begin{array}{r} 36 \\ 36 \\ \hline 72 \\ 36 \\ \hline 108 \end{array}$ cm^2

Answer 2

Here are two supermarket price tickets.

2 cm B

6 cm C
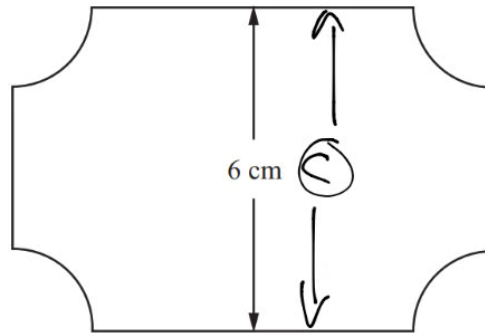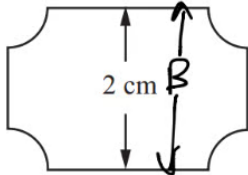
Diagrams **NOT**
accurately drawn

The two supermarket price tickets are mathematically similar.

The area of the smaller ticket is 7 cm².
Calculate the area of the larger ticket.

$$C = 6 \ , \ B = 2$$
$$C = 3 \times B$$
$$C^2 = 3^2 \times B^2$$
$$\therefore C^2 = 9 \times B^2$$

$$A_L = 9 \times A_S$$
$$\therefore A_L = 63$$

........63........ cm²

Answer 3

L and M are two mathematically similar prisms.

Diagram **NOT** accurately drawn

3 cm

8 cm

L

20 cm
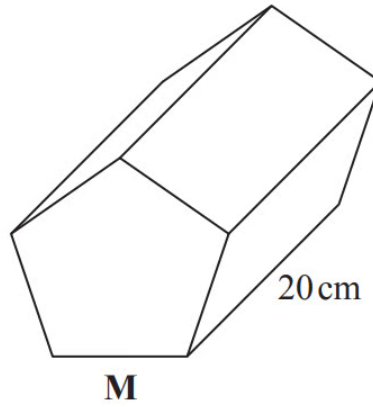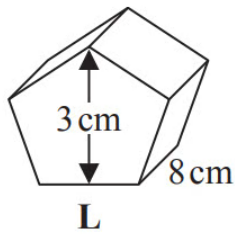
M

Prism **L** has length 8 cm.
Prism **M** has length 20 cm.

Prism **L** has height 3 cm.

(a) Work out the height of prism **M**.

L : m = 8:20

Sf = 20/8 = 2.5

3 x sf = height of m
3x 2.5 = 7.5

7.5

.... cm

Answer 4

Prism **M** has a volume of 1875 cm^3

(b) Work out the volume of prism **L**.

Volume scale factor = length sf^3

$2.5^3 = 125/8$

Vol L x Vsf = Vol M

Vol M / Vsf = Vol L

$1875 \times 8/125 = 120$

120

.... cm^3
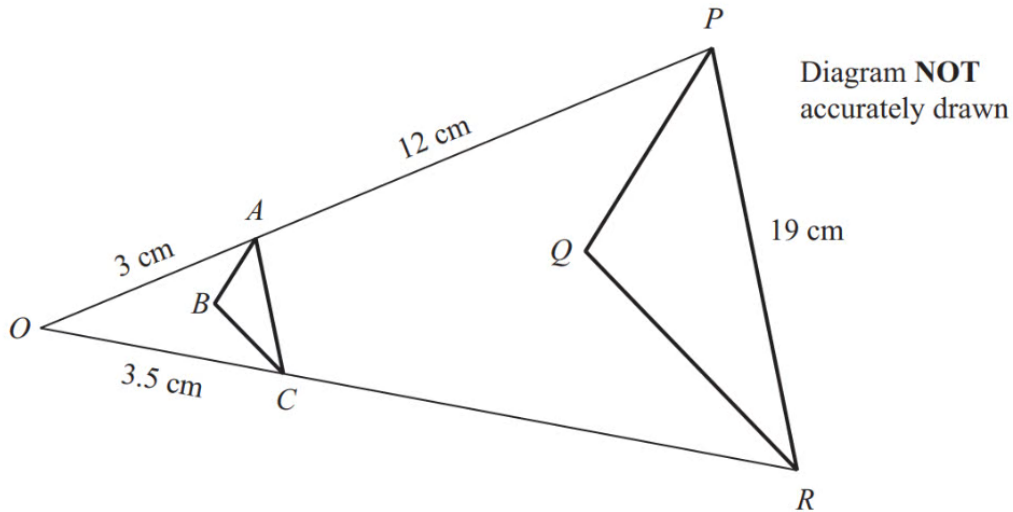
Answer 5

Triangle *PQR* is an enlargement, centre *O*, of triangle *ABC*.
OAP and *OCR* are straight lines.
OA = 3 cm.
AP = 12 cm.
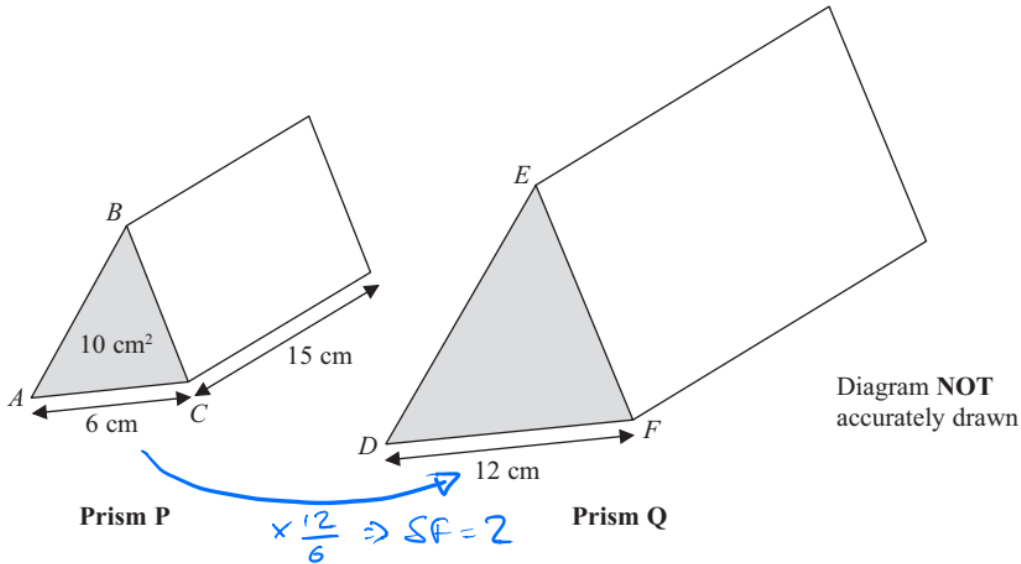*OC* = 3.5 cm.
PR = 19 cm.

(a) Work out the length of *CR*.

$$SF = \frac{12}{3} = 4 \qquad 3.5 \times 4 = 14$$

.... cm

Answer 6

P and Q are two triangular prisms that are mathematically similar.

Diagram **NOT** accurately drawn

Prism P

$\times \frac{12}{6} \Rightarrow SF = 2$

Prism Q

Prism **P** has triangle *ABC* as its cross section.
Prism **Q** has triangle *DEF* as its cross section.

$AC = 6$ cm
$DF = 12$ cm

The area of the cross section of prism **P** is 10 cm².
The length of prism **P** is 15 cm.

Work out the volume of prism **Q**.

PRISM

V = AREA OF CROSS SECTION × LENGTH

$V_Q = V_P \times$ VOLUME FACTOR

SIMILAR SHAPES

AREA FACTOR = (SCALE FACTOR)²

VOLUME FACTOR = (SCALE FACTOR)³

$V_P = 10 \times 15 = 150$ cm³

$V_Q = 150 \times 2^3$

$= 150 \times 2 \times 2 \times 2$

$= 300 \times 2 \times 2$

$= 1200$ cm³

__1200 cm³__

Answer 7

Fred is making two rectangular flower beds.

The dimensions of the larger rectangle will be three times the dimensions of the smaller rectangle.

$SF = 3$

There is going to be the same depth of soil in each flower bed.
Fred needs 180 kg of soil for the smaller flower bed.

AREA CHANGES
SO USE AREA FACTOR

Work out how much soil Fred needs for the larger flower bed.

SMALL   180 kg

SIMILAR SHAPES

AREA FACTOR $= (SCALE\ FACTOR)^2$

VOLUME FACTOR $= (SCALE\ FACTOR)^3$

$SF = 3$ } $AF = 3$

LARGE

$AMOUNT\ OF\ SOIL = 3^2 \times 180$

$= 1620\ kg$

Answer 8

Mark has made a clay model.
He will now make a clay statue that is mathematically similar to the clay model.

The model has a base area of $6\,cm^2$
The statue will have a base area of $253.5\,cm^2$

Mark used $2\,kg$ of clay to make the model.

Clay is sold in $10\,kg$ bags.
Mark has to buy all the clay he needs to make the statue.

How many bags of clay will Mark need to buy?

SIMILAR SHAPES

AREA FACTOR $= (\text{SCALE FACTOR})^2$

VOLUME FACTOR $= (\text{SCALE FACTOR})^3$

$$\text{AREA FACTOR} = \frac{253.5}{6}$$

$$\text{So SCALE FACTOR} = \sqrt{\frac{253.5}{6}} = \frac{13}{2}$$

$$\text{So VOLUME FACTOR} = \left(\frac{13}{2}\right)^3$$

$$\text{MASS OF STATUE} = 2 \times \left(\frac{13}{2}\right)^3$$

$$= 549.25 \, kg$$

$$\text{No OF BAG} = \frac{549.25}{10}$$

$$= 54.925 \text{ BAGS}$$

$$\text{WHOLE BAGS} = \underline{55} \text{ BAGS}$$

Answer 9

Solid **A** and solid **B** are mathematically similar.
The ratio of the surface area of solid **A** to the surface area of solid **B** is $4:9$

The volume of solid **B** is $405\,cm^3$.

Show that the volume of solid **A** is $120\,cm^3$.

SIMILAR SHAPES

AREA FACTOR $= (\text{SCALE FACTOR})^2$
VOLUME FACTOR $= (\text{SCALE FACTOR})^3$

$\qquad A \quad : \quad B$

AREA: $\sqrt{}\{\, 4 \quad : \quad 9 \,\}\sqrt{}$

LENGTH   CUBE$\{\, 2 \quad : \quad 3 \,\}$CUBE

VOLUME $2^3 \quad : \quad 3^3$

$\qquad\qquad 8 \quad : \quad 27$
×5   ×$\dfrac{405}{27}$ $\{$
$\qquad\qquad ? \quad : \quad 405\,\}\times\dfrac{405}{27} = 15$

VOLUME OF A $= 8 \times 15$

$\qquad\qquad = 120\,cm^3$

$\qquad\qquad ×27$
$1 : \quad 27$
$2 : \quad 54$
$4 : 108$
$8 : 216$
$16 : 432$
$-1\qquad\qquad\qquad -27$
$15 : 405$

Answer 10

ABCD and *APQR* are two similar quadrilaterals.

PQ = 9 cm.
BC = 6 cm.
AD = 5 cm.
QR = 12 cm.

9 cm

P   *Q*

Diagram **NOT**
accurately drawn

B

6 cm

C

12 cm

A

5 cm

D

R
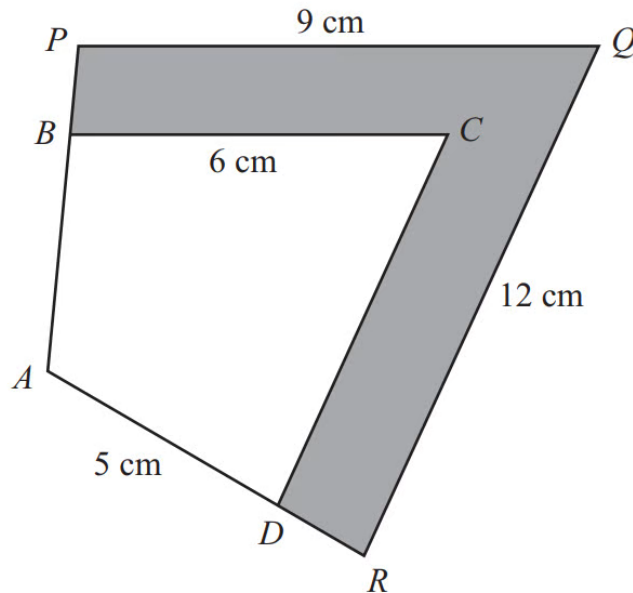
(a) Find the length of *DC*.

Scale factor big - small = 6/9
6/9 x 12 = 8

.............8............. cm

Answer 11

Here are two similar solid shapes.

A ⟵ AF = $\frac{3}{4}$ B
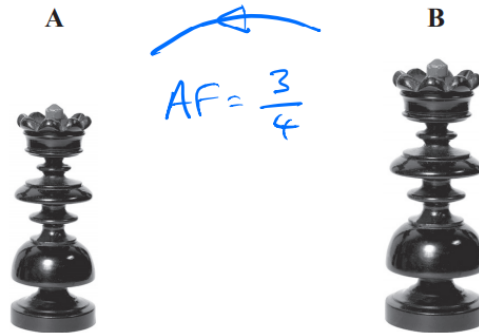
surface area of shape **A** : surface area of shape **B** = 3 : 4

The volume of shape **B** is $10\,cm^3$

Work out the volume of shape **A**.
Give your answer correct to 3 significant figures.

SIMILAR SHAPES

AREA FACTOR = (SCALE FACTOR)^2

VOLUME FACTOR = (SCALE FACTOR)^3

$AF = \frac{3}{4}$ so $SF = \sqrt{\frac{3}{4}}$

So $VF = \left(\sqrt{\frac{3}{4}}\right)^3$

$VOL_A = VF \times VOL_B$

$= \left(\sqrt{\frac{3}{4}}\right)^3 \times 10$

$= 6.49519...$

≥ 5
ROUND UP

$= 6.50\,cm^3$

Answer 12

B — 18 cm — C

$x \dfrac{18}{12}$

12 cm

E — F

Diagram **NOT**
accurately drawn

5 cm

36 cm^2
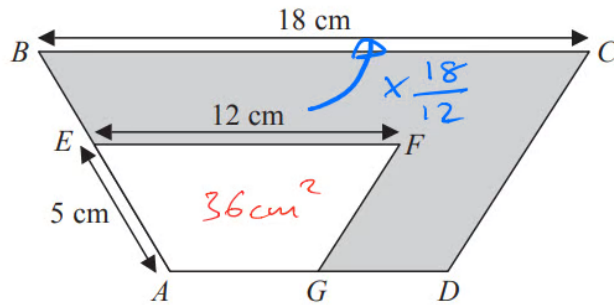
A   G   D

ABCD and AEFG are mathematically similar trapeziums.
AE = 5 cm
EF = 12 cm
BC = 18 cm

↳ USE SCALE FACTOR.

(a) Work out the length of AB.

$SF = \dfrac{18}{12} = \dfrac{6 \times 3}{6 \times 2}$

$= \dfrac{3}{2}$

$AB = AE \times SF$

$= \dfrac{5}{1} \times \dfrac{3}{2}$

$= \dfrac{15}{2} \text{ cm} \left(7 \cdot 5 \text{ cm} \right)$

Answer 13

Cone **A** and cone **B** are mathematically similar.
The ratio of the volume of cone **A** to the volume of cone **B** is 27 : 8

The surface area of cone **A** is $297\,cm^2$

Show that the surface area of cone **B** is $132\,cm^2$

SIMILAR SHAPES

AREA FACTOR $= (SCALE\ FACTOR)^2$
VOLUME FACTOR $= (SCALE\ FACTOR)^3$

$$A \quad : \quad B$$

$$VOLUME: \quad 27 \quad : \quad 8$$

$$VF = \frac{8}{27}$$

$$SO \quad SF = \sqrt[3]{\frac{8}{27}}$$

$$SF = \frac{2}{3}$$

$$SO \quad AF = \left(\frac{2}{3}\right)^2$$

$$AF = \frac{4}{9}$$

$$AREA\ OF\ B = \frac{4}{9} \times AREA\ OF\ A$$

$$= \frac{4}{9} \times 297$$

$$= 132\,cm^2$$

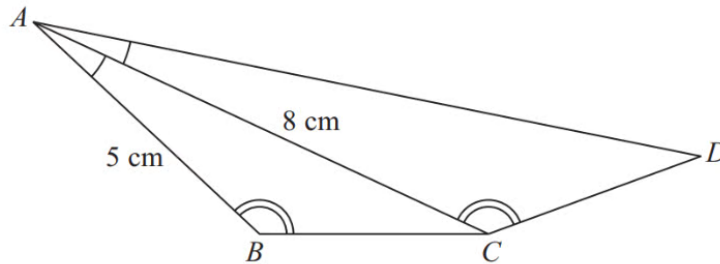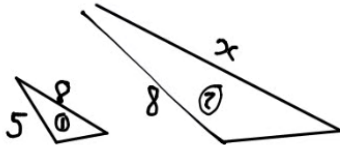## Answer 14

Triangles *ABC* and *ACD* are similar.

Diagram **NOT** accurately drawn

8 cm

5 cm

Angle *BAC* = angle *CAD*.
Angle *ABC* = angle *ACD*.
AB = 5 cm and *AC* = 8 cm.

(a) Calculate the length of *AD*.

Break down into Two similar triangles

$\frac{8}{5}$ = Scale factor

$8 \times \frac{8}{5} = x = \frac{64}{5} = 12.8$

12.8 .... cm

Answer 15

Triangles ABC and ACD are similar.

A

Diagram **NOT**
accurately drawn

8 cm

5 cm

D

B C

Angle BAC = angle CAD.
Angle ABC = angle ACD.
AB = 5 cm and AC = 8 cm.

(a) Calculate the length of AD.

x

8   ⑦

8

5   ①

Break down into Two similar triangles

$\dfrac{8}{5}$ = Scale factor

$8 \times \dfrac{8}{5} = x = \dfrac{64}{5} = 12.8$

......12.8...................... cm