EXAM PAPERS PRACTICE

Sequences

Model Answers

Question 1

Here are the first four terms of a sequence.

23 17 11 5

(a) Find the next term. [1]

Based on the pattern, each term is 6 less than the previous term.
Therefore, the next term would be $5 - 6 = -1$.

(b) Find the *n*th term.

$$-6n + 29$$

[2]

Question 2

7, 5, 3, 1, – 1, …

(a) Find the next term in this sequence. [1]

Based on the pattern, each term is 2 less than the previous term. Therefore, the next term in the sequence would be $-1-2=-3$.
Here is the complete sequence:
$7, 5, 3, 1, -1, -3, \ldots$

(b) Find the *n*th term of the sequence. [2]

The first term is 7 and each subsequent term takes away another 2 . This gives us
$u_1 = 7$
$u_2 = 7 - 2$
$u_3 = 7 - 2 \times 2$
$u_4 = 7 - 2 \times 3$
We can see that in general we can write
$u_n = 7 - 2(n-1)$
This can also be written as:
$u_n = 9 - 2n$

Question 3

Find the *n*th term of each sequence.

(a) 4, 8, 12, 16, 20, ... [1]

The sequence is just multiples of 4 .

$4 = 4 \times 1, \quad 8 = 4 \times 2, \quad 12 = 4 \times 3$

The n-th term is: **4n**

(b) 11, 20, 35, 56, 83, ... [2]

The relation is not linear, as the common difference changes.

We guess that the relation is quadratic with general form $an^2 + bn + c$.

Use given numbers to form three equations for $n = 1, n = 2$ and $n = 3$.

$a + b + c = 11$

$4a + 2b + c = 20$

$9a + 3b + c = 35$

Subtract the first equation from the second: $3a + b = 9$

Subtract the first equation from the third: $8a + b = 24$

From these two equations, we can see that $b = 0$ and $a = 3$.

Use the first equation to calculate the value of c :

$3 + 0 + c = 11$

$c = 8$

Therefore the nth term of the sequence is

$3n^2 + 8$

(We can check that this formula holds by plugging the values for $n = 4$ and $n = 5$)

Question 4

5, 11, 21, 35, 53, ...

Find the *n*th term of this sequence. [2]

Therefore the nth term of the sequence is

$2n^2 + 3$

(We can check that this formula holds by plugging the values for $n = 4$ and $n = 5$)

Question 5

These are the first five terms of a sequence.

13 8 3 –2 –7

Find the *n*th term of this sequence. [2]

$18 - 5n$

Question 6

32 25 18 11 4

These are the first 5 terms of a sequence.

Find

(a) the 6th term, [1]

-3

(b) the *n*th term,

$u_n = -7n + 39$

[2]

(c) which term is equal to –332.

[2]

$$-7n + 39 = -332$$
$$\rightarrow -7n = -371$$
$$\rightarrow n = 53$$

Question 7

The first five terms of a sequence are shown below.

13 9 5 1 –3

Find the *n*th term of this sequence. [2]

Position: 1 2 3 4 5
Term: 13 9 5 $1-3$
In this case, to get from one term to the other we subtract 4.
We observe that the rule for getting from each position to its corresponding term is:$17-4\times$ position
For position n, the term is therefore: $17-4n$
The nth term of the sequence is: $17-4n$

Question 8

A sequence is given by $u_1=\sqrt{1}$, $u_2=\sqrt{3}$, $u_3=\sqrt{5}$, $u_4=\sqrt{7}$, …

(a) Find a formula for u_n, the *n*th term. [2]

$$u_1=\sqrt{2\times1-1}=\sqrt{1}$$
$$u_2=\sqrt{2\times2-1}=\sqrt{3}$$
$$u_3=\sqrt{2\times3-1}=\sqrt{5}$$

By looking at the terms above we deduce the following nth term:

$$u_n=\sqrt{2n-1}$$

(b) Find u_{29}. [1]

We substiute $n=29$ in the nth term above to work out u_{29}.

$$U_{29}=\sqrt{2\times29-1}=\sqrt{57}\approx7.55$$

Question 9

(a) The formula for the *n*th term of the sequence

$$1,\ 5,\ 14,\ 30,\ 55,\ 91, \ldots \quad \text{is} \quad \frac{n(n+1)(2n+1)}{6}.$$

Find the 20th term. [1]

The given formula for the nth term of the sequence is $u_n = \frac{n(n+1)(2n+1)}{6}$.

To find the 20 th term (u_{20}), substitute $n = 20$ into the formula:

$u_{20} = \frac{20(20+1)(2 \cdot 20+1)}{6}$.

Now, simplify the expression:

$u_{20} = \frac{20 \cdot 21 \cdot 41}{6}$.

Cancel out common factors (divide both the numerator and denominator by 2):

$u_{20} = \frac{10 \cdot 21 \cdot 41}{3}$.

Now, calculate the product in the numerator:

$u_{20} = \frac{8610}{3}$.

Finally, simplify the fraction:

$u_{20} = 2870$.

So, the 20 th term of the given sequence is 2870 .

(b) The *n*th term of the sequence 10, 17, 26, 37, 50, ... is $(n + 2)^2 + 1$.

Write down the formula for the *n*th term of the sequence 17, 26, 37, 50, 65, … [1]

$$((n+1)+2)^2+1$$
$$= (n+3)^2+1$$

Question 10

For each of the following sequences, write down the next term.

(a) 2, 3, 5, 8, 13, … [1]

Answer

The next term in the sequence is 21.

The sequence is the Fibonacci sequence.

(b) x^6, $6x^5$, $30x^4$, $120x^3$, … [1]

We notice the following pattern:

$$\frac{6x^6}{x} = 6x^5$$

$$\frac{5x6x^5}{x} = 30x^4$$

$$\frac{4x30x^4}{x} = 120x^3$$

$$\frac{3x120x^3}{x}$$

$$= 360x^2$$

[1]

(c) 2, 6, 18, 54, 162, ...

The given sequence is a geometric sequence where each term is obtained by multiplying the previous term by a constant factor. In this case, the common ratio (constant factor) is 3.

To find the next term, multiply the last term by the common ratio:

162 × 3 = 486.

Question 11

For the sequence $5\frac{1}{2}$, 7, $8\frac{1}{2}$, 10, $11\frac{1}{2}$, …

(a) find an expression for the *n*th term, [2]

We notice that every term is formed by adding $1\frac{1}{2}$ to the previous term.
Following this rule, the first term is formed by adding: $4 + 1\frac{1}{2}$
The second one is formed by adding: $4 + 1\frac{1}{2} \times 2$
The nth term will be
: $4 + 1\frac{1}{2}n$

(b) work out the 100th term. [1]

The 100^{th} term will be: $4 + 1\frac{1}{2} \times 100 = 4 + 100\frac{1}{2}$
$= 154$

Question 12

Write down the next term in each of the following sequences.

(a) 8.2, 6.2, 4.2, 2.2, 0.2, … [1]

In this sequence, each term is 2 less than the term before it. Thus, the next term is $0.2 - 2 =$ -1.8 .

(b) 1, 3, 6, 10, 15, … [1]

The next term in the sequence is **21**.

Question 13

1 A pattern of numbers is shown below.

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 x 20 x 6 1

Write down the value of x. [1]

In the pattern, we notice that a number is the sum of the 2 numbers immediately above it.

For example, $3 = 2 + 1$

$4 = 3 + 1$

$10 = 4 + 6$ etc.

Therefore, $x = 10 + 5 = 15$

$x = 15$

Question 14

8, 15, 22, 29, 36, ……..

A sequence of numbers is shown above.

(a) Find the 10th term of the sequence. [1]

This is an arithmetic sequence with a common difference of 7 . The first term is 8 , so the 10 th term is equal to $8+9 \cdot 7=71$.

(b) Find the *n*th term of the sequence.

To express the nth term (a_n) in terms of n, we can use the formula for the nth term of an arithmetic sequence:
$a_n=a_1+(n-1) d$,
where:
- a_n is the nth term,
- a_1 is the first term,
- n is the term number, and [1]
- d is the common difference.

In this sequence:
- $a_1=8$ (the first term),
- $d=7$ (the common difference).

Now, substitute these values into the formula:
$a_n=8+(n-1) \cdot 7$
Simplify the expression:
$a_n=8+7 n-7$.
Combine like terms:
$a_n=7 n+1$.
So, the nth term of the given sequence is $a_n=7 n+1$.

[1]

(c) Which term of the sequence is equal to 260?

Answer

We can find the answer by substituting different values of n in the formula from part 1. For the nth term to be equal to 260, we need to find the value of n such that 8+7(n-1)=260. Solving this equation, we get n=37. Therefore, the 37th term of the sequence is equal to 260.

Question 15

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The first five terms of a sequence are 4, 9, 16, 25, 36, …
Find

(a) the 10th term, [1]

$$10^{\text{th}} \text{ term } = (1+10)^2 = 121$$

(b) the *n*th term. [1]

$$\text{Nth term } = (1+n)^2$$

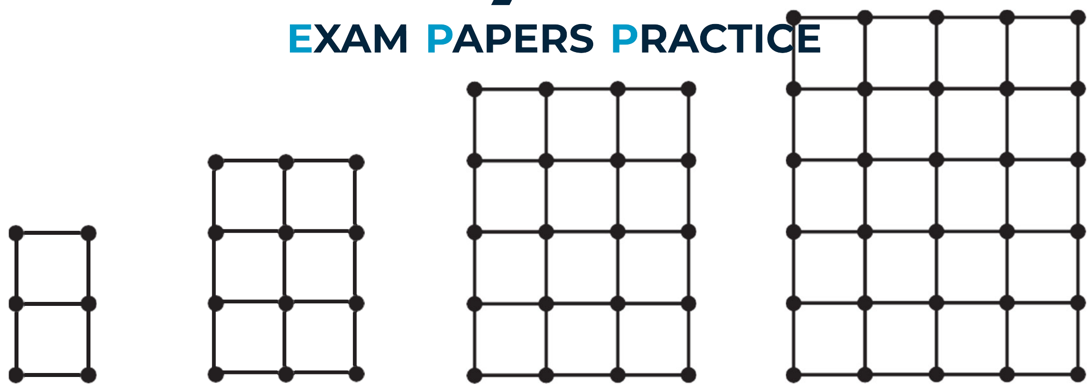


Diagram 1 Diagram 2 Diagram 3 Diagram 4

The first four Diagrams in a sequence are shown above.
Each Diagram is made from dots and one centimetre lines.
The area of each small square is 1 cm^2.

(a) Complete the table for Diagrams 5 and 6.

Diagram	1	2	3	4	5	6
Area (cm^2)	2	6	12	20	**10**	**18**
Number of dots	6	12	20	30	**12**	**18**
Number of one centimetre lines	7	17	31	49	**17**	**31**

(b) The area of Diagram n is $n(n + 1)$ cm^2.

(i) Find the area of Diagram 50. [1]

$n(n+1) = 50(50+1) = 2550 \text{ cm}^2$
Therefore, the answer is 2550 .

(ii) Which Diagram has an **area** of 930 cm^2? [1]

The area of each small square in the diagrams is 1 cm2, so we can count the number of squares in each diagram to find its area.
Diagram 1: 2 squares
Diagram 2: 6 squares
Diagram 3: 12 squares
Diagram 4: 20 squares
We can see that Diagram 3 has an area of 12 cm2, and Diagram 4 has an area of 20 cm2. So, the area of Diagram 30 is $30(30+1) = 930$ cm2.
Therefore, the diagram with an area of 930 cm2 is Diagram 30 .

(c) Find, in terms of n, the number of **dots** in Diagram n. [

To do this, we can first find the number of dots in the first few diagrams.
Diagram 1 has 6 dots.
Diagram 2 has 12 dots, which is 6 more dots than Diagram 1.
Diagram 3 has 20 dots, which is 8 more dots than Diagram 2.
Diagram 4 has 30 dots, which is 10 more dots than Diagram 3.
We can see that the number of dots in each diagram increases by 2 more dots than the previous diagram. This means that the number of dots in Diagram n is given by the arithmetic sequence $6, 8, 10, \ldots$
The general form of an arithmetic sequence is $a_1 + d(n-1)$, where a_1 is the first term, d is the common difference, and n is the term number. In this case, $a_1 = 6$ and $d = 2$.
Therefore, the number of dots in Diagram n is given by the following expression:
$6 + 2(n-1)$
This can also be written as:
$2n + 4$
Therefore, the answer is $2n + 4$.

(d) The number of one centimetre lines in Diagram n is $2n^2 + pn + 1$.

(i) Show that $p = 4$. [2]

To show that $p = 4$, we can use the fact that the number of one centimetre lines in Diagram 1 is 7 . This is because Diagram 1 has 4 horizontal lines and 3 vertical lines.
Substituting $n = 1$ into the given formula, we get:
$2 \cdot 1^2 + p \cdot 1 + 1 = 7$
Simplifying, we get:
$p + 3 = 7$
Subtracting 3 from both sides, we get:
$p = 4$
Therefore, we have shown that $p = 4$.

(ii) Find the number of one centimetre lines in Diagram 10. [1]

From the formula, we have that the number of one centimetre lines in Diagram 10 is $2 \cdot 10^2 + p.10 + 1 = 200 + 10p + 1 = 201 + 10p$.
We can find the value of p by looking at the table for Diagram 4 . The table tells us that there are 49 one centimetre lines in Diagram 4 . Substituting this into the formula, we get $2 \cdot 4^2 + p \cdot 4 + 1 = 49$, which simplifies to $33 = p$.
Therefore, the number of one centimetre lines in Diagram 10 is $201 + 10 \cdot 33 = 531$.

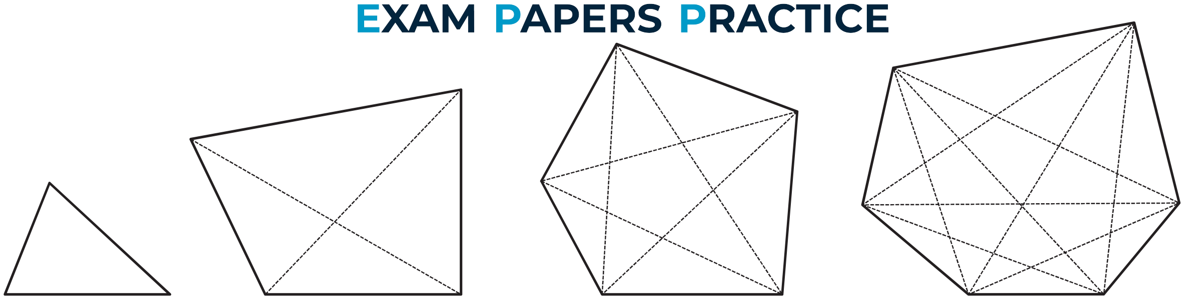
(iii) Which Diagram has 337 one centimetre lines? [3]

To find which diagram has 337 one centimeter lines, we need to solve the equation $2n^2 + pn + 1 = 337$.
First, we can subtract 1 from both sides to get $2n^2 + pn = 336$. We can then divide both sides by 2 to get $n^2 + \frac{p}{2}n = 168$.
Next, we can complete the square by taking half of the coefficient of our n term, squaring it, and adding it to both sides of the equation. The coefficient of our n term is $\frac{p}{2}$, so half of it would be $\frac{p}{4}$. Squaring this gives us $\frac{p^2}{16}$.
Therefore, we can add $\frac{p^2}{16}$ to both sides of the equation to get $n^2 + \frac{p}{2}n + \frac{p^2}{16} = 168 + \frac{p^2}{16}$. This can be rewritten as $\left(n + \frac{p}{4}\right)^2 = \left(\frac{22}{4} + \frac{p}{4}\right)^2$.
Taking the square root of both sides gives us $n + \frac{p}{4} = \pm\left(\frac{22}{4} + \frac{p}{4}\right)$. Subtracting $\frac{p}{4}$ from both sides gives us $n = \pm\frac{22}{4} \pm \frac{p}{4}$.
Since n must be an integer, the only possible solution is $n = 6$. This means that the sixth diagram has 337 one centimeter lines.
Answer: Diagram 6

(e) For **each** Diagram, the number of squares of area 1 cm^2 is A, the number of dots is D and the number of one centimetre lines is L.

Find a connection between A, D and L that is true for each Diagram. [1]

$D = 2A - 1$
and
$L = A - 1$
Substituting the value of L from the second equation into the first equation, we get:
$D = 2A - 1 = 2(A - 1) + 1 = L + 1$
Therefore, the connection between A, D, and L that is true for each Diagram is:
$D = L + 1$



The diagrams show some polygons and their diagonals.

(a) Complete the table.

Number of sides	Name of polygon	Total number of diagonals
3	triangle	0
4	quadrilateral	2
5	**pentagon**	5
6	hexagon	9
7	heptagon	14
8	**octagon**	**20**

(b) Write down the total number of diagonals in

(i) a decagon (a 10-sided polygon), [1]

The total number of diagonals in a decagon is 35.

(ii) a 12-sided polygon. [1]

The total number of diagonals in a 12-sided polygon is 54.

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(c) A polygon with n sides has a total of $\frac{1}{p}n(n-q)$ diagonals, where p and q are integers.

(i) Find the values of p and q. [3]

the values of p and q in the formula $\frac{1}{p}n(n-q)$ are $p=1$ and $q=4$.

(ii) Find the total number of diagonals in a polygon with 100 sides. [1]

The formula $\frac{1}{p}n(n-q)$ gives us the number of diagonals in a polygon with n sides if $p=2$ and$q=3$. Therefore, a polygon with 100 sides has a total of $\frac{1}{2}\cdot 100\cdot(100-3)=4850$ diagonals.
We can check this answer by using the formula we derived earlier: $\frac{n(n-3)}{2}$. Plugging in $n=100$, we get $\frac{100(100-3)}{2}=4850$. Therefore, both formulas give us the same answer.
The image shows a polygon with 100 sides and 4850 diagonals.

(iii) Find the number of sides of a polygon which has a total of 170 diagonals. [2]

the number of sides of a polygon which has a total of 170 diagonals is 20.

(d) A polygon with $n+1$ sides has 30 more diagonals than a polygon with n sides.

Find n. [1]

Let's call the polygon with n sides P_n and the polygon with n + 1 sides P_{n+1}. We know that P_{n+1} has 30 more diagonals than P_n, so we can write the following equation:
$n\_d\left(P\_\{n+1\}\right)=n\_d\left(P\_n\right)+30$
where $n_d\left(P_n\right)$ is the number of diagonals in polygon P_n.
We can use the formula for the number of diagonals in a polygon to rewrite the equation as follows:
$(n+1)(n)/2=n(n-3)/2+30$
Multiplying both sides of the equation by 2 , we get:
$(n+1)n=n(n-3)+60$
Expanding the right side of the equation, we get:
$(n+1)n=n^\wedge 2-3n+60$
Subtracting n^2-3n from both sides of the equation, we get:
$n+1=60$
Subtracting 1 from both sides of the equation, we get:
n = 59
Therefore, the polygon with n sides has 59 sides.
Answer: 59

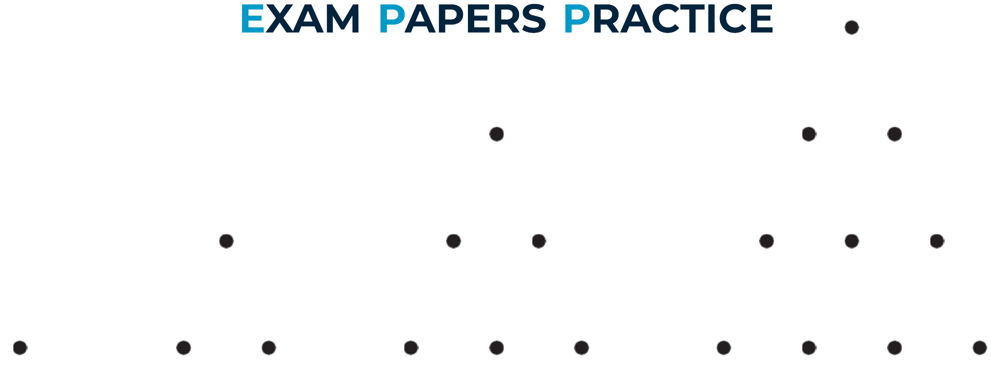

Diagram 1 Diagram 2 Diagram 3 Diagram 4

The first four terms in a sequence are 1, 3, 6 and 10.
They are shown by the number of dots in the four diagrams above.

(a) Write down the next four terms in the sequence. [2]

the next four terms in the sequence are 15, 21, 28, and 36.

(b) (i) The sum of the two consecutive terms 3 and 6 is 9. The sum of the two consecutive terms 6 and 10 is 16.

Complete the following statements using different pairs of terms.

The sum of the two consecutive terms **10** and **15** is **25**.

The sum of the two consecutive terms **15** and **21** is **36**. [1]

(ii) What special name is given to these sums? [1]

partial sums

(c) (i) The formula for the nth term in the sequence 1, 3, 6, 10… is $\frac{n(n+1)}{k}$, where k is an integer. Find the value of k.

the value of k is 2.

(ii) Test your formula when $n = 4$, **showing your working.** [1]

To test the formula when $n = 4$, we can simply plug in $n = 4$ into the formula and evaluate:
Number of diagonals $= 4(4-3)/2 = 4(1)/2 = 4/2 = 2$
This matches the number of diagonals shown in the diagram for $n = 4$, which is 2 .
Therefore, the formula for the number of diagonals in a polygon is correct.

[1]

(iii) Find the value of the 180th term in the sequence.

Answer: 16200

(d) (i) Show clearly that the sum of the nth and the $(n + 1)$th terms is $(n + 1)^2$. [3]

To show that the sum of the nth and the $(n+1)$th terms is $(n+1)^2$, we can use the following steps:
1. Let the nth term be T_n and the $(n+1)$th term be T_{n+1}.
2. We know that the sum of two consecutive terms in an arithmetic progression is equal to the average of those two terms. Therefore, we can write the following equation:
$T_n + T_{n+1} = (T_n + T_{n+1})/2$
3. Multiplying both sides of the equation by 2 , we get:
$2T_n + 2T_{n+1} = T_n + T_{n+1}$
4. Subtracting $T_n + T_{n+1}$ from both sides of the equation, we get:
$T_n = T_{n+1}$
5. This means that the nth and $(n+1)$th terms are equal. Therefore, the sum of the nth and $(n+1)$th terms is equal to twice the nth term:
Sum of nth and $(n+1)$th terms $= 2T_n$
6. We also know that the nth term is equal to $(n)(n+1)$, since the sequence is an arithmetic progression with a common difference of 1 . Therefore, we can substitute $(n)(n+1)$ for T_n in the above equation:
Sum of nth and $(n+1)$th terms $= 2(n)(n+1)$
7. Simplifying the right side of the equation, we get:
Sum of nth and $(n+1)$th terms $= (2n^2 + 2n) = (n+1)^2$
Therefore, we have shown that the sum of the nth and the $(n+1)$th terms is $(n+1)^2$.

(ii) Find the values of the two consecutive terms which have a sum of 3481. [2]

If the two consecutive terms are a and $a+1$, then we have:$a + (a+1) = 34812a + 1 = 34812a = 3480a = 1740$
Therefore, the two consecutive terms are 1740 and 1741.

1	2	3	4	5	6
7	8	9	10	11	12
13	14	15	16	17	18
19	20	21	22	23	24
25	26	27	28	29	30
31	32	33	34	35	36

A 3 by 3 square

x	b	c
d	e	f
g	h	i

can be chosen from the 6 by 6 grid above.

(a) One of these squares is

8	9	10
14	15	16
20	21	22

.

In this square, $x = 8$, $c = 10$, $g = 20$ and $i = 22$.

For this square, calculate the value of

(i) $(i - x) - (g - c)$, 4 [1]

(ii) $cg - xi$. 24 [1]

(b)

x	b	c
d	e	f
g	h	i

(i) $c = x + 2$. Write down g and i in terms of x. $x + 12, x + 14$ o.e. [2]

(ii) Use your answers to **part(b)(i)** to show that $(i - x) - (g - c)$ is constant.

$(x + 14 - x)$ and $(x + 12 - (x + 2))$

$14 - 10$ or $14 - 12 + 2$ or 4

(iii) Use your answers to **part(b)(i)** to show that $cg - xi$ is constant. [2]

$(x + 2)(x + 12) - x(x + 14)$

24

(c) The 6 by 6 grid is replaced by a 5 by 5 grid as shown.

1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20
21	22	23	24	25

A 3 by 3 square

x	b	c
d	e	f
g	h	i

can be chosen from the 5 by 5 grid.

For any 3 by 3 square chosen from this 5 by 5 grid, calculate the **value** of

(i) $(i - x) - (g - c)$, 4 [1]

(ii) $cg - xi$. 20 [1]

(d) A 3 by 3 square is chosen from an n by n grid.

(i) Write down the value of $(i - x) - (g - c)$. [1]

4

(ii) Find g and i in terms of x and n. [2]

$x + 2n$ o.e., $x + 2 + 2n$ o.e.

(iii) Find $cg - xi$ in its simplest form. [1]

$4n$

The table shows some terms of several sequences.

Term	1	2	3	4		8	
Sequence P	7	5	3	1		p	
Sequence Q	1	8	27	64		q	
Sequence R	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{3}{4}$	$\frac{4}{5}$		r	
Sequence S	4	9	16	25		s	
Sequence T	1	3	9	27		t	
Sequence U	3	6	7	−2		u	

(a) Find the values of p, q, r, s, t and u. [6]

p = 3 q = 64 r = 9 s = 25 t = 27 u = 5

(b) Find the nth term of sequence

$7-2(n-1)$

(i) P, [1]

(ii) Q, [1]

(iii) R, [1]

(iv) S, [1]

(v) T, [1]

(vi) U. [1]

(c) Which term in sequence **P** is equal to −777? [2]

The 388 th term in sequence P is equal to -777.

(d) Which term in sequence **T** is equal to 177 147? [2]

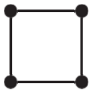

Diagram 1

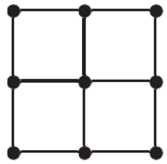

Diagram 2

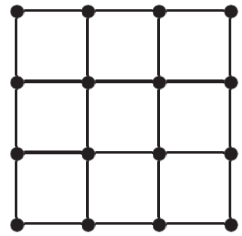

Diagram 3

The first three diagrams in a sequence are shown above.

The diagrams are made up of dots and lines. Each line is one centimetre long.

(a) Make a sketch of the next diagram in the sequence. [1]

The first diagram is a square with the side 1 unit

the second diagram has the side 2 units

the third one has the side 3 units

The next unit will be a square with 4 units side.

(b) The table below shows some information about the diagrams.

Diagram	1	2	3	4	---------	n
Area	1	4	9	16	---------	x
Number of dots	4	9	16	p	---------	y
Number of one centimetre lines	4	12	24	q	---------	z

(i) Write down the values of p and q. [2]

Looking at the diagrams we work out $p = 25$ and $q = 40$

4]

(ii) Write down each of x, y and z in terms of n

$$x = n^2 \qquad y = (n+1)^2 \qquad z = n^2 + (n+1)^2 - 1$$

(c) The **total** number of one centimetre lines in the first n diagrams is given by the expression

$$\frac{2}{3}n^3 + fn^2 + gn.$$

[1]

(i) Use $n = 1$ in this expression to show that $f + g = \frac{10}{3}$ [2]

$$f + g = \frac{10}{3}$$

3]

(ii) Use $n = 2$ in this expression to show that $4f + 2g = \frac{32}{3}$

$$4f + 2g = \frac{32}{3}$$

(iii) Find the values of f and g.

$$f = 2 \qquad g = \frac{4}{3}$$

(iv) Find the total number of one centimetre lines in the first 10 diagrams.

We substitute in the expression $f = 2, n = 10$ and $g = 4/3$.

$\frac{2}{3}10^3 + 2 \times 10^2 + 10 \times \frac{4}{3}$

$= 880$