Question 1

The second term, u_2, of a geometric sequence is 44 and the third term, u_3, is 55.

(a) Find the common ratio, r, of the sequence. [2]

(b) Find the first term of the sequence, u_1. [2]

(c) Find S_5, the sum of the first 5 terms of the sequence. [2]

a) For a geometric sequence the common ratio, r, is given by

$$r = \frac{u_2}{u_1} = \frac{u_3}{u_2} = \frac{u_4}{u_3} \dots$$

$u_2 = 44$ $u_3 = 55$

sub u_2 and u_3 into r formula

$$r = \frac{55}{44}$$

$$\boxed{r = \frac{5}{4}}$$

The second term, u_2, of a geometric sequence is 44 and the third term, u_3, is 55.

(a) Find the common ratio, r, of the sequence. $\boxed{r = \frac{5}{4}}$ [2]

(b) Find the first term of the sequence, u_1. [2]

(c) Find S_5, the sum of the first 5 terms of the sequence. [2]

b) $$r = \frac{u_2}{u_1}$$

$$\frac{5}{4} = \frac{44}{u_1}$$

rearrange for u_1

$$u_1 = \frac{44}{\left(\frac{5}{4}\right)}$$

$$\boxed{u_1 = 35.2}$$

The second term, u_2, of a geometric sequence is 44 and the third term, u_3, is 55.

(a) Find the common ratio, r, of the sequence. [2]

$\boxed{r = \frac{5}{4}}$

(b) Find the first term of the sequence, u_1. [2]

$\boxed{u_1 = 35.2}$

(c) Find S_5, the sum of the first 5 terms of the sequence. [2]

c) For a geometric sequence the sum of the first n terms is given by

$$S_n = \frac{u_1(r^n - 1)}{r - 1}$$ (in formula booklet)

sub u_1, r and $n = 5$

$$S_5 = \frac{35.2\left(\left(\frac{5}{4}\right)^5 - 1\right)}{\left(\frac{5}{4}\right) - 1}$$

$$S_5 = 288.8875$$

$$\boxed{S_5 = 289 \text{ (3sf)}}$$

Question 2

The sum of the first 16 terms of an arithmetic sequence is 920.

(a) Find the common difference, d, of the sequence if the first term is 27.5. [3]

(b) Find the first term of the sequence if the common difference, d, is 11. [3]

a) For an arithmetic sequence the sum of the first n terms is given by

$$S_n = \frac{n}{2}\left(2u_1 + (n-1)d\right)$$ (in formula booklet)

$S_{16} = 920$ $\quad$ $u_1 = 27.5$

sub in S_{16}, u_1 and $n = 16$

$$920 = \frac{16}{2}\left(2(27.5) + (16-1)d\right)$$

$$920 = 8(55 + 15d)$$

$$115 = 55 + 15d$$

$$60 = 15d$$

$$\boxed{d = 4}$$

The sum of the first 16 terms of an arithmetic sequence is 920.

(a) Find the common difference, d, of the sequence if the first term is 27.5. [3]

(b) Find the first term of the sequence if the common difference, d, is 11. [3]

b) $S_n = \frac{n}{2}(2u_1 + (n-1)d)$ (in formula booklet)

$S_{16} = 920 \qquad d = 11$

sub in S_{16}, d and $n = 16$

$920 = \frac{16}{2}(2u_1 + (16-1)(11))$

$920 = 8(2u_1 + 165)$

$115 = 2u_1 + 165$

$-50 = 2u_1$

$\boxed{u_1 = -25}$

Question 3

The sum of the first 5 terms of a geometric sequence is 461.12.

(a) Find the common ratio, r, of the sequence if the first term is 200, given that $r > 0$. [3]

(b) Find the first term of the sequence if the common ratio, r, is -2.
Give your answer correct to 2 decimal places. [3]

a) For a geometric sequence the sum of the first n terms is given by

$S_n = \frac{u_1(r^n - 1)}{r - 1}$ (in formula booklet)

$S_5 = 461.12 \qquad u_1 = 200$

sub in S_5, u_1 and $n = 5$ and solve for r using your GDC

$461.12 = \frac{200(r^5 - 1)}{r - 1}$

$\boxed{r = \frac{3}{5}}$

Alternative GDC methods

- Plot $y = 461.12$ and $y = \frac{200(x^5 - 1)}{x - 1}$ and find intersection.
- Use the algebraic solver.

The sum of the first 5 terms of a geometric sequence is 461.12.

(a) Find the common ratio, r, of the sequence if the first term is 200, given that $r > 0$.

[3]

(b) Find the first term of the sequence if the common ratio, r, is -2.
Give your answer correct to 2 decimal places.

[3]

b) $S_n = \dfrac{u_1(r^n - 1)}{r - 1}$ (in formula booklet)

$S_5 = 461.12$ $\quad r = -2$

sub in S_5, r and $n = 5$ and solve for u_1 using your GDC

$461.12 = \dfrac{u_1((-2)^5 - 1)}{(-2) - 1}$

$\boxed{u_1 = 41.92 \ (2dp)}$

Alternative GDC methods

- Plot $y = 461.12$ and $y = \dfrac{x((-2)^5 - 1)}{(-2) - 1}$ and find intersection.
- Use the algebraic solver.

Question 4

The table below shows information about the terms of four different sequences, a_n, b_n, c_n and d_n.

	$n = 1$	$n = 2$	$n = 3$	$n = 4$
a_n		12	30	
b_n		12	30	
c_n	80			10
d_n	80			10

(a) Calculate a_1, a_4 and the common difference, d, given that a_n is an arithmetic sequence.

[2]

(b) Calculate b_1, b_4 and the common ratio, r, given that b_n is a geometric sequence.

[2]

(c) Calculate c_2, c_3 and the common difference, d, given that c_n is an arithmetic sequence.

[2]

(d) Calculate d_2, d_3 and the common ratio, r, given that d_n is a geometric sequence.

[2]

a) For an arithmetic sequence the common difference, d, is given by

$d = u_2 - u_1 = u_3 - u_2 = u_4 - u_3 \ldots$

$a_2 = 12$ $\quad a_3 = 30$

sub in u_2 and u_3 into d formula

$d = 30 - 12$

$\boxed{d = 18}$

Use $d = 18$ to find a_1 and a_4

$18 = 12 - a_1$ $\quad 18 = a_4 - 30$

$\boxed{a_1 = -6}$ $\quad \boxed{a_4 = 48}$

The table below shows information about the terms of four different sequences, a_n, b_n, c_n and d_n.

	$n = 1$	$n = 2$	$n = 3$	$n = 4$
a_n		12	30	
b_n		12	30	
c_n	80			10
d_n	80			10

(a) Calculate a_1, a_4 and the common difference, d, given that a_n is an arithmetic sequence. [2]

(b) Calculate b_1, b_4 and the common ratio, r, given that b_n is a geometric sequence. [2]

(c) Calculate c_2, c_3 and the common difference, d, given that c_n is an arithmetic sequence. [2]

(d) Calculate d_2, d_3 and the common ratio, r, given that d_n is a geometric sequence. [2]

b) For a geometric sequence the common ratio, r, is given by

$$r = \frac{u_2}{u_1} = \frac{u_3}{u_2} = \frac{u_4}{u_3} \ldots$$

$b_2 = 12$ $\quad$ $b_3 = 30$

sub in b_2 and b_3 into r formula

$$r = \frac{30}{12}$$

$$\boxed{r = 2.5}$$

use $r = 2.5$ to find b_1 and b_4

$$2.5 = \frac{12}{b_1} \qquad 2.5 = \frac{b_4}{30}$$

$$\boxed{b_1 = 4.8} \qquad \boxed{b_4 = 75}$$

The table below shows information about the terms of four different sequences, a_n, b_n, c_n and d_n.

	$n = 1$	$n = 2$	$n = 3$	$n = 4$
a_n		12	30	
b_n		12	30	
c_n	80			10
d_n	80			10

(a) Calculate a_1, a_4 and the common difference, d, given that a_n is an arithmetic sequence. [2]

(b) Calculate b_1, b_4 and the common ratio, r, given that b_n is a geometric sequence. [2]

(c) Calculate c_2, c_3 and the common difference, d, given that c_n is an arithmetic sequence. [2]

(d) Calculate d_2, d_3 and the common ratio, r, given that d_n is a geometric sequence. [2]

c) The nth term formula for an arithmetic sequence is given by

$u_n = u_1 + (n-1)d$ $\quad$ (in formula booklet)

$c_1 = 80$ $\quad$ $c_4 = 10$

sub c_1 and c_4 into the nth term formula to find d.

$$10 = 80 + (4-1)d$$
$$3d = 10 - 80$$
$$3d = -70$$

$$\boxed{d = -\frac{70}{3}}$$

Use the nth term formula to find c_2 and c_3

$$c_2 = 80 + (2-1)\left(-\frac{70}{3}\right) \qquad c_3 = 80 + (3-1)\left(-\frac{70}{3}\right)$$

$$\boxed{c_2 = \frac{170}{3}} \qquad \boxed{c_3 = \frac{100}{3}.}$$

The table below shows information about the terms of four different sequences, a_n, b_n, c_n and d_n.

	$n=1$	$n=2$	$n=3$	$n=4$
a_n		12	30	
b_n		12	30	
c_n	80			10
d_n	80			10

(a) Calculate a_1, a_4 and the common difference, d, given that a_n is an arithmetic sequence.

[2]

(b) Calculate b_1, b_4 and the common ratio, r, given that b_n is a geometric sequence.

[2]

(c) Calculate c_2, c_3 and the common difference, d, given that c_n is an arithmetic sequence.

[2]

(d) Calculate d_2, d_3 and the common ratio, r, given that d_n is a geometric sequence.

[2]

d) The nth term formula for a geometric sequence is given by

$u_n = u_1 r^{n-1}$ (in formula booklet)

$d_1 = 80 \qquad d_4 = 10$

sub d_1 and d_4 into the nth term formula to find r

$10 = 80r^{4-1}$

$r^3 = \frac{1\cancel{0}}{8\cancel{0}}$

$r = \left(\frac{1}{8}\right)^{1/3}$

$\boxed{r = \frac{1}{2}}$

Use the nth term formula to find d_2 and d_3

$d_2 = 80\left(\frac{1}{2}\right)^{2-1} \qquad d_3 = 80\left(\frac{1}{2}\right)^{3-1}$

$\boxed{d_2 = 40} \qquad \boxed{d_3 = 20}$

Question 5

Students are arranged for a graduation photograph in rows which follows an arithmetic sequence. There are 20 students in the fourth row and 44 in the 10th row.

(a) (i) Find the common difference, d, of the arithmetic sequence.

(ii) Find the first term of the arithmetic sequence.

[3]

(b) Given there are 20 rows of students in the photograph, calculate how many students there are altogether.

[3]

a) i) $u_n = u_1 + (n-1)d$ (in formula booklet)

$u_4 = 20 \qquad u_{10} = 44$

$20 = u_1 + (4-1)d \qquad 44 = u_1 + (10-1)d$

$20 = u_1 + 3d$ ① $\qquad 44 = u_1 + 9d$ ②

② − ①

$$\begin{array}{r} 44 = u_1 + 9d \\ -\ 20 = u_1 + 3d \\ \hline 24 = \qquad 6d \end{array}$$

$\boxed{d = 4}$

ii) sub $d = 4$ into ① to find u_1

$20 = u_1 + 3(4)$

$20 = u_1 + 12$

$20 - 12 = u_1$

$\boxed{u_1 = 8}$

Students are arranged for a graduation photograph in rows which follows an arithmetic sequence. There are 20 students in the fourth row and 44 in the 10th row.

(a) (i) Find the common difference, d, of the arithmetic sequence.

$\boxed{d = 4}$

(ii) Find the first term of the arithmetic sequence.

$\boxed{u_1 = 8}$

[3]

(b) Given there are 20 rows of students in the photograph, calculate how many students there are altogether.

[3]

b) $S_n = \frac{n}{2}(2u_1 + (n-1)d)$ (in formula booklet)

$n = 20 \quad u_1 = 8 \quad d = 4$

sub n, u_1 and d into S_n formula

$S_{20} = \frac{20}{2}(2(8) + (20-1)(4))$

$S_{20} = 10(16 + 76)$

$S_{20} = 10(92)$

$\boxed{S_{20} = 920 \text{ students}}$

Question 6

Marie is an athlete returning to running after an injury and wants to manage the number of kilometres she runs per week. She decides to run 4 km the first week and increase this by 1.5 km each week.

(a) Find the distance Marie ran in the 10th week.

[2]

(b) Find the week in which Marie runs 26.5 km.

[3]

Marie's coach says she can start preparing for her next race once she has run a total of 220 km.

(c) Find the week in which Marie will complete this.

[3]

a) Identify the arithmetic sequence

The 10th week will be u_{10} in the sequence.

$u_n = u_1 + (n-1)d$ (in formula booklet)

$u_1 = 4 \quad d = 1.5$

sub in u_1 and d into the formula to find u_{10}.

$u_{10} = 4 + (10-1)(1.5)$

$u_{10} = 4 + 9(1.5)$

$u_{10} = 4 + 13.5$

$\boxed{u_{10} = 17.5 \text{ km}}$

Marie is an athlete returning to running after an injury and wants to manage the number of kilometres she runs per week. She decides to run 4 km the first week and increase this by 1.5 km each week.

(a) Find the distance Marie ran in the 10th week.

[2]

(b) Find the week in which Marie runs 26.5 km.

[3]

Marie's coach says she can start preparing for her next race once she has run a total of 220 km.

(c) Find the week in which Marie will complete this.

[3]

b) $u_n = u_1 + (n-1)d$ (in formula booklet)

$u_1 = 4$ $d = 1.5$ $u_n = 26.5$

sub in u_1, d and u_n into the formula to find n

$26.5 = 4 + (n-1)(1.5)$

$22.5 = (n-1)(1.5)$

$\frac{22.5}{1.5} = n-1$

$15 = n-1$

$n = 16$

$\therefore$ Marie runs 26.5 km in the 16th week.

Alternative GDC methods

- Plot $y = 26.5$ and $y = 4 + (x-1)(1.5)$ and find intersection.
- Use the algebraic solver.

Marie is an athlete returning to running after an injury and wants to manage the number of kilometres she runs per week. She decides to run 4 km the first week and increase this by 1.5 km each week.

(a) Find the distance Marie ran in the 10th week.

[2]

(b) Find the week in which Marie runs 26.5 km.

[3]

Marie's coach says she can start preparing for her next race once she has run a total of 220 km.

(c) Find the week in which Marie will complete this.

[3]

c) $S_n = \frac{n}{2}(2u_1 + (n-1)d)$ (in formula booklet)

$u_1 = 4$ $d = 1.5$ $S_n = 220$

sub in u_1, d and S_n into the formula

$220 = \frac{n}{2}(2(4) + (n-1)(1.5))$

$440 = n(8 + (n-1)(1.5))$

Put the equation into the algebraic solver in your GDC.

$n = 15.0968$

$\therefore S_{15} < 220$

$S_{16} > 220$

Marie will complete a total of 220 km during the 16th week.

Alternative GDC method

- Plot $y = 220$ and $y = \frac{x}{2}(8 + (x-1)(1.5))$ and find intersection.

EXAM PAPERS PRACTICE

Question 7a

The eighth term, u_8, of an arithmetic sequence is 18 and the common difference, d, is 2.

(a) (i) Find the first term of the arithmetic sequence.

(ii) Find the value of u_{17}.

[4]

The first and 17th terms of the arithmetic sequence are the third and fifth terms respectively of a geometric sequence.

(b) (i) Find the possible values for the common ratio, r, of the geometric sequence.

(ii) Find the first term of the geometric sequence.

[4]

a) i) $u_n = u_1 + (n-1)$ (in formula booklet)

$u_8 = 18 \qquad d = 2$

sub in u_8 and d into the formula to find u_1

$18 = u_1 + (8-1)(2)$

$18 = u_1 + 14$

$\boxed{u_1 = 4}$

ii) sub in u_1 and d into the formula to find u_{17}

$u_{17} = 4 + (17-1)(2)$

$u_{17} = 4 + 32$

$\boxed{u_{17} = 36}$

Question 7b

The eighth term, u_8, of an arithmetic sequence is 18 and the common difference, d, is 2.

(a) (i) Find the first term of the arithmetic sequence. $\boxed{u_1 = 4}$

(ii) Find the value of u_{17}. $\boxed{u_{17} = 36}$

[4]

The first and 17th terms of the arithmetic sequence are the third and fifth terms respectively of a geometric sequence.

(b) (i) Find the possible values for the common ratio, r, of the geometric sequence.

(ii) Find the first term of the geometric sequence.

[4]

b) Geometric sequence

$u_3 = 4 \qquad u_5 = 36$

$u_n = u_1 r^{n-1}$ (in formula booklet)

sub in u_3 and u_5 into formula

$4 = u_1 r^{3-1} \qquad 36 = u_1 r^{5-1}$

$4 = u_1 r^2$ ① $\qquad 36 = u_1 r^4$ ②

② ÷ ①

$\frac{36}{4} = \frac{\cancel{u_1} r^{\cancel{4}\,2}}{\cancel{u_1 r^2}}$

i) $\boxed{r = \pm 3}$

$9 = r^2$

sub r into ① to find u_1

$4 = u_1 (\pm 3)^2$

$4 = u_1 (9)$

ii) $\boxed{u_1 = \frac{4}{9}}$

NB $(+3)^2 = (-3)^2$

$9 = 9$

Question 8a

In a geometric sequence, $u_3 = 160$ and the common ratio, r, is $\frac{1}{4}$.

(a) (i) Find the first term, u_1.

(ii) Find u_6.

[4]

(b) Find the value of the infinite sum of the sequence.

[2]

The first and third terms of the geometric sequence are the seventh and ninth terms respectively of an arithmetic sequence.

(c) (i) Find the common difference, d, of the arithmetic sequence.

(ii) Find the first term of the arithmetic sequence.

[4]

a) i) $u_n = u_1 r^{n-1}$ (in formula booklet)

$u_3 = 160 \quad r = \frac{1}{4}$

sub in u_3 and r into formula to find u_1

$160 = u_1 \left(\frac{1}{4}\right)^{3-1}$

$u_1 = \dfrac{160}{\left(\frac{1}{4}\right)^2}$

$\boxed{u_1 = 2560}$

ii) sub in u_1 and r into formula to find u_6

$u_6 = 2560 \left(\frac{1}{4}\right)^{6-1}$

$u_6 = 2560 \left(\frac{1}{4}\right)^5$

$\boxed{u_6 = 2.5}$

Question 8b

In a geometric sequence, $u_3 = 160$ and the common ratio, r, is $\frac{1}{4}$.

(a) (i) Find the first term, u_1. $\boxed{u_1 = 2560}$

(ii) Find u_6.

[4]

(b) Find the value of the infinite sum of the sequence.

[2]

The first and third terms of the geometric sequence are the seventh and ninth terms respectively of an arithmetic sequence.

(c) (i) Find the common difference, d, of the arithmetic sequence.

(ii) Find the first term of the arithmetic sequence.

[4]

b) The sum of an infinite geometric sequence.

$S_\infty = \dfrac{u_1}{1-r}, \quad |r| < 1$ (in formula booklet)

$u_1 = 2560 \qquad r = \frac{1}{4}$

$S_\infty = \dfrac{2560}{1-\frac{1}{4}} = \dfrac{10240}{3} = 3413.333\ldots$

$\boxed{S_\infty = \dfrac{10240}{3} \text{ or } 3410 \text{ (3sf)}}$

Question 8c

In a geometric sequence, $u_3 = 160$ and the common ratio, r, is $\frac{1}{4}$.

(a) (i) Find the first term, u_1.

$\boxed{u_1 = 2560}$

(ii) Find u_6.

[4]

(b) Find the value of the infinite sum of the sequence.

[2]

The first and third terms of the geometric sequence are the seventh and ninth terms respectively of an arithmetic sequence.

(c) (i) Find the common difference, d, of the arithmetic sequence.

(ii) Find the first term of the arithmetic sequence.

[4]

c) i) Arithmetic sequence

$u_7 = 2560 \qquad u_9 = 160$

$u_n = u_1 + (n-1)d$ (in formula booklet)

sub u_7 and u_9 into formula

$2560 = u_1 + (7-1)d \qquad 160 = u_1 + (9-1)d$

$2560 = u_1 + 6d \quad ① \qquad 160 = u_1 + 8d \quad ②$

① − ②

$$\begin{array}{rl} & 2560 = u_1 + 6d \\ - & 160 = u_1 + 8d \\ \hline & 2400 = -2d \end{array}$$

$\boxed{d = -1200}$

ii) sub d into ① to find u_1

$2560 = u_1 + 6(-1200)$

$2560 = u_1 - 7200$

$u_1 = 2560 + 7200$

$\boxed{u_1 = 9760}$

Alternative GDC methods

- Plot ① and ② and find intersection.
- Use the simultaneous equation solver.

Question 9

A sequence can be defined by $a_n = 32 - 7n$, for $n \in \mathbb{Z}^+$.

(a) Write an expression for $a_1 + a_2 + a_3 + \cdots + a_{12}$ using sigma notation and find the value of the sum.

[3]

(b) Write an expression for $a_4 + a_5 + a_6 + \cdots + a_{15}$ using sigma notation and find the value of the sum.

[3]

a) Using sigma notation

$a_1 + a_2 + a_3 + \ldots + a_{12} = \sum_{k=1}^{12} a_k$

$\boxed{\sum_{k=1}^{12} (32 - 7k)}$

$S_n = \frac{n}{2}(2u_1 + (n-1)d)$ (in formula booklet)

$a_1 = 25 \qquad d = -7 \qquad n = 12$

sub in a_1, d and n

$S_{12} = \frac{12}{2}(2(25) + (12-1)(-7))$

$\boxed{S_{12} = -162}$

Alternative GDC method using sigma notation.

A sequence can be defined by $a_n = 32 - 7n$, for $n \in \mathbb{Z}^+$.

(a) Write an expression for $a_1 + a_2 + a_3 + \cdots + a_{12}$ using sigma notation and find the value of the sum.

[3]

(b) Write an expression for $a_4 + a_5 + a_6 + \cdots + a_{15}$ using sigma notation and find the value of the sum.

[3]

b) Using sigma notation

$a_4 + a_5 + a_6 + \ldots + a_{15} = \sum_{k=4}^{15} a_k$

$$\boxed{\sum_{k=4}^{15} (32 - 7k)}$$

$S_n = \frac{n}{2}(2u_1 + (n-1)d)$ (in formula booklet)

$a_1 = 4$ $\quad d = -7$ $\quad n = 12$

sub in a_1, d and n

$S_{12} = \frac{12}{2}(2(4) + (12-1)(-7))$

$$\boxed{S_{12} = -414}$$

Alternative GDC method using sigma notation.

Question 10

A sequence can be defined by $g_n = 4 \times 3^{n-1}$, for $n \in \mathbb{Z}^+$.

(a) Write an expression for $g_1 + g_2 + g_3 + \cdots + g_{10}$ using sigma notation and find the value of the sum.

[3]

(b) Write an expression for $g_8 + g_9 + g_{10} + \cdots + g_{18}$ using sigma notation and find the value of the sum.

[3]

a) Using sigma notation

$g_1 + g_2 + g_3 + \ldots + g_{10} = \sum_{k=1}^{10} g_k$

$$\boxed{\sum_{k=1}^{10} (4 \times 3^{k-1})}$$

$S_n = \frac{u_1(r^n - 1)}{r - 1}$ (in formula booklet)

$g_1 = 4$ $\quad r = 3$ $\quad n = 10$

sub in g_1, r and n

$S_{10} = \frac{4(3^{10} - 1)}{3 - 1}$

$S_{10} = 118\ 096$

$$\boxed{S_{10} = 118\ 000 \ (3sf)}$$

Alternative GDC method using sigma notation.

A sequence can be defined by $g_n = 4 \times 3^{n-1}$, for $n \in \mathbb{Z}^+$.

(a) Write an expression for $g_1 + g_2 + g_3 + \cdots + g_{10}$ using sigma notation and find the value of the sum.

[3]

(b) Write an expression for $g_8 + g_9 + g_{10} + \cdots + g_{18}$ using sigma notation and find the value of the sum.

[3]

b) Using sigma notation

$g_8 + g_9 + g_{10} + \ldots + g_{18} = \sum_{k=8}^{18} g_k$

$$\boxed{\sum_{k=8}^{18} (4 \times 3^{k-1})}$$

$S_n = \dfrac{u_1(r^n - 1)}{r-1}$ (in formula booklet)

$g_1 = 8748 \quad r = 3 \quad n = 11$

sub in g_1, r and n

$S_{11} = \dfrac{8748(3^{11} - 1)}{3 - 1}$

$S_{11} = 774\ 836\ 604$

$$\boxed{S_{11} = 775\ 000\ 000 \ (3sf)}$$

Alternative GDC method using sigma notation.

Question 11

The kiwi is a flightless bird and is a national treasure in New Zealand. At the start of 2021 there were approximately 68 000 kiwi left, with the population decreasing by 2% every year.

(a) Find the expected population size of kiwis in 2030 assuming the rate of decrease in kiwi population remains the same.

[3]

(b) Find the year in which the population of kiwis falls below 50 000 assuming the rate of decrease in kiwi population remains the same.

[3]

a) Identify the geometric sequence.

The common ratio, r, will be equal to the percentage of the remaining population every year (as a decimal).

Population decrease is 2% (0.02) every year.

Therefore the remaining population every year is

$100\% - 2\% = 98\%$

$1 - 0.02 = 0.98$

Hence $r = 0.98$

$u_1 = 68\ 000 \qquad r = 0.98$

Be sure to select the correct value for n.

$u_1 : 2021, \ u_2 : 2022, \ u_3 : 2023 \ldots \ \underline{u_{10} : 2030}$

$u_n = u_1 r^{n-1}$ (in formula booklet)

$u_{10} = 68000(0.98)^{10-1}$

$u_{10} = 56\ 694.84782$

The expected population of kiwis in 2030 is 56 700.

The kiwi is a flightless bird and is a national treasure in New Zealand. At the start of 2021 there were approximately 68 000 kiwi left, with the population decreasing by 2% every year.

(a) Find the expected population size of kiwis in 2030 assuming the rate of decrease in kiwi population remains the same.

[3]

(b) Find the year in which the population of kiwis falls below 50 000 assuming the rate of decrease in kiwi population remains the same.

[3]

b) $u_n = u_1 r^{n-1}$ (in formula booklet)

$u_1 = 68\ 000 \quad r = 0.98 \quad u_n < 50\ 000$

sub in u_1, r and u_n into the formula

$50\ 000 > 68000(0.98)^{n-1}$

solve the equation for n using your GDC swapping the inequality (>) to an equal sign (=)

$50000 = 68000(0.98)^{n-1}$

$n = 16.22$

$\therefore u_{16} > 50000 \qquad u_{17} < 50000$

$u_{16}: 2036 \qquad u_{17}: 2037$

The population of kiwis will fall below 50 000 in 2036.

Question 12

Aaron is working on his cycling in preparation for a triathlon event in 10 months. He cycles a total of 240 km in the first month and plans to increase this by 12.5% each month.

(a) Find the distance Aaron cycles in the fifth month of preparation.

[3]

(b) Calculate the total distance Aaron cycles until the triathlon.

[3]

a) Identify the geometric sequence

$u_n = u_1 r^{n-1}$ (in formula booklet)

$u_1 = 240 \qquad r = 1.125 \qquad n = 5$

sub in u_1, r and n

$u_5 = 240(1.125)^4$

$\boxed{u_5 = 384 \text{ km } (3sf)}$

Aaron is working on his cycling in preparation for a triathlon event in 10 months. He cycles a total of 240 km in the first month and plans to increase this by 12.5% each month.

(a) Find the distance Aaron cycles in the fifth month of preparation. [3]

(b) Calculate the total distance Aaron cycles until the triathlon. [3]

b) $S_n = \frac{u_1(r^n - 1)}{r - 1}$ (in formula booklet)

$u_1 = 240$ $r = 1.125$ $n = 10$

sub in u_1, r and n

$S_{10} = \frac{240(1.125^{10} - 1)}{1.125 - 1}$

$\boxed{S_{10} = 4310 \text{ km} \ (3sf)}$

Question 13a

A geometric sequence has $u_1 = 0.5$ and $r = 3$.

(a) Find

(i) u_4

(ii) S_5. [2]

An arithmetic sequence has the same u_4 and S_5 as the geometric sequence above.

(b) Find u_1 and d for the arithmetic sequence. [4]

a) i) $u_n = u_1 r^{n-1}$ (in formula booklet)

$u_1 = 0.5$ $r = 3$ $n = 4$

sub in u_1, r and n

$u_4 = 0.5(3)^{4-1}$

$\boxed{u_4 = 13.5}$

ii) $S_n = \frac{u_1(r^n - 1)}{r - 1}$ (in formula booklet)

$u_1 = 0.5$ $r = 3$ $n = 5$

sub in u_1, r and n

$S_5 = \frac{0.5(3^5 - 1)}{3 - 1}$

$\boxed{S_5 = 60.5}$

Question 13b

A geometric sequence has $u_1 = 0.5$ and $r = 3$.

(a) Find

(i) u_4 $\boxed{u_4 = 13.5}$

(ii) S_5. $\boxed{S_5 = 60.5}$

[2]

An arithmetic sequence has the same u_4 and S_5 as the geometric sequence above.

(b) Find u_1 and d for the arithmetic sequence.

[4]

b) $U_n = U_1 + (n-1)d$ (in formula booklet)

$S_n = \frac{n}{2}(2u_1 + (n-1)d)$ (in formula booklet)

$U_4 = 13.5$ $S_5 = 60.5$

$13.5 = U_1 + 3d$ ① $60.5 = \frac{5}{2}(2u_1 + 4d)$ ②

Input ① and ② into your GDC to solve for u_1 and d.

$\boxed{u_1 = 9.3 \quad d = 1.4}$

Alternative GDC methods

- Plot ① and ② and find intersection.
- Input ① and ② into the simultaneous equation solver.

Question 14a

Daniel and Jonah have each been given \$5000 to save for university.

Daniel invests his money in an account that pays a nominal annual interest rate of 2.24%, **compounded quarterly.**

(a) Calculate the amount Daniel will have in his account after 8 years. Give your answer to 2 decimal places.

[3]

Jonah wants to invest his money in an account such that his investment will double in 10 years. Assume the account pays a nominal annual interest of r%, **compounded half-yearly.**

(b) Determine the value of r.

[3]

a) Compound interest formula

$FV = PV\left(1 + \frac{r}{100k}\right)^{kn}$ (in formula booklet)

$PV = 5000$ $r = 2.24\%$ $k = 4$ $n = 8$

Sub PV, r, k and n into formula.

$FV = 5000\left(1 + \frac{2.24}{100(4)}\right)^{(4)(8)}$

$\boxed{FV \approx \$5978.31 \quad (2dp)}$

Question 14b

Daniel and Jonah have each been given $5000 to save for university.

Daniel invests his money in an account that pays a nominal annual interest rate of 2.24%, **compounded quarterly.**

(a) Calculate the amount Daniel will have in his account after 8 years. Give your answer to 2 decimal places.

[3]

Jonah wants to invest his money in an account such that his investment will double in 10 years. Assume the account pays a nominal annual interest of r%, **compounded half-yearly.**

(b) Determine the value of r.

[3]

b) Compound interest formula

$FV = PV\left(1+\frac{r}{100k}\right)^{kn}$ (in formula booklet)

$FV = 10\,000$ $PV = 5000$ $k = 2$ $n = 10$

Sub FV, PV, k and n into formula and solve for r using your GDC.

$10\,000 = 5000\left(1+\frac{r}{100(2)}\right)^{(2)(10)}$

$\boxed{r \approx 7.05\,\%}$

Question 15

On his 40th birthday, Robert invests $15 000 into a savings account that pays a nominal annual interest rate of 4.78%, **compounded monthly**.

(a) (i) Write an expression for the total value of the investment after n years. Give your answer to 2 decimal places.

(ii) Find the total amount in the savings account after 3 and 5 years.

[3]

(b) Find the age Robert will be when the amount of money in his account will be 1.5 times the initial amount.

[2]

a) i) Compound interest formula

$FV = PV\left(1+\frac{r}{100k}\right)^{kn}$ (in formula booklet)

$PV = 15000$ $r = 4.78\,\%$ $k = 12$

Sub PV, r and k into formula.

$\boxed{FV = 15000\left(1+\frac{4.78}{100(12)}\right)^{12n}}$

ii) Sub $n = 3$ into expression for amount after 3 years.

$FV = 15\,000\left(1+\frac{4.78}{100(12)}\right)^{(12)(3)}$

$\boxed{FV \approx \$17\,307.94 \quad (2dp)}$

Sub $n = 5$ into expression for amount after 5 years.

$FV = 15\,000\left(1+\frac{4.78}{100(12)}\right)^{(12)(5)}$

$\boxed{FV \approx \$19\,040.64 \quad (2dp)}$

On his 40th birthday, Robert invests \$15 000 into a savings account that pays a nominal annual interest rate of 4.78%, **compounded monthly**.

(a) (i) Write an expression for the total value of the investment after n years. Give your answer to 2 decimal places.

(ii) Find the total amount in the savings account after 3 and 5 years.

[3]

(b) Find the age Robert will be when the amount of money in his account will be 1.5 times the initial amount.

[2]

b) Compound interest formula

$FV = PV\left(1+\frac{r}{100k}\right)^{kn}$ (in formula booklet)

$FV = 1.5(15\,000) = 22\,500$

$PV = 15\,000$ $r = 4.78\%$ $k = 12$

Sub FV, PV, r and k into formula and solve for n using your GDC.

$22\,500 = 15\,000\left(1+\frac{4.78}{100(12)}\right)^{12n}$

$n = 8.5$

$\therefore$ 8 years and 6 months

$\boxed{\text{Robert will be 48 years and 6 months old.}}$

Question 16

The sum of the first two terms of a geometric sequence is 15.3 and the sum of the infinite geometric sequence is 30. Find the positive value of the common ratio, r.

[6]

The sum of a finite geometric sequence.

$S_n = \frac{u_1(r^n-1)}{(r-1)} = \frac{u_1(1-r^n)}{(1-r)}$ (in formula booklet)

Sub $S_2 = 15.3$ into formula.

$15.3 = \frac{u_1(1-r^2)}{(1-r)}$ — ①

The sum of an infinite geometric sequence.

$S_\infty = \frac{u_1}{1-r}$, $|r|<1$ (in formula booklet)

Sub $S_\infty = 30$ into formula.

$30 = \frac{u_1}{1-r}$ $\therefore u_1 = 30(1-r)$

Sub $u_1 = 30(r-1)$ into ①.

$15.3 = \frac{30\cancel{(1-r)}(1-r^2)}{\cancel{(1-r)}}$ cancel

$15.3 = 30(1-r^2)$ rearrange

$r^2 = 1 - \frac{15.3}{30}$ $\sqrt{\ }$

$r = \sqrt{1-\frac{15.3}{30}}$

$\boxed{r = 0.7}$