

# IB Maths: AI HL

## Sequences & Series

### Topic Questions

**These practice questions can be used by students and teachers and is Suitable for IB Maths AI HL Topic Questions**

Course	IB Maths
Section	1. Number & Algebra
Topic	1.3 Sequences & Series
Difficulty	Medium

**Level: IB Maths**

**Subject: IB Maths AI HL**

**Board: IB Maths**

**Topic: Sequences & Series**

### Question 1

The second term,  $u_2$ , of a geometric sequence is 44 and the third term,  $u_3$ , is 55.

(a) Find the common ratio,  $r$ , of the sequence.

[2 marks]

(b) Find the first term of the sequence,  $u_1$ .

[2 marks]

(c) Find  $S_5$ , the sum of the first 5 terms of the sequence.

[2 marks]

### Question 2

The sum of the first 16 terms of an arithmetic sequence is 920.

(a) Find the common difference,  $d$ , of the sequence if the first term is 27.5.

[3 marks]

(b) Find the first term of the sequence if the common difference,  $d$ , is 11.

[3 marks]

### Question 3

The sum of the first 5 terms of a geometric sequence is 461.12.

(a) Find the common ratio,  $r$ , of the sequence if the first term is 200, given that  $r > 0$ .

[3 marks]

(b) Find the first term of the sequence if the common ratio,  $r$ , is  $-2$ .

Give your answer correct to 2 decimal places.

[3 marks]

### Question 4

The table below shows information about the terms of four different sequences  $a_n$ ,  $b_n$ ,  $c_n$  and  $d_n$ .

	$n = 1$	$n = 2$	$n = 3$	$n = 4$
$a_n$		12	30	
$b_n$		12	30	
$c_n$	80			10
$d_n$	80			10

(a) Calculate  $a_1$ ,  $a_4$  and the common difference,  $d$ , given that  $a_n$  is an arithmetic sequence.

[2 marks]

(b) Calculate  $b_1$ ,  $b_4$  and the common ratio,  $r$ , given that  $b_n$  is a geometric sequence.

[2 marks]

(c) Calculate  $c_2$ ,  $c_3$  and the common difference,  $d$ , given that  $c_n$  is an arithmetic sequence.

[2 marks]

(d) Calculate  $d_2$ ,  $d_3$  and the common ratio,  $r$ , given that  $d_n$  is a geometric sequence.

[2 marks]

### Question 5

Students are arranged for a graduation photograph in rows which follows an arithmetic sequence. There are 20 students in the fourth row and 44 in the 10th row.

- (a) (i) Find the common difference,  $d$ , of the arithmetic sequence.
- (ii) Find the first term of the arithmetic sequence.

[3 marks]

(b) Given there are 20 rows of students in the photograph, calculate how many students there are altogether

[3 marks]

### Question 6

Marie is an athlete returning to running after an injury and wants to manage the number of kilometres she runs per week. She decides to run 4 km the first week and increase this by 1.5 km each week.

(a) Find the distance Marie ran in the 10th week.

[2 marks]

(b) Find the week in which Marie runs 26.5 km.

[3 marks]

Marie's coach says she can start preparing for her next race once she has run a total of 220 km.

(c) Find the week in which Marie will complete this.

[3 marks]

### Question 7

The eighth term,  $u_8$ , of an arithmetic sequence is 18 and the common difference,  $d$ , is 2.

(a) (i) Find the first term of the arithmetic sequence.

(ii) Find the value of  $u_{17}$ .

[4 marks]

The first and 17th terms of the arithmetic sequence are the third and fifth terms respectively of a geometric sequence.

- (b) (i) Find the possible values for the common ratio,  $r$ , of the geometric sequence.
- (ii) Find the first term of the geometric sequence.

[4 marks]

### Question 8

In a geometric sequence,  $u_3 = 160$  and the common ratio,  $r$ , is  $\frac{1}{4}$ .

- (a) (i) Find the first term,  $u_1$ .
- (ii) Find  $u_6$ .

[4 marks]

(b) Find the value of the infinite sum of the sequence.

[2 marks]

The first and third terms of the geometric sequence are the seventh and ninth terms respectively of an arithmetic sequence.

- (c) (i) Find the common difference,  $d$ , of the arithmetic sequence.
- (ii) Find the first term of the arithmetic sequence.

[4 marks]

### Question 9

A sequence can be defined by  $a_n = 32 - 7n$ , for  $n \in \mathbb{Z}^+$ .

(a) Write an expression for  $a_1 + a_2 + a_3 + \dots + a_{12}$  using sigma notation and find the value of the sum.

[3 marks]

(b) Write an expression for  $a_4 + a_5 + a_6 + \dots + a_{15}$  using sigma notation and find the value of the sum.

[3 marks]

### Question 10

A sequence can be defined by  $g_n = 4 \times 3^{n-1}$ , for  $n \in \mathbb{Z}^+$ .

(a) Write an expression for  $g_1 + g_2 + g_3 + \dots + g_{10}$  using sigma notation and find the value of the sum.

[3 marks]

(b) Write an expression for  $g_8 + g_9 + g_{10} + \dots + g_{18}$  using sigma notation and find the value of the sum.

[3 marks]

### Question 11

The kiwi is a flightless bird and is a national treasure in New Zealand. At the start of 2021 there were approximately 68 000 kiwi left, with the population decreasing by 2% every year.

- (a) Find the expected population size of kiwis in 2030 assuming the rate of decrease in kiwi population remains the same.

[3 marks]

- (b) Find the year in which the population of kiwis falls below 50 000 assuming the rate of decrease in kiwi population remains the same.

[3 marks]

### Question 12

Aaron is working on his cycling in preparation for a triathlon event in 10 months. He cycles a total of 240 km in the first month and plans to increase this by 12.5% each month.

- (a) Find the distance Aaron cycles in the fifth month of preparation.

[3 marks]

- (b) Calculate the total distance Aaron cycles until the triathlon.

[3 marks]



### Question 13

A geometric sequence has  $u_1 = 0.5$  and  $r = 3$ .

(a) Find

(i)  $u_4$

(ii)  $S_5$ .

[2 marks]

An arithmetic sequence has the same  $u_4$  and  $S_5$  as the geometric sequence above.

(b) Find  $u_1$  and  $d$  for the arithmetic sequence.

[4 marks]

### Question 14

The sum of the first two terms of a geometric sequence is 15.3 and the sum of the infinite geometric sequence is 30. Find the positive value of the common ratio,  $r$ .

[6 marks]