EXAM PAPERS PRACTICE

GCSE OCR Math J560

Sequence Quadratic

Answers

"We will help you to achieve A Star "

Answer 1

Here are the first five terms of an arithmetic sequence.

4   9   14   19   24
+5   +5   +5   +5   +5

(a) Find, in terms of n, an expression for the nth term of this sequence.

$$n^{th} \text{ TERM} = 5n + k \longrightarrow k \text{ "MAKES IT WORK"}$$

$$\text{WHEN } n = 1 \quad n^{th} \text{ TERM} = 4$$

$$\text{So } 5 \times 1 + k = 4$$
$$ -5 -5$$

$$k = -1 \longrightarrow n^{th} \text{ TERM} = 5n - 1$$

Answer 2

Here are the first 5 terms of a quadratic sequence.

$$1 \qquad 3 \qquad 7 \qquad 13 \qquad 21$$

Find an expression, in terms of n, for the nth term of this quadratic sequence.

$$1 \quad 3 \quad 7 \quad 13 \quad 21$$
$$2 \quad 4 \quad 6 \quad 8$$
$$2 \quad 2 \quad 2$$

$$\frac{2a}{2} = \frac{2}{2}$$

$$a = 1$$
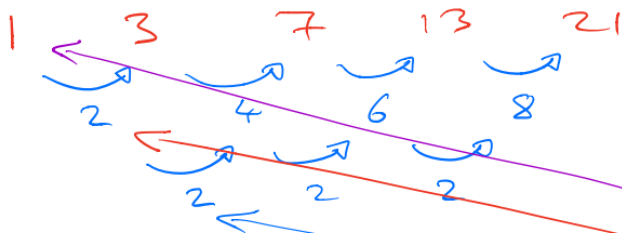
$$3a + b = 2$$

$$3 + b = 2$$
$$-3 \qquad -3$$
$$b = -1$$

$$a + b + c = 1$$
$$1 - 1 + c = 1$$
$$c = 1$$

QUADRATIC SEQUENCES

n^{TH} TERM $= an^2 + bn + c$

$n = 1 \qquad n = 2 \qquad n = 3$

$a + b + c \qquad 4a + 2b + c \qquad 9a + 3b + c$

$3a + b \qquad\qquad 5a + b$

$2a$

COMPARE HIGHLIGHTED TERMS WITH DIFFERENCES IN YOUR SEQUENCE TO FIND a, b AND c

$$n^{th} \text{ TERM} = n^2 - n + 1$$

Answer 3

(b) The 3rd term of this sequence is 21 and the 6th term is 96.

Find the value of a and the value of b.
You must show all your working.

$n = 6:$ 6TH TERM $= a \times 6^2 + b \times 6$

$= 36a + 6b$

$\left\{ \begin{array}{l} n=3: \quad 9a + 3b = 21 \quad —① \\ n=6: \quad 36a + 6b = 96 \quad —② \end{array} \right.$

LINEAR SIMULTANEOUS EQUATIONS
USE "ELIMINATION" METHOD

$2 \times ①$ $18a + 6b = 42$ —③

$②-③$ $\dfrac{18a}{18} = \dfrac{54}{18}$

$a = 3$

$\rightarrow ①$ $9 \times 3 + 3b = 21$
$\quad\quad -27 \quad\quad\quad -27$

$\dfrac{3b}{3} = \dfrac{-6}{3}$

$b = -2$

Answer 4

The nth term of a number sequence is $n^2 + 1$

Write down the first three terms of the sequence.

$n = 1 :$ $1^2 + 1$ $= \underline{\underline{2}}$

$n = 2 :$ $2^2 + 1$ $= \underline{\underline{5}}$

$n = 3 :$ $3^2 + 1$ $= \underline{\underline{10}}$

Answer 5

Here are the first five terms of a different sequence.

$$2 \qquad 2 \qquad 0 \qquad -4 \qquad -10$$

An expression for the nth term of this sequence is $3n - n^2$

(b) Write down, in terms of n, an expression for the nth term of a sequence whose first five terms are

$$4 \qquad 4 \qquad 0 \qquad -8 \qquad -20$$

$$n^{th} \text{ TERM} = \left(3n - n^2\right) \times 2$$

$$= 2\left(3n - n^2\right)$$

Answer 6

The nth term of a sequence is $an^2 + bn$.

(a) Write down an expression, in terms of a and b, for the 3rd term.

$$n = 3: \qquad 3^{RD} \text{ TERM} = a \times 3^2 + b \times 3$$

$$= 9a + 3b$$

Answer 7

The nth term of a sequence is given by $an^2 + bn$ where a and b are integers. → "WHOLE NUMBERS"

The 2nd term of the sequence is –2
The 4th term of the sequence is 12

(a) Find the 6th term of the sequence.

LINEAR SIMULTANEOUS EQUATIONS
USE "ELIMINATION" METHOD

$n = 2$: $4a + 2b = -2$ — ①

$n = 4$: $16a + 4b = 12$ — ②

$2 \times ①$ $8a + 4b = -4$ — ③

② − ③ $\dfrac{8a}{8} = \dfrac{16}{8}$

$\underline{a = 2}$

→ ① $4 \times 2 + 2b = -2$
      $-8 \qquad\qquad -8$

$\dfrac{2b}{2} = \dfrac{-10}{2}$

$\underline{b = -5}$

So

n^{TH} Term $= 2n^2 - 5n$

6TH Term $= 2 \times 6^2 - 5 \times 6$

$= 72 - 30$

$= \underline{42}$

Answer 8

Find the nth term of each sequence.

(a) 4, 8, 12, 16, 20, ...
     $+4 \quad +4 \quad +4 \quad +4$
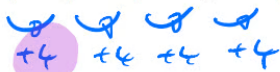
LINEAR

n^{TH} Term $= 4n + C$

$n = 1$: $4 = 4 \times 1 + C$
      $-4 \qquad\quad -4$

$0 = C$ SO n^{TH} Term $= \underline{4n}$

Answer 9

Find the nth term of each sequence.

(a) 7,   13,   19,   25,   31,   …

$+6$ $+6$ $+6$ $+6$

Find the differences between the terms

Common Difference $= 6$

So: n^{th} Term $= 6n + c$
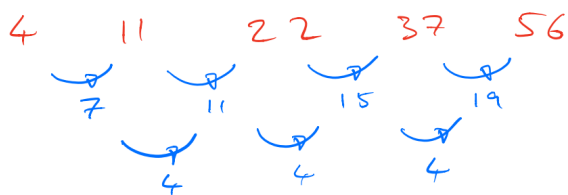
Use 1st term to find c: $7 = 6 \times 1 + c$

$c = 1$

So: n^{th} Term $= 6n + 1$

Answer 10

Here are the first five terms of a sequence.

$$4 \qquad 11 \qquad 22 \qquad 37 \qquad 56$$

Find an expression, in terms of n, for the nth term of this sequence.

$$4 \qquad 11 \qquad 22 \qquad 37 \qquad 56$$

$7 \qquad 11 \qquad 15 \qquad 19$

$4 \qquad 4 \qquad 4$

QUADRATIC SEQUENCES

n^{TH} TERM $= an^2 + bn + c$

$n = 1 \qquad n = 2 \qquad n = 3$

$a + b + c \qquad 4a + 2b + c \qquad 9a + 3b + c$

$3a + b \qquad 5a + b$

$2a$

COMPARE HIGHLIGHTED TERMS WITH DIFFERENCES IN YOUR SEQUENCE TO FIND a, b AND c

2nd DIFFERENCES ARE EQUAL

So QUADRATIC SEQUENCE

$\dfrac{2a}{2} = \dfrac{4}{2}$

$a = 2$

$3a + b = 7$

$3 \times 2 + b = 7$

$-6 \qquad -6$

$b = 1$

$a + b + c = 4$

$2 + 1 + c = 4$

$-3 \qquad -3$

$c = 1$

$$n^{TH} \text{ TERM} = 2n^2 + n + 1$$

Answer 11

Here are the first five terms of a different quadratic sequence.

$$0 \qquad 2 \qquad 6 \qquad 12 \qquad 20$$

(b) Find an expression, in terms of n, for the nth term of this sequence.

$$0 \quad 2 \quad 6 \quad 12 \quad 20$$
$$\quad 2 \quad 4 \quad 6 \quad 8$$
$$\quad\quad 2 \quad 2 \quad 2$$
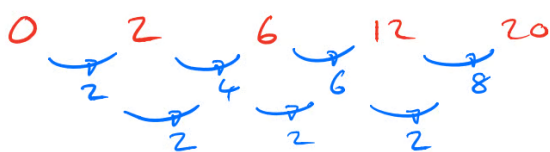
NOW $\dfrac{2a}{2} = \dfrac{2}{2}$ SO $\underline{a = 1}$

AND $3 \times 1 + b = 2$ SO $\underline{b = -1}$
$\quad\quad -3 \quad\quad\quad -3$

AND $1 + (-1) + C = 0$ SO $\underline{C = 0}$

n^{TH} TERM $= \underline{n^2 - n}$

QUADRATIC SEQUENCES

n^{TH} TERM $= an^2 + bn + c$

$n = 1 \qquad n = 2 \qquad n = 3$

$a + b + c \qquad 4a + 2b + c \qquad 9a + 3b + c$

$\qquad\qquad 3a + b \qquad\qquad 5a + b$

$\qquad\qquad\qquad 2a$

COMPARE HIGHLIGHTED TERMS WITH DIFFERENCES IN YOUR SEQUENCE TO FIND a, b AND c

Answer 12

(b) $11, \quad 20, \quad 35, \quad 56, \quad 83, \quad ....$
$\quad +9 \quad +15 \quad +21 \quad +27$
$\quad\quad +6 \quad +6 \quad +6$
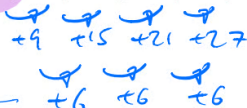
QUADRATIC $\quad \dfrac{2a}{2} = \dfrac{6}{2}$

$a = 3$

$3 \times 3 + b = 9$
$\quad -9 \qquad -9$
$\quad\quad b = 0$

$3 + 0 + C = 11$
$-3 \qquad\qquad -3$
$\quad\quad C = 8$

n^{th} TERM $= \underline{3n^2 + 8}$

QUADRATIC SEQUENCES

n^{TH} TERM $= an^2 + bn + c$

$n = 1 \qquad n = 2 \qquad n = 3$

$a + b + c \qquad 4a + 2b + c \qquad 9a + 3b + c$

$\qquad 3a + b \qquad\qquad 5a + b$

$\qquad\qquad 2a$

COMPARE HIGHLIGHTED TERMS WITH DIFFERENCES IN YOUR SEQUENCE TO FIND a, b AND c

Answer 13

(b) $9, \quad 16, \quad 25, \quad 36, \quad 49, \quad \ldots$
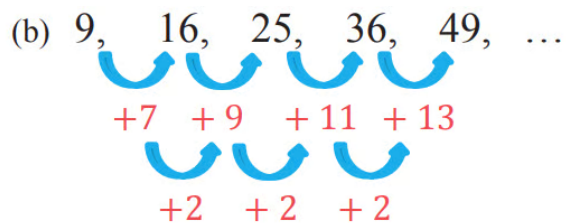
$+7 \quad +9 \quad +11 \quad +13$ Find the differences between the terms

$+2 \quad +2 \quad +2$ …and the second differences

The second differences are constant so it is a Quadratic sequence.

Looking at the sequence again notice that the numbers are all square numbers:

n:	1	2	3	4	5
n^{th} Term:	3^2	4^2	5^2	6^2	7^2

And there is a clear relationship (3 is 2 more than 1, 4 is 2 more than 2 etc.:

$$n^{\text{th}} \text{ Term} = (n+2)^2$$