

IB Maths: AA HL

Reciprocal & Rational Functions

Topic Questions

These practice questions can be used by students and teachers and is Suitable for IB Maths AA HL Topic Questions

Course	IB Maths
Section	2. Functions
Topic	2.5 Reciprocal & Rational Functions
Difficulty	Medium

Level: IB Maths

Subject: IB Maths AA HL

Board: IB Maths

Topic: Reciprocal & Rational Functions

Question 1

Let $f(x) = \frac{2x+1}{x-4}, x \neq 4$.

(a) For the graph of f , find the equation of:

- (i) the vertical asymptote
- (ii) the horizontal asymptote.

[3 marks]

(b) Find $f^{-1}(x)$.

[2 marks]

(c) Write down the equation of the vertical asymptote to the graph of $f^{-1}(x)$.

[1 mark]

Question 2

Consider $f(x) = \frac{ax+b}{3x+c}$, for $x \neq -\frac{c}{3}$, where $a, b, c \in \mathbb{Z}$.

$y = -1$ and $x = 2$ are the equations of the asymptotes of the graph of f . Point A $\left(-2, -\frac{2}{3}\right)$ lies on the graph.

Find the values of a, b and c .

[6 marks]

Question 3

Consider the function f defined by $f(x) = \frac{5}{x+5} - 2$, $x \in \mathbb{R}$, $x \neq p$.

(a) Write down the value of p .

[1 mark]

(b) Write down the equation of the horizontal asymptote to the graph of $y = f(x)$.

[1 mark]

(c) Show that $\frac{3}{x+5} - 2 = \frac{ax+b}{x+5}$, where a and b are constants to be determined.

[2 marks]

(d) Sketch the graph of $y = f(x)$.

[3 marks]

Question 4

Let $f(x) = \frac{4x-2}{2x+5}$, for $x \neq -\frac{5}{2}$.

(a) For the graph of f , find the coordinates of

(i) the x -intercept

(ii) the y -intercept.

[3 marks]

(b) For the graph of f , find the equation of

- (i) the vertical asymptote
- (ii) the horizontal asymptote.

[3 marks]

Question 5

Consider the function f defined by $f(x) = \frac{2(3x-1)}{(x+3)(x-2)}$, $x \in \mathbb{R}$, $x \neq -3, 2$.

(a) Find the coordinates of the points where the graph of $y = f(x)$ intersects the coordinate axes.

[2 marks]

(b) Express $f(x)$ as partial fractions.

[3 marks]

(c) Hence find the equation of the horizontal asymptote to the graph of $y = f(x)$.

[2 marks]

Question 6

Consider the function $f(x) = \frac{4x-12}{x^2-4x-5}$, $x \in \mathbb{R}$, $x \neq -1, 5$.

(a) Find the coordinates of the points where the graph of $y = f(x)$ intersects the

- (i) x -axis
- (ii) y -axis.

[2 marks]

(b) Write down the equations of

- (i) the vertical asymptotes
- (ii) the horizontal asymptote

to the graph of $y = f(x)$.

[3 marks]

(c) By considering the value of f for large positive and large negative values of x , sketch the graph of f . Be sure to indicate clearly the points of intersection with the coordinate axes, as well as any asymptotes.

[4 marks]

Question 7

Consider the function $f(x) = \frac{x^2+5x+6}{x+1}$, $x \in \mathbb{R}$, $x \neq -1$.

(a) Find the coordinates of the points where the graph of $y = f(x)$ intersects the

- (i) x -axis,
- (ii) y -axis.

[3 marks]

(b) Write down the equation of the vertical asymptote to the graph of $y = f(x)$.

[1 mark]

(c) (i) Show that $\frac{x^2+5x+6}{x+1} = x + a + \frac{b}{x+1}$, where a and b are constants to be determined.

(ii) Hence write down the equation of the oblique asymptote to the graph of $y = f(x)$.

[4 marks]

(d) Sketch the graph of $y = f(x)$. Be sure to indicate clearly the points of intersection with the coordinate axes, as well as any asymptotes.

[3 marks]

Question 8

Let f be a function defined by $f(x) = 3x - 4 + \frac{2}{x-1}$, $x \in \mathbb{R}$, $x \neq 1$.

(a) Write down

(i) the value of p

(ii) the equation of the vertical asymptote to the graph of $y = f(x)$

(iii) the equation of the oblique asymptote to the graph of $y = f(x)$.

[3 marks]

(b) Show that $f(x)$ can be written in the form $\frac{ax^2+bx+c}{x-1}$, where a , b and c are constants to be determined.

[2 marks]

(c) Use an algebraic method to show that the graph of $y = f(x)$ does not cross the x -axis.

[3 marks]

(d) Sketch the graph of $y = f(x)$ and hence write down the range of the function f .

[3 marks]