Question 1

Let $f(x) = \frac{2x+1}{x-4}, x \neq 4$.

(a) For the graph of f, find the equation of:

(i) the vertical asymptote

(ii) the horizontal asymptote. [3]

(b) Find $f^{-1}(x)$. [2]

(c) Write down the equation of the vertical asymptote to the graph of $f^{-1}(x)$. [1]

a) i) $f(x)$ is undefined when the denominator = 0.

$x - 4 = 0$

$x = 4$

$\boxed{\text{Vertical asymptote: } x = 4}$

ii) As x tends towards $\pm\infty$

$f(x)$ tends towards 2.

$\lim_{x \to \pm\infty} f(x) = \frac{2(\pm\infty)+1}{(\pm\infty)-4}$

$\boxed{\text{Horizontal asymptote: } y = 2}$

Let $f(x) = \frac{2x+1}{x-4}, x \neq 4$.

(a) For the graph of f, find the equation of:

(i) the vertical asymptote

(ii) the horizontal asymptote. [3]

(b) Find $f^{-1}(x)$. [2]

(c) Write down the equation of the vertical asymptote to the graph of $f^{-1}(x)$. [1]

b) Find $f^{-1}(x)$.

$y = f(x)$

$y = \frac{2x+1}{x-4}$ swap x and y

$x = \frac{2y+1}{y-4}$

$x(y-4) = 2y+1$

$xy - 4x = 2y + 1$

$xy - 2y = 4x + 1$

$y(x-2) = 4x + 1$

$y = \frac{4x+1}{x-2}$

$\boxed{f^{-1}(x) = \frac{4x+1}{x-2}}$

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Question 2

Consider $f(x) = \frac{ax+b}{3x+c}$, for $x \neq -\frac{c}{3}$, where $a, b, c \in \mathbb{Z}$.

$y = -1$ and $x = 2$ are the equations of the asymptotes of the graph of f. Point A $\left(-2, -\frac{2}{3}\right)$ lies on the graph.

Find the values of a, b and c.

[6]

Find c by using vertical asymptote, $x = 2$

denominator = 0 at $x = 2$

$3(2) + c = 0$ $\qquad \boxed{\therefore c = -6}$

Find a by using horizontal asymptote, $y = -1$.

$\lim_{x \to \pm\infty} f(x) = -1 = \frac{a}{3}$ $\qquad \boxed{\therefore a = -3}$

Find b by using $A(-2, -\frac{2}{3})$.

$f(-2) = \frac{-3(-2)+b}{-12} = \frac{6+b}{-12} = -\frac{2}{3}$ $\qquad \boxed{\therefore b = 2}$

Question 3

Consider the function f defined by $f(x) = \frac{3}{x+5} - 2,\ x \in \mathbb{R},\ x \neq p$.

(a) Write down the value of p.

[1]

a) The denominator of the fraction cannot be zero.

$\boxed{p = -5}$

(b) Write down the equation of the horizontal asymptote to the graph of $y = f(x)$.

[1]

(c) Show that $\frac{3}{x+5} - 2 = \frac{ax+b}{x+5}$, where a and b are constants to be determined.

[2]

(d) Sketch the graph of $y = f(x)$.

[3]

EXAM PAPERS PRACTICE

Consider the function f defined by $f(x) = \frac{3}{x+5} - 2,\ x \in \mathbb{R},\ x \neq p$.

(a) Write down the value of p. $\boxed{p = -5}$

[1]

(b) Write down the equation of the horizontal asymptote to the graph of $y = f(x)$.

[1]

(c) Show that $\frac{3}{x+5} - 2 = \frac{ax+b}{x+5}$, where a and b are constants to be determined.

[2]

(d) Sketch the graph of $y = f(x)$.

[3]

b) $\lim_{x \to \pm\infty} \frac{3}{x+5} = 0$, so as $x \to \pm\infty$, $f(x) \to -2$

$\boxed{y = -2}$

Consider the function f defined by $f(x) = \frac{3}{x+5} - 2,\ x \in \mathbb{R},\ x \neq p$.

(a) Write down the value of p. $\boxed{p = -5}$

$\boxed{y = -2}$

[1]

(b) Write down the equation of the horizontal asymptote to the graph of $y = f(x)$.

[1]

(c) Show that $\frac{3}{x+5} - 2 = \frac{ax+b}{x+5}$, where a and b are constants to be determined.

[2]

(d) Sketch the graph of $y = f(x)$.

[3]

c) $\frac{3}{x+5} - 2 = \frac{3}{x+5} - 2\left(\frac{x+5}{x+5}\right)$

$= \frac{3 - 2(x+5)}{x+5}$

$= \frac{3 - 2x - 10}{x+5}$

$\boxed{= \frac{-2x-7}{x+5} \quad (a = -2,\ b = -7)}$

Consider the function f defined by $f(x) = \frac{3}{x+5} - 2,\ x \in \mathbb{R},\ x \neq p$.

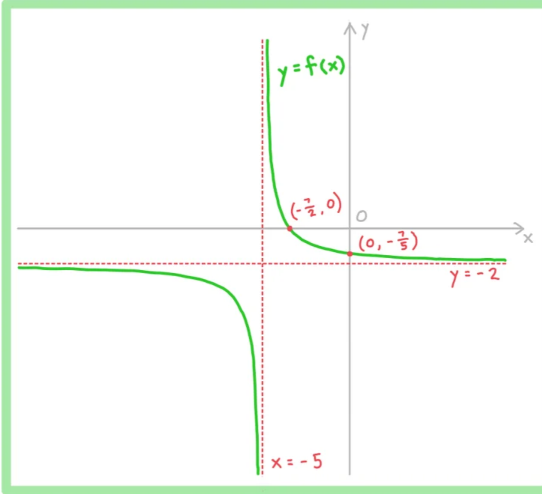
(a) Write down the value of p. $\boxed{p = -5}$ [1]

(b) Write down the equation of the horizontal asymptote to the graph of $y = f(x)$. $\boxed{y = -2}$ [1]

(c) Show that $\frac{3}{x+5} - 2 = \frac{ax+b}{x+5}$, where a and b are constants to be determined. $\boxed{\frac{-2x-7}{x+5}}$ [2]

(d) Sketch the graph of $y = f(x)$. [3]

d)



The question doesn't specifically ask for asymptotes and axes intercepts, but it's always a good idea to include them when sketching a graph.

Also note that this graph is a transformation of the graph of $y = \frac{1}{x}$.

Question 4

Let $f(x) = \frac{4x-2}{2x+5}$, for $x \neq -\frac{5}{2}$.

(a) For the graph of f, find the coordinates of

(i) the x-intercept

(ii) the y-intercept. [3]

a) i) x-intercept $\rightarrow f(x) = 0$

$f(x) = \frac{4x-2}{2x+5} = 0 \rightarrow 4x - 2 = 0 \quad \therefore x = \frac{1}{2}$

$\boxed{x\text{-intercept}: \left(\frac{1}{2}, 0\right)}$

ii) y-intercept $\rightarrow x = 0,\ f(0)$

$f(0) = \frac{4(0)-2}{2(0)+5} = -\frac{2}{5}$

$\boxed{y\text{-intercept}: \left(0, -\frac{2}{5}\right)}$

(b) For the graph of f, find the equation of

(i) the vertical asymptote

(ii) the horizontal asymptote. [3]

Let $f(x) = \frac{4x-2}{2x+5}$, for $x \neq -\frac{5}{2}$.

(a) For the graph of f, find the coordinates of

(i) the x-intercept

(ii) the y-intercept. [3]

(b) For the graph of f, find the equation of

(i) the vertical asymptote

(ii) the horizontal asymptote. [3]

b) i) Vertical asymptote is the value of x that makes the denominator equal zero.

$$\boxed{x = -\frac{5}{2}}$$

ii) Horizontal asymptote is the value $f(x)$ tends towards as x tends towards $\pm\infty$.

$$\lim_{x \to \pm\infty} f(x) = \frac{4}{2} = 2$$

$$\boxed{y = 2}$$

Question 5

Consider the function f defined by $f(x) = \frac{2(3x-1)}{(x+3)(x-2)}$, $x \in \mathbb{R}$, $x \neq -3, 2$.

(a) Find the coordinates of the points where the graph of $y = f(x)$ intersects the coordinate axes. [2]

(b) Express $f(x)$ as partial fractions. [3]

(c) Hence find the equation of the horizontal asymptote to the graph of $y = f(x)$. [2]

a) Set $f(x) = 0$ to find x-intercept:

$$\frac{2(3x-1)}{(x+3)(x-2)} = 0 \implies 2(3x-1) = 0 \implies x = \frac{1}{3}$$

$\implies (\frac{1}{3}, 0)$ is the x-intercept

Then $f(0)$ gives the y-intercept:

$$f(0) = \frac{2(-1)}{(3)(-2)} = \frac{1}{3}$$

$\implies (0, \frac{1}{3})$ is the y-intercept

$$\boxed{(\tfrac{1}{3}, 0) \text{ and } (0, \tfrac{1}{3})}$$

Consider the function f defined by $f(x) = \frac{2(3x-1)}{(x+3)(x-2)}$, $x \in \mathbb{R}$, $x \neq -3, 2$.

(a) Find the coordinates of the points where the graph of $y = f(x)$ intersects the coordinate axes. [2]

(b) Express $f(x)$ as partial fractions. [3]

(c) Hence find the equation of the horizontal asymptote to the graph of $y = f(x)$. [2]

b) $\frac{6x-2}{(x+3)(x-2)} = \frac{A}{x+3} + \frac{B}{x-2} = \frac{A(x-2)+B(x+3)}{(x+3)(x-2)}$

$\Longrightarrow A(x-2)+B(x+3) = 6x-2$

Let $x=-3 \Longrightarrow -5A = -20 \Longrightarrow A = 4$

Let $x = 2 \Longrightarrow 5B = 10 \Longrightarrow B = 2$

$$\boxed{f(x) = \frac{4}{x+3} + \frac{2}{x-2}}$$

Consider the function f defined by $f(x) = \frac{2(3x-1)}{(x+3)(x-2)}$, $x \in \mathbb{R}$, $x \neq -3, 2$.

(a) Find the coordinates of the points where the graph of $y = f(x)$ intersects the coordinate axes. [2]

(b) Express $f(x)$ as partial fractions. $\boxed{f(x) = \frac{4}{x+3} + \frac{2}{x-2}}$ [3]

(c) Hence find the equation of the horizontal asymptote to the graph of $y = f(x)$. [2]

c) $\frac{4}{x+3}$ and $\frac{2}{x-2}$ both go to zero as x becomes large in the positive or negative direction.

Therefore f(x) also goes to zero as x becomes large in the positive or negative direction.

The horizontal asymptote is $y = 0$.

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Question 6

Consider the function $f(x) = \frac{4x-12}{x^2-4x-5}$, $x \in \mathbb{R}$, $x \neq -1, 5$.

(a) Find the coordinates of the points where the graph of $y = f(x)$ intersects the

(i) x-axis

(ii) y-axis.

[2]

(b) Write down the equations of

(i) the vertical asymptotes

(ii) the horizontal asymptote

to the graph of $y = f(x)$.

[3]

(c) By considering the value of f for large positive and large negative values of x, sketch the graph of f. Be sure to indicate clearly the points of intersection with the coordinate axes, as well as any asymptotes.

[4]

a) (i) Set $f(x) = 0$ to find x-intercept:

$\frac{4x-12}{x^2-4x-5} = 0 \implies 4x - 12 = 0 \implies x = 3$

$\boxed{(3, 0)}$

(ii) Then $f(0)$ gives the y-intercept:

$f(0) = \frac{-12}{-5} = \frac{12}{5}$

$\boxed{\left(0, \frac{12}{5}\right)}$ $= (0, 2.4)$

Consider the function $f(x) = \frac{4x-12}{x^2-4x-5}$, $x \in \mathbb{R}$, $x \neq -1, 5$.

(a) Find the coordinates of the points where the graph of $y = f(x)$ intersects the

(i) x-axis $\boxed{(3, 0)}$

(ii) y-axis. $\boxed{\left(0, \frac{12}{5}\right)}$

[2]

(b) Write down the equations of

(i) the vertical asymptotes

(ii) the horizontal asymptote

to the graph of $y = f(x)$.

[3]

(c) By considering the value of f for large positive and large negative values of x, sketch the graph of f. Be sure to indicate clearly the points of intersection with the coordinate axes, as well as any asymptotes.

[4]

b) (i) Find the x-values that make the denominator equal to zero.

$x^2 - 4x - 5 = (x+1)(x-5) = 0$ when $x = -1$ or 5

$\boxed{x = -1 \text{ and } x = 5}$

Note that these are the values excluded from the domain of the function.

(ii) Consider behaviour as $x \to \pm\infty$

$\frac{4x-12}{x^2-4x-5} = \frac{4/x - 12/x^2}{1 - 4/x - 5/x^2}$ Multiply by $\frac{1/x^2}{1/x^2}$

As x becomes large in the positive or negative direction that goes to $\frac{0-0}{1-0-0} = 0$.

$\boxed{y = 0}$

Consider the function $f(x)=\frac{4x-12}{x^2-4x-5}$, $x\in\mathbb{R}$, $x\neq -1, 5$.

(a) Find the coordinates of the points where the graph of $y=f(x)$ intersects the

(i) x-axis $\boxed{(3,0)}$

(ii) y-axis. $\boxed{\left(0,\frac{12}{5}\right)}$

[2]

(b) Write down the equations of

(i) the vertical asymptotes $\boxed{x=-1 \text{ and } x=5}$

(ii) the horizontal asymptote $\boxed{y=0}$

to the graph of $y=f(x)$.

[3]

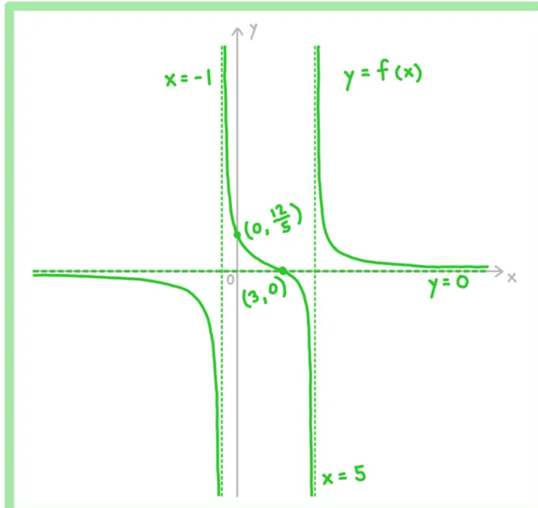
(c) By considering the value of f for large positive and large negative values of x, sketch the graph of f. Be sure to indicate clearly the points of intersection with the coordinate axes, as well as any asymptotes.

[4]

c) Need to find where $f(x)$ is positive and negative:

$x<-1$	$-1<x<3$	$3<x<5$	$x>5$
$f(x)<0$	$f(x)>0$	$f(x)<0$	$f(x)>0$
$f(-2)=-\frac{20}{7}$	$f(0)=\frac{12}{5}$	$f(4)=-\frac{4}{5}$	$f(6)=\frac{12}{7}$

For example: $f(-2)=-\frac{20}{7}$ $f(0)=\frac{12}{5}$ $f(4)=-\frac{4}{5}$ $f(6)=\frac{12}{7}$



Question 7

Consider the function $f(x)=\frac{x^2+5x+6}{x+1}$, $x\in\mathbb{R}$, $x\neq -1$.

(a) Find the coordinates of the points where the graph of $y=f(x)$ intersects the

(i) x-axis,

(ii) y-axis.

[3]

(b) Write down the equation of the vertical asymptote to the graph of $y=f(x)$.

[1]

(c) (i) Show that $\frac{x^2+5x+6}{x+1}=x+a+\frac{b}{x+1}$, where a and b are constants to be determined.

(ii) Hence write down the equation of the oblique asymptote to the graph of $y=f(x)$.

[4]

(d) Sketch the graph of $y=f(x)$. Be sure to indicate clearly the points of intersection with the coordinate axes, as well as any asymptotes.

[3]

a) (i) Set $f(x)=0$ to find x-intercept:

$$\frac{x^2+5x+6}{x+1}=0 \implies x^2+5x+6=(x+2)(x+3)=0$$

$$\implies x=-2 \text{ or } x=-3$$

$$\boxed{(-2,0) \text{ and } (-3,0)}$$

(ii) Then $f(0)$ gives the y-intercept:

$$\frac{(0)^2+5(0)+6}{(0)+1}=6$$

$$\boxed{(0,6)}$$

Consider the function $f(x) = \frac{x^2+5x+6}{x+1}$, $x \in \mathbb{R}$, $x \neq -1$.

(a) Find the coordinates of the points where the graph of $y = f(x)$ intersects the

(i) x-axis, $\boxed{(-2, 0) \text{ and } (-3, 0)}$

(ii) y-axis. $\boxed{(0, 6)}$

[3]

(b) Write down the equation of the vertical asymptote to the graph of $y = f(x)$.

[1]

(c) (i) Show that $\frac{x^2+5x+6}{x+1} = x + a + \frac{b}{x+1}$, where a and b are constants to be determined.

(ii) Hence write down the equation of the oblique asymptote to the graph of $y = f(x)$.

[4]

(d) Sketch the graph of $y = f(x)$. Be sure to indicate clearly the points of intersection with the coordinate axes, as well as any asymptotes.

[3]

b) Find the x-value that makes the denominator equal to zero.

$x + 1 = 0$ when $x = -1$

Note that this is the value excluded from the domain of the function.

$\boxed{x = -1}$

Consider the function $f(x) = \frac{x^2+5x+6}{x+1}$, $x \in \mathbb{R}$, $x \neq -1$.

(a) Find the coordinates of the points where the graph of $y = f(x)$ intersects the

(i) x-axis, $\boxed{(-2, 0) \text{ and } (-3, 0)}$

(ii) y-axis. $\boxed{(0, 6)}$

[3]

(b) Write down the equation of the vertical asymptote to the graph of $y = f(x)$.

$\boxed{x = -1}$

[1]

(c) (i) Show that $\frac{x^2+5x+6}{x+1} = x + a + \frac{b}{x+1}$, where a and b are constants to be determined.

(ii) Hence write down the equation of the oblique asymptote to the graph of $y = f(x)$.

[4]

(d) Sketch the graph of $y = f(x)$. Be sure to indicate clearly the points of intersection with the coordinate axes, as well as any asymptotes.

[3]

c) (i) $\frac{x^2+5x+6}{x+1} = x + a + \frac{b}{x+1}$

$\Longrightarrow \left(x + a + \frac{b}{x+1}\right)(x+1) = x^2 + 5x + 6$

$\Longrightarrow x^2 + \underbrace{(a+1)}_{a+1=5} x + \underbrace{(a+b)}_{a+b=6} = x^2 + 5x + 6$

$\Longrightarrow a = 4, \ b = 2$

$\boxed{\frac{x^2+5x+6}{x+1} = x + 4 + \frac{2}{x+1}}$

(ii) When x becomes large in the positive or negative direction, $\frac{2}{x+1}$ goes to zero.

$\boxed{y = x + 4}$

Consider the function $f(x) = \frac{x^2+5x+6}{x+1}$, $x \in \mathbb{R}$, $x \neq -1$.

(a) Find the coordinates of the points where the graph of $y = f(x)$ intersects the

(i) x-axis, $\boxed{(-2, 0) \text{ and } (-3, 0)}$

(ii) y-axis. $\boxed{(0, 6)}$

[3]

(b) Write down the equation of the vertical asymptote to the graph of $y = f(x)$.

$\boxed{x = -1}$

[1]

(c) (i) Show that $\frac{x^2+5x+6}{x+1} = x + a + \frac{b}{x+1}$, where a and b are constants to be determined.

(ii) Hence write down the equation of the oblique asymptote to the graph of $y = f(x)$.

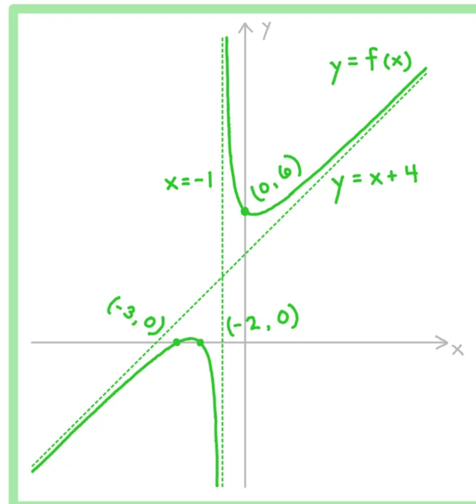
$\boxed{y = x + 4}$

[4]

(d) Sketch the graph of $y = f(x)$. Be sure to indicate clearly the points of intersection with the coordinate axes, as well as any asymptotes.

[3]

d)



Question 8

Let f be a function defined by $f(x) = 3x - 4 + \frac{2}{x-1}$, $x \in \mathbb{R}$, $x \neq p$.

(a) Write down

(i) the value of p

(ii) the equation of the vertical asymptote to the graph of $y = f(x)$

(iii) the equation of the oblique asymptote to the graph of $y = f(x)$.

[3]

a) (i) $\boxed{p = 1}$ The denominator of the fraction cannot be zero.

(ii) $\boxed{x = 1}$

(iii) When x becomes large in the positive or negative direction, $\frac{2}{x-1}$ goes to zero.

$\boxed{y = 3x - 4}$

(b) Show that $f(x)$ can be written in the form $\frac{ax^2+bx+c}{x-1}$, where a, b and c are constants to be determined.

[2]

(c) Use an algebraic method to show that the graph of $y = f(x)$ does not cross the x-axis.

[3]

(d) Sketch the graph of $y = f(x)$ and hence write down the range of the function f.

[3]

EXAM PAPERS PRACTICE

Let f be a function defined by $f(x) = 3x - 4 + \frac{2}{x-1}$, $x \in \mathbb{R}$, $x \neq p$.

(a) Write down

(i) the value of p

(ii) the equation of the vertical asymptote to the graph of $y = f(x)$

(iii) the equation of the oblique asymptote to the graph of $y = f(x)$.

[3]

(b) Show that $f(x)$ can be written in the form $\frac{ax^2+bx+c}{x-1}$, where a, b and c are constants to be determined.

[2]

(c) Use an algebraic method to show that the graph of $y = f(x)$ does not cross the x-axis.

[3]

(d) Sketch the graph of $y = f(x)$ and hence write down the range of the function f.

[3]

b) $3x - 4 + \frac{2}{x-1} = \frac{(3x-4)(x-1)+2}{x-1}$

$= \frac{3x^2 - 3x - 4x + 4 + 2}{x-1}$

$\boxed{f(x) = \frac{3x^2 - 7x + 6}{x-1}}$

Let f be a function defined by $f(x) = 3x - 4 + \frac{2}{x-1}$, $x \in \mathbb{R}$, $x \neq p$.

(a) Write down

(i) the value of p

(ii) the equation of the vertical asymptote to the graph of $y = f(x)$

(iii) the equation of the oblique asymptote to the graph of $y = f(x)$.

[3]

(b) Show that $f(x)$ can be written in the form $\frac{ax^2+bx+c}{x-1}$, where a, b and c are constants to be determined.

$\boxed{f(x) = \frac{3x^2 - 7x + 6}{x-1}}$

[2]

(c) Use an algebraic method to show that the graph of $y = f(x)$ does not cross the x-axis.

[3]

(d) Sketch the graph of $y = f(x)$ and hence write down the range of the function f.

[3]

c) $\frac{3x^2 - 7x + 6}{x-1} = 0$

$\Longrightarrow 3x^2 - 7x + 6 = 0$

The discrimant is *

$(-7)^2 - 4(3)(6) = -23 < 0$

$\Longrightarrow$ The equation has no real roots

Therefore $y = f(x)$ does not cross the x-axis.

* Can also complete the square here:

$3\left(x - \frac{7}{6}\right)^2 + \frac{23}{12} = 0$

$\Longrightarrow \left(x - \frac{7}{6}\right)^2 = -\frac{23}{36}$

$\Longrightarrow$ No real solutions

EXAM PAPERS PRACTICE

Let f be a function defined by $f(x) = 3x - 4 + \frac{2}{x-1}$, $x \in \mathbb{R}$, $x \neq p$.

(a) Write down

(i) the value of p

(ii) the equation of the vertical asymptote to the graph of $y = f(x)$ $\boxed{x = 1}$

(iii) the equation of the oblique asymptote to the graph of $y = f(x)$. $\boxed{y = 3x - 4}$

[3]

(b) Show that $f(x)$ can be written in the form $\frac{ax^2+bx+c}{x-1}$, where a, b and c are constants to be determined.

[2]

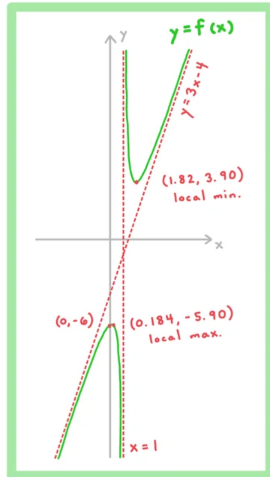
(c) Use an algebraic method to show that the graph of $y = f(x)$ does not cross the x-axis.

[3]

(d) Sketch the graph of $y = f(x)$ and hence write down the range of the function f.

[3]

d) (i)



Non-exact values given to 3 s.f.

(ii) The range is (to 3 s.f.):

$(f(x): f(x) \leq -5.90) \cup (f(x): f(x) \geq 3.90)$