EXAM PAPERS PRACTICE

Question 1

Two independent random variables X and Y follow binomial distributions, where $X \sim B(5, 0.3)$ and $Y \sim B(11, 0.45)$.

(a) Find

(i) $P(X = 2)$

(ii) $P(Y \leq 6)$.

[4]

(b) Calculate

(i) $E(2X - Y)$

(ii) $Var(3X - 2Y)$.

[4]

a) Use your GDC

i) $\boxed{\text{binomPDF}(5, 0.3, 2) = 0.3087}$

ii) $\boxed{\text{binomCDF}(11, 0.45, 0, 6) = 0.826}$

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b)

Linear transformation of a single random variable	$E(aX+b) = aE(X)+b$ $Var(aX+b) = a^2 Var(X)$
Linear combinations of n independent random variables, $X_1, X_2, \ldots, X_n$	$E(a_1X_1 \pm a_2X_2 \pm \ldots \pm a_nX_n) = a_1E(X_1) \pm a_2E(X_2) \pm \ldots \pm a_nE(X_n)$ $Var(a_1X_1 \pm a_2X_2 \pm \ldots \pm a_nX_n)$ $= a_1^2 Var(X_1) + a_2^2 Var(X_2) + \ldots + a_n^2 Var(X_n)$

i) $E(X) = 5 \times 0.3 = 1.5$, $E(Y) = 11 \times 0.45 = 4.95$

$E(2X - Y) = 2E(X) - E(Y) = 2(1.5) - 4.95$

$\boxed{E(2X - Y) = -1.95}$

ii) $Var(X) = 5 \times 0.3 \times 0.7 = 1.05$

$Var(Y) = 11 \times 0.45 \times 0.55 = 2.7225$

$Var(3X - 2Y) = 3^2 \times 1.05 + 2^2 \times 2.7225$

$\boxed{Var(3X - 2Y) = 20.34}$

EXAM PAPERS PRACTICE

Question 2

A game is played with two fair spinners. Each spinner is divided into three sections numbered 1, 2 and 3. A player's score is obtained by spinning both spinners simultaneously and adding together the numbers that they land on.

(a) Complete the table below for the probability distribution of the game.

Score, X	2	3	4	5	6
$P(X = x)$	$1/9$	$2/9$	$3/9$	$2/9$	$1/9$

[2]

(b) Find the expected score, $E(X)$.

[2]

Jian Wei wants to award prizes such that a player receives $3 for the score that they achieve.

(c) Find the expected prize money for the game.

[2]

a) Min score is 2 (1+1) and max score is 6 (3+3) and there are 9 possible outcomes.

score = 2 $\rightarrow (1+1) = \left(\frac{1}{3}\right)^2 = \frac{1}{9}$

score = 3 $\rightarrow (1+2)$ and $(2+1) = 2\left(\frac{1}{3}\right)^2 = \frac{2}{9}$

score = 4 $\rightarrow (2+2), (1+3)$ and $(3+1) = 3\left(\frac{1}{3}\right)^2 = \frac{3}{9}$

score = 5 $\rightarrow (2+3)$ and $(3+2) = 2\left(\frac{1}{3}\right)^2 = \frac{2}{9}$

score = 6 $\rightarrow (3+3) = \left(\frac{1}{3}\right)^2 = \frac{1}{9}$

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[2]

b) $E(X) = \sum x P(X = x)$ (in formula booklet)

$E(X) = (2)\left(\frac{1}{9}\right) + (3)\left(\frac{2}{9}\right) + (4)\left(\frac{3}{9}\right) + (5)\left(\frac{2}{9}\right) + (6)\left(\frac{1}{9}\right)$

$\boxed{E(X) = 4}$

A game is played with two fair spinners. Each spinner is divided into three sections numbered 1, 2 and 3. A player's score is obtained by spinning both spinners simultaneously and adding together the numbers that they land on.

(a) Complete the table below for the probability distribution of the game.

Score, X					
$P(X = x)$					

[2]

(b) Find the expected score, $E(X)$.

$\boxed{E(X) = 4}$

[2]

Jian Wei wants to award prizes such that a player receives $3 for the score that they achieve.

(c) Find the expected prize money for the game.

[2]

c) $\boxed{\text{expected prize money} = 3 \times 4 = \$12}$

Question 3

Dasha plays two games. When playing game A, Dasha has an equal chance of scoring 2, 3 or 5 points. When playing game B, Dasha has a 25% chance of scoring 1 or 2 and a 50% chance of scoring 5.

(a) For game B find the expected score.

[3]

The scores for both games are added together.

(b) Find the expected total.

[3]

a) $E(B) = 0.25 \times 1 + 0.25 \times 2 + 0.5 \times 5$

$\boxed{E(B) = 3.25}$

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$\boxed{E(B) = 3.25}$

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The scores for both games are added together.

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[3]

b) $E(A) = \frac{1}{3}(2) + \frac{1}{3}(3) + \frac{1}{3}(5) = \frac{10}{3}$

$E(T) = E(A) + E(B)$

$E(T) = \frac{10}{3} + 3.25 = \frac{79}{12}$

$\boxed{E(T) = 6.58 \quad (3s.f.)}$

Question 4

A random variable has $E(X) = 23$ and $Var(X) = 1.5$.

Find

(i) $E(X - 6)$

(ii) $E(-2X + 5)$

(iii) $Var(X + 7)$

(iv) $Var(3X - 3)$

[6]

$E(aX + b) = aE(X) - b$ (in formula booklet)

$Var(aX + b) = a^2 Var(X)$ (in formula booklet)

i) $\boxed{E(X - 6) = 23 - 6 = 17}$

ii) $\boxed{E(-2X + 5) = -2(23) + 5 = -41}$

iii) $\boxed{Var(X + 7) = 1.5}$

iv) $\boxed{Var(3X - 3) = 3^2(1.5) = 13.5}$

Question 5

A scientist is studying a population of komodo dragons and has found that the length of the dragons follows a normal distribution. The mean length, of a male dragon is 2.59 m with standard deviation of 0.18 m. For a female dragon the mean length is 2.28 m with standard deviation of 0.11 m.

(a) Find the probability that the length of a female komodo dragon selected at random will be greater than 2.4 m.

[2]

a) $\boxed{\text{normCDF}(2.4, \infty, 2.28, 0.11) = 0.138 \text{ (3s.f.)}}$

Four male komodo dragons are selected at random.

(b) Find

(i) the expected total length of the dragons

(ii) the variance of the length of the dragons.

[4]

(c) Hence find the probability that the total length of 4 randomly selected male dragons will be greater than 11 m.

[3]

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(c) Hence find the probability that the total length of 4 randomly selected male dragons will be greater than 11 m.

[3]

c) $X \sim N(10.36, 0.1296)$

$\boxed{P(X>11) = \text{normCDF}(11, \infty, 10.36, \sqrt{0.1296}) = 0.0377}$

Question 6

A cinema chain sells 3 sizes of popcorn at the food counter. When a container is filled with popcorn its mass follows a normal distribution. The mean and variance of the mass of each size of container when filled with popcorn is shown in the table below.

	Mean (g)	Variance (g^2)
Small	60	4
Medium	160	169
Large	250	441

(a) Find the probability that a large container selected at random contains between 210 g and 270 g of popcorn.

[2]

Raoul buys 1 small bag of popcorn and 3 medium bags.

(b) With reference to the total amount of popcorn that Raoul has purchased, find

(i) the mean mass

(ii) the variance.

[4]

(c) Hence find the standard deviation of the total amount of popcorn that Raoul has purchased.

[1]

a) $\boxed{\text{normCDF}(210, 270, 250, 21) = 0.801}$

A cinema chain sells 3 sizes of popcorn at the food counter. When a container is filled with popcorn its mass follows a normal distribution. The mean and variance of the mass of each size of container when filled with popcorn is shown in the table below.

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i) $E(S + M_1 + M_2 + M_3) = E(S) + E(M_1) + E(M_2) + E(M_3)$

$E(S + M_1 + M_2 + M_3) = 60 + 3(160)$

$\boxed{E(S + M_1 + M_2 + M_3) = 540g}$

ii) $Var(S + M_1 + M_2 + M_3) = Var(S) + Var(M_1) + Var(M_2) + Var(M_3)$

$Var(S + M_1 + M_2 + M_3) = 4 + 3(169)$

$\boxed{Var(S + M_1 + M_2 + M_3) = 511g^2}$

A cinema chain sells 3 sizes of popcorn at the food counter. When a container is filled with popcorn its mass follows a normal distribution. The mean and variance of the mass of each size of container when filled with popcorn is shown in the table below.

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Small	60	4
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[1]

c) $\boxed{\sigma = \sqrt{511} = 22.6g \ (3s.f.)}$

Question 7

A company manufactures individual chocolates. The distribution of the mass of these chocolates can be modelled as a normal distribution with mean mass 11 g and variance 2.25 g^2.

A chocolate with a mass of less than a g is too small to sell.

(a) Given that the probability a chocolate is too small to sell is 0.05, find the value of a.

[2]

a) $\boxed{\text{invNorm}(0.05,\ 11,\ 1.5) = 8.532\ldots = 8.53\text{g} \quad (3 \text{ s.f.})}$

Chocolates are sold in bags of 8.

(b) Find the mean weight of a bag of chocolates.

[2]

(c) Find the variance of a bag of chocolates.

[2]

(d) Find the probability that the average mass of a chocolate in a bag is less than or equal to a g.

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[2]

Chocolates are sold in bags of 8.

(b) Find the mean weight of a bag of chocolates.

[2]

b) $\boxed{8E(M) = 8(11) = 88\text{g}}$

(c) Find the variance of a bag of chocolates.

[2]

(d) Find the probability that the average mass of a chocolate in a bag is less than or equal to a g.

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[3]

c) $\boxed{8\text{Var}(M) = 8^2(2.25) = 144 \text{ g}^2}$

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Chocolates are sold in bags of 8.

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[2]

(c) Find the variance of a bag of chocolates.

[2]

(d) Find the probability that the average mass of a chocolate in a bag is less than or equal to a g.

[3]

d) $B \sim N(88, 18)$

average of 8 boxes = $8a = 8(8.532...) = 68.261...$

$\boxed{\text{normCDF}(-\infty, 68.261..., 88, \sqrt{18}) = 0.00000164 \text{ (3 s.f.)}}$

Question 8

Julie is eating in a sushi restaurant where the individual plates are transported through the restaurant on a conveyor belt. Julie's two favourite dishes are ngiri and edamame beans and the number of plates of these foods that pass Julie follow Poisson distributions. On average, one ngiri plate passes Julie every 10 seconds and one plate of edamame beans passes her every 25 seconds.

(a) Write down

(i) how many ngiri plates pass Julie in 2 minutes

(ii) how many plates of edamame beans pass Julie in 2 minutes.

[3]

(b) Hence find the probability that 15 or fewer of her favourite dishes pass Julie in a 2 minute interval.

[3]

a) i) 2 mins = 120 s = 12 × 10s

$\boxed{\text{Ngiri} = 1 \times 12 = 12 \text{ plates}}$

ii) 2 mins = 120 s = 4.8 × 25s

$\boxed{\text{Edamame} = 1 \times 4.8 = 4.8 \text{ plates}}$

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(a) Write down

(i) how many ngiri plates pass Julie in 2 minutes

$\boxed{\text{Ngiri} = 12 \text{ plates}}$

(ii) how many plates of edamame beans pass Julie in 2 minutes.

$\boxed{\text{Edamame} = 4.8 \text{ plates}}$

[3]

(b) Hence find the probability that 15 or fewer of her favourite dishes pass Julie in a 2 minute interval.

[3]

b) $N \sim Po(12)$, $E \sim Po(4.8)$

Let $T = N + E \longrightarrow T \sim Po(12 + 4.8) = T \sim Po(16.8)$

$\boxed{P(T \leq 15) = 0.390 \quad (3\text{s.f.})}$

Question 9

Matt and Hannah both like to go for a run each morning. The distance that Matt runs each day can be modelled by a random variable $M \sim N(3.2, 0.8^2)$ and the distance that Hannah runs can be modelled by a random variable $H \sim N(4.7, 0.5^2)$. All distances are measured in kilometres.

The variables H and M are independent of each other.

(a) On a day chosen at random, find the probability that Hannah will run a distance of at least 5 km.

[2]

(b) For 7 randomly selected runs find the probability that the total distance run by Hannah will exceed 30 km.

[3]

(c) Find the probability that, on a day chosen at random, Matt runs further than Hannah.

[5]

a) $\boxed{\text{normCDF}(5, \infty, 4.7, 0.5) = 0.275 \ (3 \text{ s.f.})}$

Matt and Hannah both like to go for a run each morning. The distance that Matt runs each day can be modelled by a random variable $M \sim N(3.2, 0.8^2)$ and the distance that Hannah runs can be modelled by a random variable $H \sim N(4.7, 0.5^2)$. All distances are measured in kilometres.

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(c) Find the probability that, on a day chosen at random, Matt runs further than Hannah.

[5]

b) $7E(H) = 7(4.7) = 32.9$

$7Var(H) = 7 \times 0.5^2 = 1.75$

$\boxed{P(7H > 30) = 0.9858...}$

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[5]

c) $M > H \rightarrow M - H > 0 \rightarrow X = M - H$

$P(X > 0) = P(M - H > 0)$

$E(X) = E(M) - E(H) = 3.2 - 4.7 = -1.5$

$Var(X) = var(M) - var(H) = 0.8^2 - 0.5^2 = 0.39$

$X \sim N(-1.5, 0.39)$

$\boxed{P(X > 0) = 0.008155... = 0.00816 \ (3 \text{s.f.})}$

Question 10

A computer game has two levels. It is found that the time taken for a player to complete level 1 is normally distributed with mean 110 seconds and standard deviation 23 seconds. The time taken for a player to complete level 2 is normally distributed with a mean 196 seconds and standard deviation 27 seconds.

(a) Find the probability that, for a randomly chosen player, the time taken to complete level 1 will be between 97 and 105 seconds.

[2]

(b) Find the probability that the length of time to complete level 2 for a randomly chosen player is more than twice as long as it takes to complete level 1 for another randomly chosen player.

[6]

a) $\boxed{\text{normCDF}(97, 105, 110, 23) = 0.1279... = 0.128 \text{ (3 s.f.)}}$

A computer game has two levels. It is found that the time taken for a player to complete level 1 is normally distributed with mean 110 seconds and standard deviation 23 seconds. The time taken for a player to complete level 2 is normally distributed with a mean 196 seconds and standard deviation 27 seconds.

(a) Find the probability that, for a randomly chosen player, the time taken to complete level 1 will be between 97 and 105 seconds.

[2]

(b) Find the probability that the length of time to complete level 2 for a randomly chosen player is more than twice as long as it takes to complete level 1 for another randomly chosen player.

[6]

b) $P(B > 2A) = P(T > 0)$, where $T = B - 2A$

$E(T) = E(B - 2A) = E(B) - 2E(A)$

$E(T) = 196 - 2(110) = -24$

$\text{var}(T) = \text{var}(B - 2A) = \text{var}(B) + 2^2\text{var}(A)$

$\text{var}(T) = 27^2 + 4 \times 23^2 = 2845$

$\boxed{P(T > 0) = 0.3263... = 0.326 \text{ (3 s.f.)}}$