# Radioactive Decay

## Mark Schemes

| | |
|---|---|
| Course | HL IB Physics |
| Section | Nuclear & Quantum Physics |
| Topic | Radioactive Decay |
| Difficulty | Medium |



To be used by all students preparing for HL IB Physics
Students of other boards may also find this useful

1

The correct answer is **C** because:

- **Activity** means the **rate of decay**
- Element **P** starts with an unknown number of atoms, $N$
    - In one half-life, nucleus P decays so that
        - $N \rightarrow \frac{1}{2}N$
    - **P** decayed at a **rate** of $\frac{1}{2}N\,T^{-1}$
- **Q** has the same **activity**, so it also decays at a rate of $\frac{1}{2}N\,T^{-1}$
    - Element Q starts with $3N$ atoms
    - Therefore, in one half-life for Q the number of decays can be written as $3N \rightarrow \frac{3}{2}N$
    - Three times as many decays occur for Q as for P
    - Therefore the half-life must be $3T$



Questions like this one come up again and again. To start with you will probably need to solve them step-by-step as shown above. Once you have got the hang of it you will see the pattern and be able to answer them very quickly.

2

The answer is **B** because:

- The known quantities in the question are that:
    - Half-life, $T$ = 10 days
    - Number of days passed, $t$ = 25 days
- Therefore 2.5 half-lives have passed
    - Each half-life reduces the amount by 50%, therefore determine the remaining percentage after each half-life:

| time / days | % remaining |
|---|---|
| 0 | 100 |
| 10 | 50 |
| 20 | 25 |
| 30 | 12.5 |

- The percentage remaining after 25 days will be approximately halfway between the 20 and 30 day points
- Take the average, $\frac{25 + 12.5}{2} = 18.75$

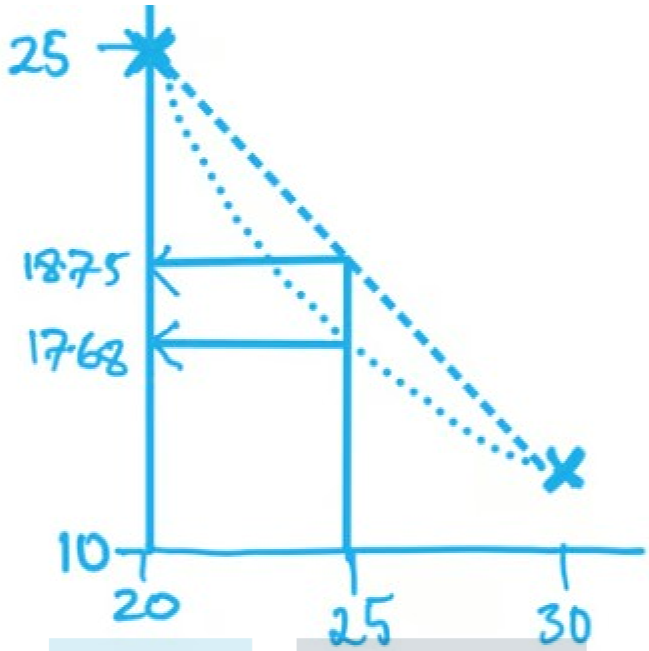
The question asks for an approximation, which you can get to without the use of a calculator as shown.

If you were to use a calculator, you could solve the problem using the equations in the data booklet:

- The decay constant for this sample $\lambda = \frac{ln2}{T_{1/2}} = \frac{0.693}{10} = 0.0693\ day^{-1}$
- The nuclei remaining, $N = N_0 e^{-\lambda t} = 100 \times e^{(-0.0693 \times 25)} = 17.68\ \%$

You should notice that the first method comes close but does not reach as accurate an answer as the calculation using the exponential function.

Decay curves are exactly that, curves, so you cannot expect to see a perfectly linear result. The graph below shows how the two are close, but will never be the same.




But the approximation method is still useful, and very quick.

3

The correct answer is **D** because

- The parent nuclide is reduced by half with each successive half-life
  - The simplest way to model these is noting down successive fractions

| $T_{1/2}$ | fraction |
|---|---|
| 0 | 1 |
| 1 | 1/2 |
| 2 | 1/4 |
| 3 | 1/8 |
| 4 | 1/16 |
| 5 | 1/32 |

- After five half-lives the fraction of the **parent remaining** is $\frac{1}{32}$
  - The **daughter** nuclide makes up the **rest**
  - $1 - \frac{1}{32} = \frac{31}{32}$



**A** is incorrect as this is the fraction of the parent nuclei remaining after four half-lives.

**B** is incorrect as this is the fraction of the parent nuclide remaining.

**C** is incorrect as this is the fraction of the daughter nuclides remaining after four half-lives.

4

The correct answer is **C** because

- Anti-neutrinos have a continuous energy spectrum. This is known because:
  - Anti-neutrinos are produced in beta-minus decay
  - In beta-minus decay a neutron decays into a proton and an electron, releasing the binding energy inside the nucleus

- Since the proton remains in the nucleus, all of the binding energy **could** be converted into the kinetic energy of the ejected electron
    - However, experiments show that these ejected electrons have a **range** of energies
- The 'missing' energy is the kinetic energy of the anti-neutrino which is also ejected from the nucleus
    - Since the ejected electrons have a range of energies, the anti-neutrino also has to have a range
    - Hence the value of the energy of a neutrino is continuous
    - So, only **C** or **D** can be correct
- Alpha particles have a discrete energy spectrum. This is known because:
    - All of the decay energy goes into emitting an alpha particle
    - No other decay takes place when an alpha particle is emitted from a large nuclei
    - Experimental evidence shows that the energies of alpha particles emitted from nuclei are discrete
    - This is a key piece of evidence to suggest that nuclei also exist in discrete energy levels
    - So, only **C** can be correct



5

The correct answer is **B** because:

- The discrete kinetic energy levels of alpha particles and gamma-ray photos have led to the discovery of emission spectra
    - This realisation led to theories of discrete energy levels of nuclei and atoms
    - The existence of discrete energy levels explains and led to the use of, emission and absorption spectra to identify elements
    - So, only **B** or **D** can be correct
- Beta-particles have a continuous range of kinetic energies and this led to the discovery of the neutrino

- Neutrinos were theorised by Pauli to explain the 'missing energy' problem seen in beta-minus decay
- Electrons emitted from the nucleus were shown to have a **range** of energies
- However, due to the mass defect when a neutron changes to a proton and an electron, it was clear that a **specific** amount of kinetic energy had to be accounted for
- The neutrino, with its range of kinetic energies, filled this gap
- So, only **B** can be correct

6

The correct answer is **C** because:

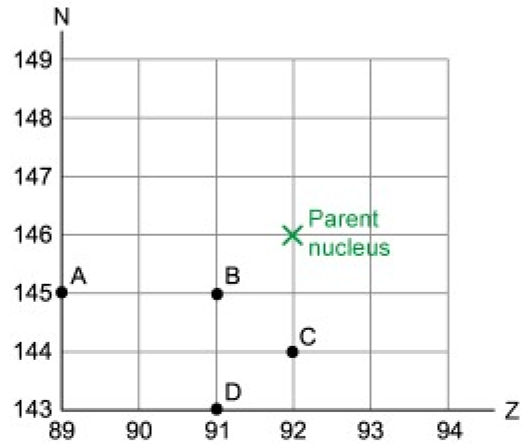
- The law of radioactive decay states: $A = A_0 e^{-\lambda t}$
- Decay constant: $\lambda = \frac{\ln 2}{t_{1/2}}$
- Combining both expressions: $A = A_0 e^{-\left(\frac{\ln 2}{t_{1/2}}\right)t}$
- Activity after $t$ = 2.5 days: $A = 10 \times e^{\frac{2.5 \ln 2}{1}} = 1.8 \text{ Bq}$



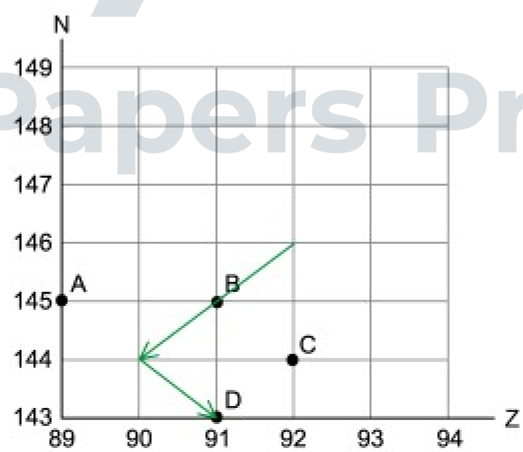
7

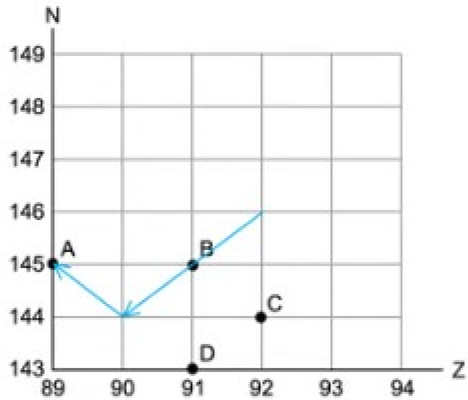
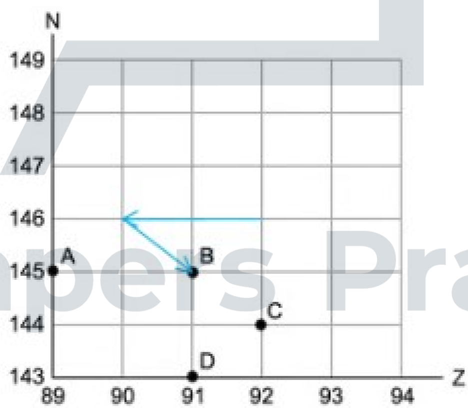
The correct answer is **D** because:

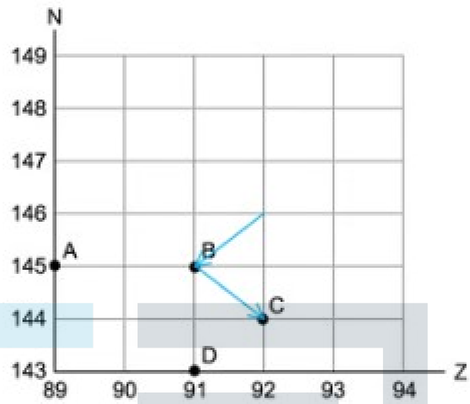
- $^{238}_{92}U$ means that the uranium nucleus contains 92 protons and 146 neutrons
  - The number of neutrons is calculated by subtracting the atomic number from the nucleon number
  - Neutron number = 238 – 92 = 146
- We can then plot the position of uranium-238 (parent nucleus) on the N-Z graph



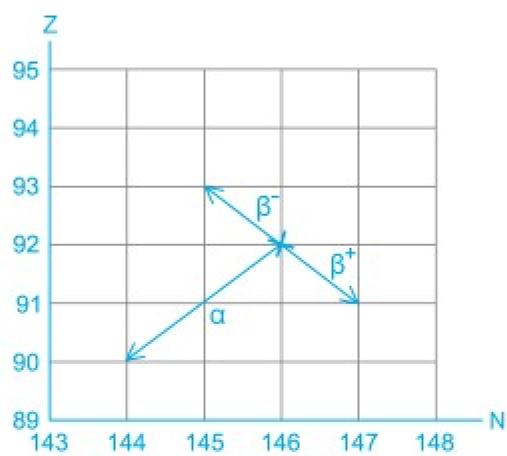

- When a nucleus undergoes alpha decay, an alpha particle is emitted
  - An alpha particle consists of two neutrons and two protons and can be written as $^{4}_{2}\alpha$
- The N and Z values of the nucleus both decrease by two
  - **Note:** Gamma radiation does not affect the number of nucleons
- Beta radiation decreases neutron number by one but increases proton number by one
  - A beta particle is an electron and can be written as $^{0}_{-1}\beta$



| | |
|---|---|
| **A** is incorrect as | this represents alpha decay followed by positron, or $\beta^+$, decay   |
| **B** is incorrect as | this represents alpha decay which neglects to decrease the number of neutrons, followed by beta decay   |

| | |
|---|---|
| **C** is incorrect as | this would be obtained if the proton and neutron numbers were to only decrease by one during the alpha decay. This is then followed by a beta decay   |

To be able to answer these questions correctly, you need to recall that alpha decay is the emission of two neutrons and two protons (a helium-4 nucleus) and that beta decay involves a neutron changing into a proton. This is a diagonal right on the diagram above, but watch out – the axes may not always be this way round! It's better to understand what's going on in each decay as opposed to memorising the paths on an N-Z graph.




Alternatively, you could write out an equation to determine the N and Z values and then find the correct point on the graph

$$^{238}_{92}U \xrightarrow{^{4}_{2}\alpha} {}^{234}_{90}Th \xrightarrow{^{0}_{-1}\beta} {}^{234}_{91}Pa$$

So protactinium, or $^{234}_{91}Pa$ (don't worry, you wouldn't be expected to know its symbol) contains 91 protons and 234 – 91 = 143 neutrons. So, N = 143 and Z = 91

8

The correct answer is **B** because:

- The 80 g sample of living tissue has an activity of 20 Bq
  - Therefore, 20 g of living tissue has an activity of 5 Bq
- After one half-life, the activity is 2.5 Bq
- After two half-lives, the activity is 1.25 Bq – this is the activity of the ancient sample
- The age of the sample is, therefore, equal to two half-lives:
  - 2 × 6000 years = 12 000 years

**A** is incorrect as this answer is obtained by halving the half-life as opposed to doubling it.

**C** is incorrect as this answer is three half-lives instead of two.



**D** is incorrect as this answer has not accounted for the fact that the living sample is four times larger in mass. This means that four half-lives have been calculated instead of two.

Any living sample of this tissue will have an activity of roughly 0.25 Bq per gram. This is because the carbon-14 atoms are constantly being replaced by new nuclei from the elephant's diet. Once the organism has died, those nuclei are no longer being replaced and the activity begins to decrease as nuclei decay.

The trick in this question is to spot that the mass of the living sample is four times larger than the mass of the ancient sample. You have to correct this!

If 80 g of living tissue has an activity of 20 Bq, 20g of living tissue has an activity of 5 Bq. Mass of a material and its activity are **directly proportional**.

This activity of 5 Bq has halved twice to give the ancient activity of 1.25 Bq, so two half-lives must have passed since the elephant was last alive.

9

The correct answer is **A** because:

- Since fluorine-18 is a positron emitter, we would expect to see a positron, or beta-plus particle ${}^{0}_{+1}\beta$, on the right-hand side of the decay equation
  - This eliminates option **B**
- In order to conserve lepton number, the positron must be accompanied by an electron neutrino, $v_e$
  - This eliminates option **D**
- The atomic number decreases but the nucleon number remains the same, as a proton changes into a neutron during this process
  - Therefore, we would expect the total nucleon and proton numbers on both sides of the decay equation to be the same
- This is shown by option **A**



$${}^{18}_{9}F \rightarrow {}^{18}_{8}O + {}^{0}_{+1}\beta + v_e$$

Nucleon number: 18 = 18 + 0 + 0 ✓**conserved**

Proton number: 9 = 8 + 1 + 0 ✓**conserved**

**B** is incorrect as a beta-minus particle (an electron) is emitted whereas the question refers to a positron i.e. beta-plus decay.

**C** is incorrect as the atomic number of the nucleus does not change. All other aspects are correct but the atomic number of oxygen does not reflect the fact that a proton has changed into a neutron.

**D** is incorrect as the neutrino emitted in beta-plus decay should be an electron neutrino. **D** shows an antineutrino being emitted. This does not conserve lepton number.

Being aware of the changes that occur in the nucleus during beta-plus and beta-minus decay is crucial to spotting mistakes in atomic number in decay equations.

These decay equations also link closely to the topic on particle conservation laws.

Checking whether lepton number is conserved is a very helpful method for double-checking your answer when it comes to beta decay, as neutrinos or antineutrinos must be emitted alongside the beta particles

$$^{18}_{9}F \rightarrow ^{18}_{8}O + ^{0}_{+1}\beta + v_e$$

When considering just the nuclei, the left side of option **A** has a lepton number of zero. A positron has a lepton number of -1 as it is an antiparticle, so it must be accompanied by a neutrino with a lepton number of +1 to ensure that lepton number is conserved.

10

The correct answer is **D** because:

- When an alpha decay occurs, the following process takes place:
  - $^{231}_{91}Pa \rightarrow ^{227}_{89}Ac + \ ^{4}_{2}\alpha$
- Protactinium-231 will decay to become Actinium-227 (**Note:** you don't need to know the name of these elements) and an alpha particle
  - An alpha particle contains 4 nucleons and 2 protons (effectively a helium-4 nucleus)
- Following the alpha decay, the Actinium-227 will further decay through beta-minus decay:
  - $^{227}_{89}Ac \rightarrow ^{227}_{90}Th + \ ^{0}_{-1}\beta + \bar{v_e}$
- Actinium-227 will decay to become Thorium-227, an electron (a beta-minus particle) and an antineutrino
- Therefore, the final answer is 90 protons and 227 nucleons
  - $^{231}_{91}Pa \rightarrow A + \ ^{4}_{2}\alpha \rightarrow B + \ ^{0}_{-1}\beta^{-} + \bar{v_e}$
- Alternatively, you can calculate the nucleon number of **B** as follows:
  - 231 – 4 – 0 = 227

- And the proton number of **B** as follows:
  - 91 – 2 – (–1) = 90

**A** is incorrect as the nucleon number has only reduced by two, whereas four nucleons are emitted during alpha decay. This indicates that the neutrons have been neglected in the alpha decay step. Additionally, the increase in atomic number caused by beta decay has not been accounted for.

**B** is incorrect as the same misconception about alpha decay from option **A** is present, but this time there is a subsequent increase in atomic number as a result of beta-minus decay has been included.

**C** is incorrect as the reduction in nucleon number is correct, but the increase in atomic number due to a neutron becoming a proton in beta-minus decay has not been included.

If you are not very confident in your understanding of multi-step decays, it is useful to write out each step of the decay so you don't miss anything! It is important to know that alpha particles contain four nucleons, two of which are protons, allowing you to write the first step correctly:

$$^{231}_{91}Pa \rightarrow \, ^{227}_{89}Ac + \, ^{4}_{2}\alpha$$

After this step, it is once again important to be aware of the changes within the nucleus during beta decay: a neutron changes into a proton – this is because the nucleus was unstable due to an excess of neutrons. This change is shown by increasing the atomic number but keeping the nucleon number the same.

Once you are more confident with multi-step decays, you can quickly determine new nucleon and proton numbers with a quick calculation, as shown at the end of the worked solution.

11

The correct answer is **C** because:

- Beta radiation has a relatively high ionising power
  - This means that it has a short range in air – typically only a few metres

- This eliminates options **B** and **D**
- At a range of more than 1.5 m, the count rate is still not zero as background radiation is always present
  - This eliminates option **A**

**A** is incorrect as the count rate drops to zero after a certain distance – this cannot happen as there will always be a count rate from the background radiation.

**B** is incorrect as the count rate in this graph decreases very little with a metre distance – beta radiation only has a range of a few metres to the count rate should reduce a lot at a distance of 1.5 m, this graph is likely to represent gamma radiation.

**D** is incorrect as the count rate in this graph decreases to the background rate at a distance of only a few centimetres – this graph is likely to represent alpha radiation.

It is important that you are aware of the different ranges of the three types of ionising radiation – alpha, beta and gamma:

- Alpha's range is short because it is highly ionising – the particles ionise molecules in the air before they can travel very far
- Beta's range is slightly longer as the particles are less ionising and are moving faster
- Gamma radiation is a form of electromagnetic radiation – this interacts very little with molecules in the air and has the longest range




12

The correct answer is **C** because:

- After every half-life passes, the number of radioactive nuclei remaining also halves
- If 10% of the sample is unstable at the start of the experiment, once the percentage reduces to 2.5%, this means two half-lives have passed

10% → Half-life → 5% → Half-life → 2.5%

- Similarly, the initial count rate is 180 cpm
- After two half-lives, the count rate will be 45 cpm
  - 180 cpm → 90 cpm represents 1 half life
  - 90 cpm → 45 cpm represents 1 half life

In questions like these, you have to be aware that after every half-life, the amount of radioactive material remaining **halves**. Students can fall into the trap of thinking that a set percentage of radioactive material is **subtracted** after every half-life, but this is incorrect.

You needed to spot that the half-life of the unstable nuclide was 2 hours and then determine the count rate after 4 hours i.e. two half-lives.

It is also worth noting that the number of unstable nuclei, the activity, count rate and mass of a radioactive substance **all** halve after each half-life. This is because they are all directly proportional.



13

The correct answer is **D** because:

- The source is an alpha emitter because:
  - There is a large reduction in count rate when paper is used as a shield
  - Alpha radiation is the only form of ionising radiation that cannot penetrate paper
- The source is **not** a beta emitter because:
  - Beta radiation can penetrate paper but it cannot penetrate aluminium

- While there is a slight reduction in count rate between when a paper shield is replaced with an aluminium shield, 4 cpm (counts per minute) is insignificant and is most likely due to natural fluctuations in background radiation
- The source is a gamma emitter because:
    - Even with aluminium in front of the source, the count rate is 524 cpm
    - Gamma radiation is the only form of ionising radiation that can penetrate aluminium
    - Additionally, the count rate is only reduced when thick lead blocks the source and the intensity of gamma radiation is reduced when passing through thick dense material
- Therefore, the source emits both $\alpha$ and $\gamma$ radiation

**A** is incorrect as there is little difference in count rate between a paper shield and an aluminium shield, implying that beta radiation is not being emitted.

**B** is incorrect as radiation is still making it through to the Geiger counter even with a paper tube – the source must be emitting more than just alpha radiation.

**C** is incorrect as the source is an alpha emitter as the count rate reduces with a paper shield and alpha radiation is not included in this answer.



It is worth noting that the background count rate naturally fluctuates a little, so we cannot attribute small changes in count rate to the radiation from the source.

14

The correct answer is **D** because:

- Isotopes of the same element always have the same atomic number – the number of protons
    - Atomic number = nucleon number – neutron number
- **A**, **B** and **C** all have an atomic number of 92
- Whereas option **D**:
    - Atomic number = 239 – 146 = 93

- **D** is the only nucleus with an atomic number of 93, so it is a different element from the other nuclei

**A** is incorrect as atomic number = 233 – 141 = 92.

**B** is incorrect as atomic number = 235 – 143 = 92.

**C** is incorrect as atomic number = 238 – 146 = 92.

For questions like this one, you must be aware that when nuclei have the same number of protons (atomic number), they are the same element by definition. The neutron number may be different, but these are just isotopes of the same element.

15

The correct answer is **C** because:

- Point C has the greatest binding energy per nucleon; this means that the nuclei in this region are the most stable

**A** is incorrect as point A has the lightest nuclei and these have a low binding energy per nucleon.

**B** is incorrect as point B is the point of the nuclei Helium-4, which, while particularly stable given the small size of the nucleus, is not the most stable nuclei on the binding energy curve.

**D** is incorrect as point D is the point on the binding energy graph where the heaviest nuclei occur; they are less stable than those that are at point C, having lower binding energy per nucleon values.

16

The correct answer is **A** because:

- Boron-11 has a proton number of 5 and a nucleon number of 11
    - Therefore, it is made up of 5 protons and 6 neutrons

- The mass of those components should be added together as a reference point for when binding energy does not occur to hold them together in a nucleus
- The actual mass of the Boron-11, $m_B$, should be subtracted to find only the mass equivalent $\Delta m$ of the binding energy
  - Therefore, $\Delta m = 5m_p + 6m_N - m_B$
- The binding energy $E = \Delta mc^2$, therefore:
  - $E = (5m_p + 6m_N - m_B)c^2$

**B** is incorrect as the mass of 5 protons, 6 neutrons and Boron-11 have been added. This would not be equal to the mass equivalent of the binding energy value.

**C** is incorrect as the mass of 5 protons and 11 neutrons have been added together as the reference mass. However, Boron-11 only has 6 neutrons.

**D** is incorrect as the mass of 6 protons and 5 neutrons should be added together as the reference mass. However, these values should be switched since Boron must have 5 protons and since it is Boron-11, this means it will have 11 – 5 = 6 neutrons.

17



The correct answer is **D** because:

- The binding energy of each nucleon is 7.98 MeV per nucleon for Oxygen-16
- This can be approximated as 8 MeV for convenience
- Since there are 16 nucleons in this atom, this means that there will be about 8 × 16 = 128 MeV which is close to 127.7 MeV, which is the correct answer

18

The correct answer is **B** because:

- The mass-energy equivalence value comes from the data booklet in the form of unified atomic mass unit:
  - Unified atomic mass unit ($u$) = $1.661 \times 10^{-27}$ kg = 931.5 MeV $c^{-2}$
  - We can approximate this to 1000 MeV $c^{-2}$
- To determine the number of atomic mass units which make up the mass defect of helium-4, approximate as follows:
  - The mass ratio is: $\frac{5.04 \times 10^{-29}}{1.661 \times 10^{-27}} \approx \frac{5 \times 10^{-29}}{2 \times 10^{-27}} = 2.5 \times 10^{-2}$
- Therefore, since the energy equivalence can be found to be $(2.5 \times 10^{-2}) \times 1000 = (2.5 \times 10^{-2}) \times 10^{3} = 25$ MeV
  - Therefore, approximations lead us to be nearest to option B, which is correct

19

The correct answer is **C** because:

- Since the two smaller mass nuclei with mass $m$ bond together in fusion, the product will need binding energy to hold the larger single nuclei together
- Therefore, the mass of the larger nuclei must be less than the original two masses



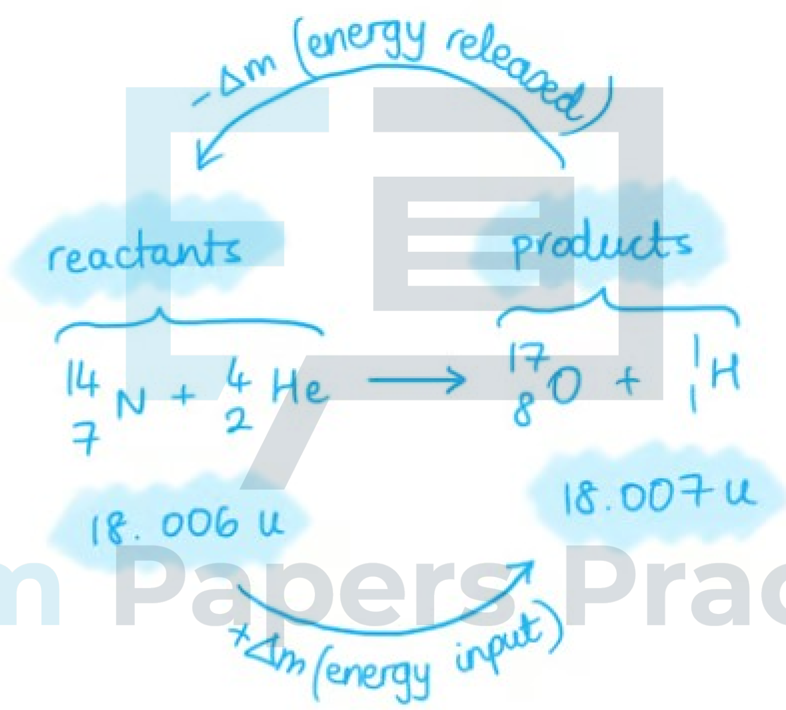
20

The correct answer is **C** because:

- The mass of the products ($^{17}_{8}O + ^{1}_{1}H$) is 0.001 u larger than the mass of the reactants ($^{14}_{7}N + ^{4}_{2}He$)
  - Therefore, the mass defect $\Delta m$ = 0.001 u
- Since 1 u is equivalent to 931.5 MeV
  - 001 u = 0.9315 MeV ≈ 1 MeV

- The law of conservation of mass-energy requires that the kinetic energy of the reactants therefore exceeds the kinetic energy of the products by about 1 MeV
    - Therefore, C is correct

Using the terms 'reactants' for the things on the left-hand side of an equation, and 'products' for the things on the right-hand side, enables you to describe the transfer of energy, using the mass defect. This is sketched out below:

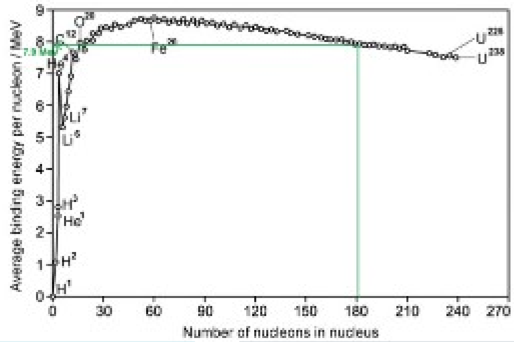



You should be able to see that there is 'extra' energy (in the form of a mass defect) on the left-hand side, with the reactants: therefore, the kinetic energy of the reactants exceeds that of the products.

21

The correct answer is **B** because:

- The mass number of the nucleus is 181
  - Therefore, there are 181 nucleons in the nucleus
- You can determine the average binding energy per nucleon of a nucleus with 181 neutrons using the graph, as shown below:




- The average binding energy per nucleon = $\frac{total\ binding\ energy}{number\ of\ nucleons}$
  - This means the total binding energy = average binding energy per nucleon × number of nucleons
- Therefore, if the average binding energy per nucleon is approximately 7.9 MeV for this nucleus, then the total binding energy = 7.9 × 181 = 1429.9 MeV
  - This means that 1429.9 MeV ≈ 1430 MeV of energy is released when the nucleus $^{181}_{70}X$ forms
- Hence B is correct



Remember to check the axis of a binding energy graph. Sometimes, the energy is given as the total binding energy (which is the energy released when nucleons fuse together, or equivalently, the energy required to separate them fully), and sometimes the energy given is the average binding energy **per nucleon**.

22

The correct answer is **B** because:

- In order to separate a nucleus into its constituent nucleons, energy is required
  - In other words, work must be done

- The total amount of energy required to separate a nucleus into its constituent nucleons is the binding energy of the nucleus

**A** is incorrect as the energy equivalent of the mass of the nucleus is not the same as the energy required to separate nucleons in the nucleus.

**C** is incorrect as the binding energy is the energy required to separate nucleons from each other in the nucleus. Therefore, it is the energy required to overcome the strong nuclear force, which is the force that binds the nucleons together in a nucleus.

**D** is incorrect as the average energy required to remove (separate) a single nucleon from the nucleus is called the binding energy per nucleon.