

Wednesday 3 June 2026 – Afternoon

A Level Mathematics A

H240/01 Pure Mathematics

Time allowed: 2 hours

You must have:

- the Printed Answer Booklet
- a scientific or graphical calculator

QP

INSTRUCTIONS

- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
 - Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined page at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
 - Fill in the boxes on the front of the Printed Answer Booklet.
 - Answer **all** the questions.
 - Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
 - Give non-exact numerical answers correct to **3** significant figures unless a different degree of accuracy is specified in the question.
 - The acceleration due to gravity is denoted by $g \text{ m s}^{-2}$. When a numerical value is needed use $g = 9.8$ unless a different value is specified in the question.
- You can write on this Question Paper but it will **not** be sent for marking.

INFORMATION

- The total mark for this paper is **100**.
- The marks for each question are shown in brackets [].
- This document has **8** pages.

ADVICE

- Read each question carefully before you start your answer.

Formulae

A Level Mathematics A (H240)

Arithmetic series

$$S_n = \frac{1}{2}n(a + l) = \frac{1}{2}n(2a + (n-1)d)$$

Geometric series

$$S_n = \frac{a(1-r^{n+1})}{1-r}$$

$$S_\infty = \frac{a}{1-r} \quad \text{for } |r| < 1$$

Binomial series

$$(a+b)^n = a^n + {}^nC_1 a^{n-1}b + {}^nC_2 a^{n-2}b^2 + \dots + {}^nC_r a^{n-r}b^r + \dots + b^n \quad |b/a| < 1$$

where ${}^nC_r = {}^nC_r = \frac{n!}{r!(n-r)!}$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \dots + \frac{n(n-1)\dots(n-r+1)}{r!}x^r + \dots \quad |x| < 1, n \in \mathbb{R}$$

Differentiation

$f(x)$	$f'(x)$
$\tan x$	$\sec^2 x$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\operatorname{cosec} x$	$-\cot x \operatorname{cosec} x$

Quotient rule $y = \frac{u}{v}, \frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$

Differentiation from first principles

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

Integration

$$\int \frac{1}{f(x)} dx = \ln|f(x)| + c$$

$$\int f(x) x^a dx = \frac{1}{a+1} f(x) x^{a+1} + c \quad (a \neq -1)$$

Integration by parts $\int u \frac{dv}{dx} dx = uv - \int v \frac{du}{dx} dx$

Small angle approximations

$$\sin i \approx i, \cos i \approx 1 - \frac{1}{2}i^2, \tan i \approx i \quad \text{where } i \text{ is measured in radians}$$

Trigonometric identities

$$\sin(A+B) = \sin A \cos B + \cos A \sin B$$

$$\cos(A+B) = \cos A \cos B - \sin A \sin B$$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

Numerical methods

Trapezium rule: $\int_a^b y dx \approx \frac{1}{2}h [y_0 + y_n + 2(y_1 + y_2 + \dots + y_{n-1})]$, where $h = \frac{b-a}{n}$

The Newton-Raphson iteration for solving $f(x)=0$: $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

Probability

$$P(A+B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cap B) = P(A)P(B) \text{ or } P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Standard deviation

$$\sqrt{\frac{\sum (x - \bar{x})^2}{n}} = \sqrt{\frac{\sum x^2}{n} - \bar{x}^2} \text{ or } \sqrt{\frac{\sum f x^2}{\sum f} - \bar{x}^2}$$

The binomial distribution

If $X \sim B(n, p)$ then $P(X=x) = {}^n C_x p^x (1-p)^{n-x}$, mean of X is np , variance of X is $np(1-p)$

Hypothesis test for the mean of a normal distribution

If $X \sim N(\mu, \sigma^2)$ then $\frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \sim N(0, 1)$

Percentage points of the normal distribution

If Z has a normal distribution with mean 0 and variance 1 then, for each value of p , the table gives the value of z such that $P(Z \leq z) = p$.

p	0.75	0.90	0.95	0.975	0.99	0.995	0.9975	0.999	0.9995
z	0.674	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291

Kinematics

Motion in a straight line

$$v = u + at$$

$$s = ut + \frac{1}{2}at^2$$

$$v^2 = u^2 + 2as$$

$$s = ut + \frac{1}{2}at^2$$

$$s = vt - \frac{1}{2}at^2$$

Motion in two dimensions

$$\mathbf{v} = \mathbf{u} + \mathbf{a}t$$

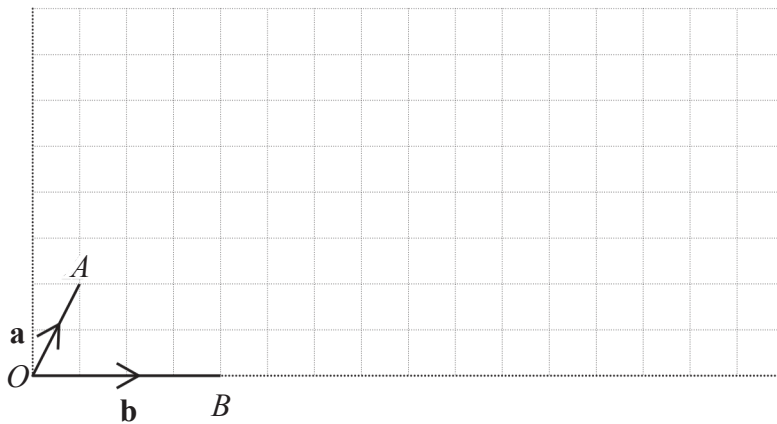
$$\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$$

$$\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$$

$$\mathbf{s} = \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2$$

- 1 (a) (i) Express $x^2-6x+11$ in the form $(x-a)^2+b$ where a and b are constants. [2]
- (ii) Hence state the equation of the line of symmetry of the curve $y=x^2-6x+11$. [1]
- (b) (i) Find the discriminant of $px^2+(2p-1)x+p$, where p is a constant. Give your answer in a simplified form in terms of p . [2]
- (ii) Given that $px^2+(2p-1)x+p=0$ has equal roots, find the value of p . [2]
- 2 (a) A digit sum is the sum of all digits in a number, for example 31 has a digit sum of 4.
A student claims that all two digit prime numbers have a digit sum that is even.
- (i) Prove by exhaustion that this is true for all prime numbers between 10 and 20. [3]
- (ii) Find a counterexample to disprove that the statement is true for all two digit prime numbers. [1]
- (b) Prove that $(2n+5)^2-(2n+3)^2$ is always a multiple of 8 for any integer n . [3]

3



The diagram shows the points A and B , which have position vectors $\mathbf{a}$ and $\mathbf{b}$ with respect to an origin O .

The points C and D have position vectors, with respect to the origin O , such that $\mathbf{c}=\mathbf{a}+3\mathbf{b}$ and $\mathbf{d}=3\mathbf{a}+\mathbf{b}$.

- (a) Mark and label the points C and D on the diagram in the **Printed Answer Booklet**. [2]
- (b) (i) Find the following vectors. Give your vectors in a simplified form in terms of $\mathbf{a}$ and $\mathbf{b}$. [3]
- $\overrightarrow{AB}$
 - $\overrightarrow{DC}$
- (ii) Hence explain why $ABCD$ is a trapezium. [1]

4 In this question you must show detailed reasoning.

Solve the following equations.

(a) $5e^{2x+1}=7$ [2]

(b) $\log_{10}(2x^3+27)=1+3\log_{10}x$ [4]

5 The curve $y=ax^2+bx^2$ has a gradient of 22 at the point (4, 32).

Determine the values of the constants a and b . [6]

6 A curve has equation $f(x) = x^3 - 16x$.

(a) (i) Find $\int_0^k y^k f(x) dx$, where k is a positive constant, giving your answer in terms of k . [3]

(ii) Show that $\int_{-k}^k y^k f(x) dx = 0$. [1]

(b) Use the result from part (a)(ii) to explain what can be deduced about the curve $y = f(x)$. [1]

(c) Find the total area enclosed by the curve $y=f(x)$ and the x -axis. [3]

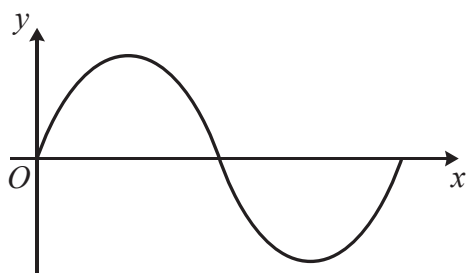
7 (a) Show that the constant term in the binomial expansion of $a3x^2 + \frac{2k^6}{x}$ is 2160. [3]

(b) Find the constant term in the binomial expansion of $\sqrt[5]{\frac{ax+xk}{5}}$. [3]

(c) Explain why the constant term in the expansion of $a3x^2 + \frac{2k^6}{x}$ is not the product of your answers to part (a) and part (b). [1]

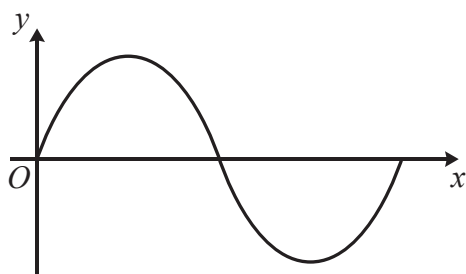
- 8 (a) **Figure 1** shows part of the curve $y = a \sin bx$, where x is in radians, and a and b are positive constants. The maximum point on this part of the curve is at $(r, 3)$.

Figure 1



- (i) State the coordinates of the minimum point on this part of the curve. [2]
- (ii) Find the values of a and b . [2]
- (b) **Figure 2** shows part of a different curve, $y = p \sin qx$, where x is in radians, and p and q are positive constants.

Figure 2



- (i) On the copy of **Figure 2** in the **Printed Answer Booklet** sketch the curve $y = p \cos qx$, for the same domain of x . [2]
- (ii) Determine the x -coordinates of the points of intersection of the part of the curve $y = p \sin qx$ shown in **Figure 2** and $y = p \cos qx$. Give your answers in an exact form in terms of p and/or q . [4]

- 9 Scientists are monitoring the population of a colony of seals. A model is suggested for the number of seals, P , at time t years after the start of the study.

In the model $P = 180 + 5t - 145e^{0.015t}$.

- (a) Find the initial number of seals. [1]
- (b) Find the increase in the number of seals over the first 10 years. [2]
- (c) Determine, using calculus, the maximum number of seals. [5]
- (d) Explain what this model predicts will happen in the long term. [1]

- 10 A curve has equation $y = \frac{3\ln x + 1}{4x - 1}$.

- (a) Find an expression for $\frac{dy}{dx}$. [3]

The curve $y = \frac{3\ln x + 1}{4x - 1}$ has a stationary point at P .

- (b) Show that the x -coordinate of P satisfies the equation $x = \frac{8x - 3}{12\ln x}$. [3]
- (c) Show by calculation that the equation in part (b) has a root between 1.6 and 1.7. [3]

- (d) (i) Use the iterative formula

$$x_{n+1} = \frac{8x_n - 3}{12\ln x_n}$$

with a starting value of $x_1 = 1.65$ to find the value of the x -coordinate of P correct to 3 significant figures. You must show the result of each iteration. [3]

- (ii) Hence give the coordinates of P correct to 3 significant figures. [1]
- (e) A student draws a diagram to show how the iterative process used in part (d)(i) converges to the root found.

State the type of convergence shown in the student's diagram. Give a reason for your answer. [2]

Turn over for questions 11 and 12

11 In this question you must show detailed reasoning.

(a) Show that $\tan i + \cot i = 2 \operatorname{cosec} 2i$. [4]

(b) Hence solve the equation $\tan i + \cot i = 4$, for $0^\circ < i < 360^\circ$. [4]

(c) Given that $\sin 2\left(\tan^{-1} \frac{1}{2} - \cot^{-1} 3\right) = \frac{1}{3}$, determine the exact value of $\cos i$. [3]

12 Use integration by parts twice to show that

$$\int e^{2x} \sin 3x \, dx = \frac{1}{13} e^{2x} (A \sin 3x + B \cos 3x) + c,$$

where A and B are integers to be determined. [8]

END OF QUESTION PAPER



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