OCR
Oxford Cambridge and RSA

Friday 20 June 2025 – Afternoon

A Level Further Mathematics A

Y545/01 Additional Pure Mathematics

Time allowed: 1 hour 30 minutes

You must have:
- the Printed Answer Booklet
- the Formulae Booklet for A Level Further Mathematics A
- a scientific or graphical calculator

QP

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INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give non-exact numerical answers correct to **3** significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. When a numerical value is needed use $g = 9.8$ unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep in the centre or recycle it.

INFORMATION
- The total mark for this paper is **75**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **4** pages.

ADVICE
- Read each question carefully before you start your answer.

1 **(a)** A student claims that the order of 5 modulo 37 is 5.

Without calculation, explain why this student's claim must be incorrect. **[1]**

(b) Show that the order of 5 modulo 37 is 36. **[3]**

2 The points P, Q, R and S have coordinates (1, 2, 4), (3, 1, 6), (4, 8, −2) and (5, 5, 5) respectively.

(a) Find, in exact form, the area of triangle PQR. **[3]**

(b) Find, in exact form, the volume of tetrahedron $PQRS$. **[3]**

(c) Use your answers to part **(a)** and part **(b)** to deduce the shortest distance from S to the plane containing P, Q and R. Give your answer in exact form.

[You may use the formula $V = \frac{1}{3}Ah$, where V is the volume of a pyramid, A is the area of one of its bases and h is the height relative to the base.] **[2]**

3 Let $N = 10a + b$ and $M = a - 3b$, where a and b are positive integers and $0 \leqslant b \leqslant 9$.

(a) By considering $3N + M$, prove that $31 \mid N$ if and only if $31 \mid M$. **[4]**

(b) Use a procedure based on the result of part **(a)** to show that $31 \mid 7\,167\,758$. **[2]**

4 **In this question you must show detailed reasoning.**

The surface S is given by the equation $z = 4x^2y - 3xy^2 - 5x$ for all real values of x and y.

(a) The point $P(1, 1, -4)$ lies on S.

Find the equation of the tangent plane to S at P. **[4]**

(b) The point Q is a stationary point of S with positive x- and y-coordinates.

(i) Find, in exact form, the coordinates of Q. **[5]**

(ii) Find the value of the determinant of **H**, the Hessian matrix of S, at Q. **[4]**

(iii) Deduce the nature of the stationary point Q. **[1]**

5 **(a)** Determine the solution of the linear congruence $3x \equiv 7 \pmod{13}$. Give your answer in the form $x \equiv a \pmod{13}$, where $0 \leqslant a < 13$. **[2]**

(b) Hence or otherwise, determine the solution of the simultaneous linear congruences $3x \equiv 7 \pmod{13}$, $7x \equiv 3 \pmod{31}$. **[4]**

6 In this question, the set S is $\{2, 4, 6, 8\}$, and $\times_{10}$ is the operation of multiplication modulo 10.

(a) The group G consists of the set S, under the operation $\times_{10}$.

(i) Complete the Cayley table for G given in the Printed Answer Booklet. **[1]**

(ii) State the identity element of G. **[1]**

(iii) State the order of each non-identity element of G. **[2]**

(b) The group H consists of all 2×2 matrices of the form $\begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}$, where $a, b \in S$, under the operation of matrix multiplication (using $\times_{10}$ to multiply elements of S).

(i) State, with justification, the order of H. **[2]**

(ii) State also the identity element of H. **[1]**

(iii) State all the elements of H of order 2. **[2]**

(iv) Using the result of part **(a)(iii)**, or otherwise, explain why all the (non-identity) elements which do not have order 2 must have order 4. **[3]**

(v) State whether or not H is cyclic. You must give a reason for your answer. **[1]**

7 **In this question you must show detailed reasoning.**

A curve is defined parametrically by $x = \cos^3 t$, $y = \sin^3 t$, for $0 \leqslant t \leqslant \frac{1}{2}\pi$. When the curve is rotated through 2π about the x-axis, a surface of revolution is formed with surface area A.

Determine the exact value of A. **[8]**

8 **(a)** The members of the family of the sequences $\{G_n\}$ satisfy the **first-order** recurrence relation $G_{n+1} = 1.25 - qG_n$, for $n \geqslant 0$,
where q is a positive constant.

(i) Determine, in terms of q, the general solution of the first-order recurrence relation. **[3]**

(ii) Hence explain why, when q is close to zero, G_n is approximately constant and equal to 1.25. **[2]**

(iii) In the case where $G_1 = 1.22$ and $G_2 = 1.2378$, find the values of q and G_0. **[3]**

(b) The population of a species of fish F_n is being modelled by the **second-order** recurrence relation

$$F_{n+2} = 1.25F_{n+1} - rF_{n+1}F_n,$$

where r is a positive constant and n is the number of weeks since the start of the breeding season.

You are given that the value of r is such that the model predicts that the population approaches a limiting value, L.

(i) Determine L in terms of r. **[2]**

(ii) Determine what happens if a term of the sequence is greater than $\frac{5}{4r}$. **[2]**

(iii) Explain why the modified recurrence relation $F_{n+2} = \text{INT}(1.25F_{n+1} - rF_{n+1}F_n)$ may be preferable when modelling this fish population. **[1]**

(iv) When $r = 0.0001$, both $F_{n+2} = 1.25F_{n+1} - rF_{n+1}F_n$ and $F_{n+2} = \text{INT}(1.25F_{n+1} - rF_{n+1}F_n)$ predict that the fish population approaches a limiting value.

By finding the limiting value for each model, when $F_0 = 2450$ and $F_1 = 2475$, compare the two models for the fish population. **[3]**

END OF QUESTION PAPER