OCR
Oxford Cambridge and RSA

Wednesday 18 June 2025 – Afternoon

A Level Further Mathematics A

Y544/01 Discrete Mathematics

Time allowed: 1 hour 30 minutes

You must have:
- the Printed Answer Booklet
- the Formulae Booklet for A Level Further Mathematics A
- a scientific or graphical calculator

QP

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give non-exact numerical answers correct to **3** significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. When a numerical value is needed use $g = 9.8$ unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION
- The total mark for this paper is **75**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **8** pages.

ADVICE
- Read each question carefully before you start your answer.

1 A coach operator offers sightseeing tours of a city. The sightseeing tour starts and ends at stop A, and there are tourist attractions at B, C, D, E, F and G. The table below shows the distances in metres that the coach must travel between each of the seven stops.

	A	B	C	D	E	F	G
A	–	120	100	230	260	150	200
B	120	–	180	210	200	250	280
C	100	180	–	235	190	170	270
D	230	210	235	–	170	185	210
E	260	200	190	170	–	210	270
F	150	250	170	185	210	–	195
G	200	280	270	210	270	195	–

The coach operator wants to find a route which starts and ends at A, visiting every tourist attraction once.

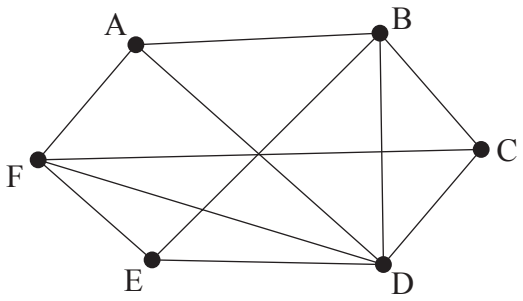
(a) State the standard network problem that they must solve. **[1]**

(b) Use the nearest neighbour method to find an upper bound for the length of the coach operator's route starting at A. **[2]**

(c) Beginning by removing stop A to reduce the network, use an algorithm to determine a lower bound for the length of the coach operator's route. **[4]**

(d) By referring to your lower bound, comment on the length of the route found by the nearest neighbour method. **[1]**

2 **(a)** Graph G is shown below



(i) Show that graph G is **not** bipartite. **[2]**

(ii) Use Kuratowski's Theorem to show that graph G is **not** planar. **[3]**

(iii) Determine whether graph G is simply connected. **[1]**

(b) The adjacency matrix for a digraph H is shown below

From \ To	J	K	L	M	N	O
J	0	1	1	0	1	0
K	2	0	1	1	0	0
L	1	1	0	1	1	1
M	0	0	1	1	0	1
N	1	0	1	0	0	1
O	0	1	0	1	1	0

(i) Write down the indegree and outdegree of node L. **[2]**

(ii) Determine whether digraph H is simply connected. **[1]**

3 Hiro uses a sorting algorithm to sort the heights in cm of 8 people into **ascending** order.

At the end of the first pass through the algorithm, Hiro ends up with the list below:

142 165 152 135 171 154 172 182

(a) If Hiro used the **shuttle sort** algorithm, write down the possibilities for the original list. **[1]**

Hiro actually used the **bubble sort** algorithm starting at the left-hand side.

(b) Complete the second and third passes using the bubble sort algorithm.
You should show the list that results at the end of each pass. **[2]**

In total, Hiro needed 5 passes to complete the bubble sort algorithm.

In total, the **shuttle sort** algorithm would take 12 comparisons to sort the original list into ascending order.

(c) Using the total number of comparisons for each algorithm, compare the efficiency of bubble sort and shuttle sort for sorting the heights of the 8 people. **[3]**

(d) It takes a computer $\mathbf{2.4 \times 10^{-3}}$ **seconds** to sort a list of 300 numbers into ascending order using the bubble sort algorithm.

Find the approximate length of a list the computer could sort in 1 minute. Give your answer to **1** significant figure. **[2]**

4 The table shows the activities involved in a project, together with the immediate predecessors and the duration of each activity in hours.

Activity	Immediate Predecessors	Duration (hours)
A	–	5
B	–	4
C	A	3
D	B	3
E	D	2
F	B	5
G	C, E	2
H	D	5
I	G	3
J	C, E, H	1
K	D, F	2

(a) (i) By constructing an activity network for the project, determine the minimum time in which the project can be completed. **[5]**

(ii) State the critical activities. **[1]**

(b) Calculate the interfering float for each non-critical activity. **[3]**

Each activity requires one worker. When an activity is started it must be completed without interruption.

(c) Use the diagram in the Printed Answer Booklet to determine the minimum number of workers needed to complete the project in the minimum completion time.

Each column in the diagram represents 1 hour. For each worker, write the letter of the activity they are doing in each box, or leave the box blank if the worker is resting for that hour. You may not need to use all the rows or columns in the table. **[3]**

(d) Activity C takes an additional t hours, but all other timings are unchanged.

Determine the range of values of t that result in an increase in the minimum project completion time. **[2]**

5 The table below shows the pay-off for Player A in a two-person zero-sum game between Player A and Player B. Each player has three options; P, Q and R represent the options for Player A and X, Y and Z represent the options for player B.

	X	Y	Z
P	1	2	−1
Q	3	0	2
R	−1	4	−2

(a) Write down the maximum possible pay-off to Player B if Player A plays option P. **[1]**

(b) Determine whether or not the game is stable. **[4]**

(c) Show that there are no Nash Equilibrium points for the game. **[2]**

(d) The entry in one cell in the zero-sum game above can be **increased** so that there is a Nash Equilibrium point.

- Identify the cell that should be chosen.
- Write down the smallest value that it could take. **[2]**

(e) Show that the original game can be reduced to a game with a 3×2 pay-off matrix using dominance. **[1]**

The entry in cell (Q,Y) in the reduced game is changed to a number **less than −4**. This new number is denoted by m.
You are given that the value of the reduced game to Player B with this change is 0.4.

(f) (i) Determine the optimal mixed strategy for Player B. **[6]**

(ii) Find the corresponding value of m. **[1]**

6 A furniture company makes three types of wardrobes.

Each **Type A** wardrobe requires 4 hours of machine time, 6 hours of craftsman time and 2 hours of finishing time.

Each **Type B** wardrobe requires 3 hours of machine time, 5 hours of craftsman time and 11 hours of finishing time.

Each **Type C** wardrobe requires 5 hours of machine time, 4 hours of craftsman time and 4 hours of finishing time.

Each month, there is a maximum of 200 hours of machine time, 160 hours of craftsman time and 144 hours of finishing time.

The company makes **£16** profit on each **Type A** wardrobe, **£20** profit on each **Type B** wardrobe and **£14** profit on each **Type C** wardrobe. The company wants to make as much profit P as possible.

Let x, y and z denote the number of Type A, Type B and Type C wardrobes made each month.

(a) Write down the objective function for this problem. **[1]**

(b) Complete the table in the Printed Answer Booklet to represent the problem as an initial simplex tableau. You should use the first blank row for your objective function. **[5]**

(c) (i) Show the calculations used to find the pivot element for the first iteration of the simplex algorithm. **[1]**

(ii) Carry out **one** iteration of the simplex algorithm. You should give each entry in exact form. **[3]**

Multiple iterations of the simplex algorithm are carried out until the objective function has been maximised. The final tableau is shown below.

	P	x	y	z	r	s	t	RHS
Row 1	1	0	5	0	0	2.25	1.25	540
Row 2	0	0	−8.5	0	1	−0.375	−0.875	k
Row 3	0	1	−1.5	0	0	0.25	−0.25	4
Row 4	0	0	3.5	1	0	−0.125	0.375	34

(d) Explain how the values in Row 1 indicate that the objective function has been maximised. **[1]**

(e) Determine the value of k. **[2]**

7 Charlie has a set of 100 cards.
On each card one of the integers from 1 to 100 is printed without any repeats.

(a) Use the inclusion-exclusion principle to determine how many of Charlie's cards show a number that is a multiple of 2, 3 or 5. **[3]**

Charlie then chooses one or more cards at random and arranges them next to each other to form a number with n digits where $n \geqslant 1$.

(b) Determine the total number of different arrangements of cards that will produce a 3-digit number. **[3]**

END OF QUESTION PAPER