

## Tuesday 3 June 2025 – Afternoon

### A Level Further Mathematics A

#### Y541/01 Pure Core 2

Time allowed: 1 hour 30 minutes

**You must have:**

- the Printed Answer Booklet
- the Formulae Booklet for A Level Further Mathematics A
- a scientific or graphical calculator

QP



### INSTRUCTIONS

- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give non-exact numerical answers correct to **3** significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by  $g \text{ m s}^{-2}$ . When a numerical value is needed use  $g = 9.8$  unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

### INFORMATION

- The total mark for this paper is **75**.
- The marks for each question are shown in brackets [ ].
- This document has **8** pages.

### ADVICE

- Read each question carefully before you start your answer.

**1 In this question you must show detailed reasoning.**

Vectors  $\mathbf{a}$  and  $\mathbf{b}$  are given by  $\mathbf{a} = \begin{pmatrix} 2 \\ -4 \\ 3 \end{pmatrix}$  and  $\mathbf{b} = \begin{pmatrix} -1 \\ 3 \\ -2 \end{pmatrix}$ .

Determine, in either order

- $\mathbf{a} \cdot \mathbf{b}$
- $\mathbf{a} \times \mathbf{b}$ .

[3]

**2** You are given that  $-7 - 5i$  is one root of the equation  $x^3 + 10x^2 + 18x - 296 = 0$ .

(a) Write down another complex root of the equation  $x^3 + 10x^2 + 18x - 296 = 0$ . [1]

(b) Using your answer to part (a), express  $x^3 + 10x^2 + 18x - 296$  as a product of a real linear factor and a real quadratic factor. [3]

**3** Matrices  $\mathbf{A}$  and  $\mathbf{B}$  are given by  $\mathbf{A} = \begin{pmatrix} 1 & -3 \\ 4 & 8 \end{pmatrix}$  and  $\mathbf{B} = \begin{pmatrix} -3 & -3 \\ 1 & 2 \end{pmatrix}$ .

(a) Find the matrix  $\mathbf{AB}$ . [1]

(b) Verify that  $\det(\mathbf{AB}) = \det(\mathbf{A}) \times \det(\mathbf{B})$ . [2]

(c) Use matrices  $\mathbf{A}$  and  $\mathbf{B}$  to demonstrate that matrix multiplication is **not** commutative. [2]

The transformation represented by matrix  $\mathbf{A}$  is denoted by T.

(d) Show that the point  $(2, -5)$  is **not** an invariant point under T. [2]

(e) Find the matrix which represents the inverse transformation of T. [2]

**4 In this question you must show detailed reasoning.**

Determine the sum of all cube numbers from 216 to 512 000 inclusive. [4]

**5 In this question you must show detailed reasoning.**

(a) Use an algebraic method to determine the two square roots of  $-3 + (4\sqrt{7})i$ .

Give your answers in the form  $a + bi$  where  $a$  and  $b$  are exact. [5]

(b) State the relationship between the two arguments of the two square roots found in part (a). [1]

**6 One of the regions bounded by two polar curves,  $C_1$  and  $C_2$ , is used to model the face of a flat earring.**

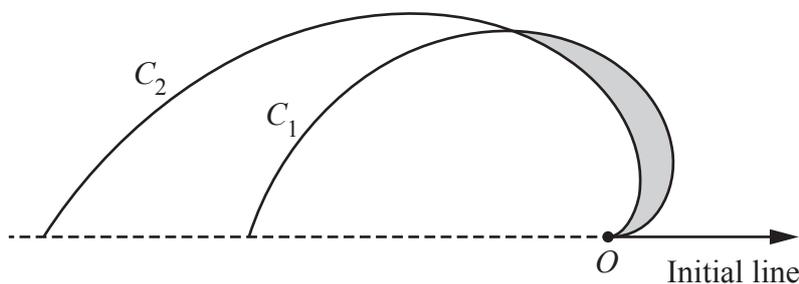
The polar equations of  $C_1$  and  $C_2$  are

$$C_1: r = 2\theta$$

$$C_2: r = \theta^2$$

where  $0 \leq \theta \leq \pi$ .

The curves  $C_1$  and  $C_2$  are shown in the diagram below with the region used to model the earring shaded.



You are given that  $C_1$  and  $C_2$  intersect at the pole  $O$ .

(a) Find the other point of intersection of  $C_1$  and  $C_2$ . Give your answer in polar coordinates. [2]

(b) **In this question you must show detailed reasoning.**

Determine the area of the face of the earring. [4]

7 In this question you must show detailed reasoning.

(a) Express  $\frac{-4x^2 + 5x - 17}{x^3 - x^2 + 3x - 3}$  in partial fractions. [6]

(b) Hence determine the exact value of  $\int_{\sqrt{3}}^3 \frac{-4x^2 + 5x - 17}{x^3 - x^2 + 3x - 3} dx$ . [4]

8 A function  $f(x)$  is defined by  $f(x) = x \sinh 2x$ .

(a) Prove by induction that  $\frac{d^{2n} f}{dx^{2n}} = 4^n (x \sinh 2x + n \cosh 2x)$  for  $n \geq 0$  where  $\frac{d^0 f}{dx^0}$  is defined as being equal to  $f(x)$ . [6]

(b) Using the formula given in part (a), determine the exact value of the coefficient of  $x^8$  in the Maclaurin series for  $x \sinh 2x$ . [3]

(c) Use the Maclaurin series for  $e^x$  to verify your answer to part (b). [3]

- 9 When light hits a certain photo-sensitive cell, at time  $t = 0$ , there is an electrical response in the cell which is denoted by  $y(t)$  where both  $y$  and  $t$  are measured in suitable units. A student wishes to model this response,  $y$ .

In an attempt to model  $y$ , the student sets up the differential equation

$$\frac{d^2y}{dt^2} + 6\frac{dy}{dt} + 9y = 10e^{-3t} \quad (*)$$

which is subject to the following conditions.

- $y = 0$  when  $t = 5$
  - $y \geq 0$  for all  $t \geq 0$
- (a) By substituting into a suitable differential equation, **verify** that  $y = (A + Bt)e^{-3t}$  is a complementary function of the differential equation (\*). [2]
- (b) Determine the particular solution for  $y$  in terms of  $t$ . [8]
- (c) Using your answer to part (b), find the value of  $y$  immediately after the light hits the cell. [1]

The cell can be considered to be operating properly if  $y < 4$  when  $t = 1$ .

- (d) Discuss whether the cell can be inferred to be operating properly. [2]

10 (a) Find  $\frac{1}{2}e^{-u}$  as a percentage of  $\sinh u$  for the following values of  $u$ .

- $u = 2$
- $u = 5$

[1]

(b) Find, as a percentage of  $\sinh u$ , the difference between  $\sinh u$  and  $\cosh u$  for the following values of  $u$ .

- $u = 2$
- $u = 5$

[1]

A function  $f$  is defined for all integers  $n$  by  $f(n) = \sinh(0.01n) - 5\cosh(0.005n) - 9\tanh n$ .

(c) **In this question you must show detailed reasoning.**

With the help of suitable approximations, use an algebraic method to determine the smallest value of  $n$  for which  $f(n) > 100$ . You should **verify** your answer, once found, by direct calculation. You may assume that the required value of  $n$  is large.

[6]

**END OF QUESTION PAPER**

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