

## Monday 12 May 2025 – Afternoon

### AS Level Further Mathematics A

#### Y531/01 Pure Core

Time allowed: 1 hour 15 minutes



**You must have:**

- the Printed Answer Booklet
- the Formulae Booklet for AS Level Further Mathematics A
- a scientific or graphical calculator

**QP**

#### INSTRUCTIONS

- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined page at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give non-exact numerical answers correct to **3** significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by  $g \text{ m s}^{-2}$ . When a numerical value is needed use  $g = 9.8$  unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

#### INFORMATION

- The total mark for this paper is **60**.
- The marks for each question are shown in brackets [ ].
- This document has **4** pages.

#### ADVICE

- Read each question carefully before you start your answer.

- 1 (a) The complex number  $z$  is such that  $|z| = 7$  and  $\arg(z) = 2.2$  radians.  
Express  $z$  in cartesian form. [3]
- (b) Use an algebraic method to determine the exact square roots of  $1 + (4\sqrt{3})i$ . [5]

- 2 Two vectors,  $\mathbf{a}$  and  $\mathbf{b}$ , are given by  $\mathbf{a} = \begin{pmatrix} 2 \\ -3 \\ 13 \end{pmatrix}$  and  $\mathbf{b} = \begin{pmatrix} -4 \\ 6 \\ p \end{pmatrix}$  where  $p$  is a constant.

(a) Find expressions in terms of  $p$  for each of the following.

- $\mathbf{a} \cdot \mathbf{b}$
- $\mathbf{a} \times \mathbf{b}$

[3]

(b) Hence or otherwise find the value of  $p$  in each of the following cases.

- $\mathbf{a}$  and  $\mathbf{b}$  are perpendicular
- $\mathbf{a}$  and  $\mathbf{b}$  are parallel

[2]

- 3 The roots of the equation  $2x^2 + 3x + 5 = 0$  are denoted by  $\alpha$  and  $\beta$ .

(a) Write down the value of  $\alpha + \beta$  and the value of  $\alpha\beta$ .

[2]

(b) Using the answers to part (a) determine the value of each of the following.

- $\alpha^2 + \beta^2$
- $\frac{1}{\alpha} + \frac{1}{\beta}$

[4]

- 4 Two transformations,  $T_A$  and  $T_B$ , are represented by matrices  $\mathbf{A}$  and  $\mathbf{B}$  respectively.

The matrix  $\mathbf{A}$  is given by  $\mathbf{A} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ .

- (a) (i) Describe the transformation  $T_A$ . [1]  
 (ii) Explain geometrically why  $\mathbf{A}^{-1} = \mathbf{A}$ . [1]

The matrix  $\mathbf{B}$  is given by  $\mathbf{B} = \frac{1}{2} \begin{pmatrix} 1 & -\sqrt{3} \\ \sqrt{3} & 1 \end{pmatrix}$ .

- (b) Describe the transformation  $T_B$ . [2]

The transformation  $T_C$  is equivalent to  $T_A$  followed by  $T_B$ .

- (c) Determine the single matrix which represents  $T_C$ . [2]

- 5 The locus  $L$  is defined by  $L = \{z : z \in \mathbb{C}, |z - (20 + 15i)| \leq 7\}$ .

- (a) On the Argand diagram in the Printed Answer Booklet, sketch and label  $L$ . [2]  
 (b) Determine the value of  $z \in L$  for which the value of  $|z|$  is smallest. Give your answer in cartesian form. [3]  
 (c) Determine the largest value of  $\arg(z)$  for  $z \in L$ . [3]

- 6 The equations of two lines,  $l_1$  and  $l_2$ , are  $l_1 : \mathbf{r} = \begin{pmatrix} 16 \\ -1 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -27 \\ -19 \end{pmatrix}$  and  $l_2 : \mathbf{r} = \begin{pmatrix} 3 \\ 10 \\ -10 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ 10 \\ 10 \end{pmatrix}$ .

- (a) Show that  $l_1$  and  $l_2$  intersect at a single point,  $P$ , giving the coordinates of  $P$ . [5]

$O$  is the origin of the coordinate system. The point  $Q$  lies on the line segment  $OP$ .

- (b) Comment on the claim that the distance  $OQ$  is less than 100. [2]

7 Prove by induction that  $n! > 20^n$  for all integers  $n \geq 52$ . [5]

8 Matrix  $\mathbf{A}$  is given by  $\mathbf{A} = \begin{pmatrix} 5+a & 5 & 1 \\ 3 & 13+a & 6 \\ a-4 & -20 & -9 \end{pmatrix}$  where  $a$  is a constant and all entries of  $\mathbf{A}$  are integers.

The transformation represented by  $\mathbf{A}$  is applied to a shape of volume 8 units.  
The image shape has volume 40 units and the orientation of the image is reversed.

Determine the image under  $\mathbf{A}$  of the point  $(1, 2, 3)$ . [7]

9 (a) Prove that if  $z$  is any complex number then  $z(z^*) = |z|^2$ . [1]

(b) Prove that if  $w$  and  $z$  are any complex numbers then  $(wz)^* = w^*z^*$ . [2]

The complex number  $v$  is defined by  $v = (10 + 3i)(9 + 4i)$ .

(c) **In this question you must show detailed reasoning.**

Express  $v$  in the form  $a + bi$  where  $a$  and  $b$  are real. [2]

(d) **In this question you must show detailed reasoning.**

By considering  $v(v^*)$ , write 10573 as the product of two prime factors. [3]

## END OF QUESTION PAPER

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