OCR
Oxford Cambridge and RSA

Wednesday 18 June 2025 – Afternoon

A Level Further Mathematics B (MEI)

Y434/01 Numerical Methods

Time allowed: 1 hour 15 minutes

You must have:
- the Printed Answer Booklet
- the Formulae Booklet for Further Mathematics B (MEI)
- a scientific or graphical calculator

QP

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined page at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give your final answers to a degree of accuracy that is appropriate to the context.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION
- The total mark for this paper is **60**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **12** pages.

ADVICE
- Read each question carefully before you start your answer.

1 The table shows some values of x and the associated values of $f(x)$.

x	1.9	2.0	2.1
$f(x)$	0.21692	0.2	0.18484

(a) Determine an estimate of $f'(2.0)$ using the forward difference method. **[2]**

(b) Determine an estimate of $f'(2.0)$ using the central difference method. **[2]**

2 The numbers p and q are approximated by

$P = 323$ and $Q = 162$.

P has been found by **rounding** p to the nearest whole number.

Q has been found by **chopping** q to the nearest whole number.

(a) **(i)** Find the maximum possible relative error in using P to approximate p. **[1]**

(ii) Find the maximum possible relative error in using Q to approximate q. **[1]**

(b) Determine the range of possible values of $R = \frac{200}{p-2q}$. **[3]**

(c) Explain why your answer to part **(b)** is so large. **[1]**

3 Approximations to $\int_{0.5}^{1.3} \sqrt{1+x^3}\,dx$ using the midpoint rule, the trapezium rule and Simpson's rule with $n = 1$ and $n = 2$ are shown in the table. The table is incomplete.

n	M_n	T_n	S_{2n}
1			1.081111
2	1.074256		

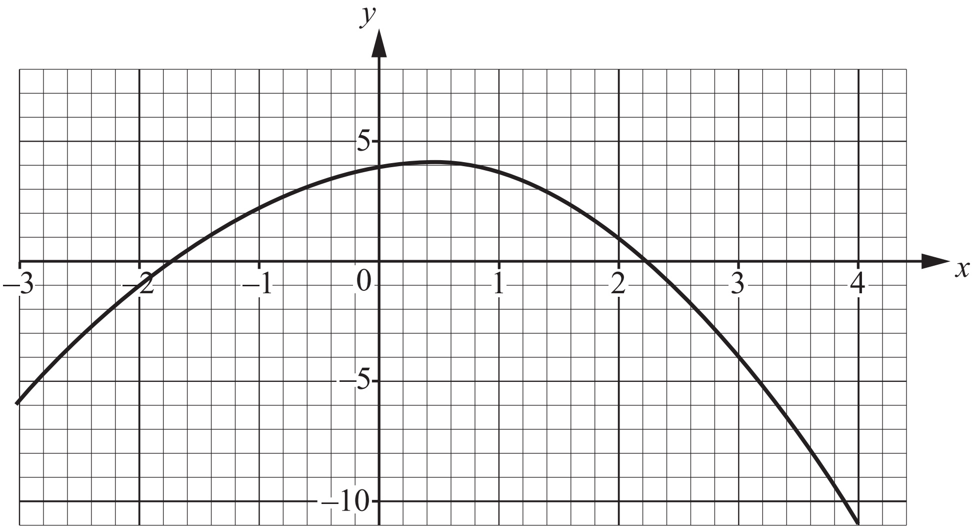
(a) Complete the copy of the table in the **Printed Answer Booklet**. Give your answers to **6** decimal places. **[4]**

(b) **Without doing any further calculations**, state the value of $\int_{0.5}^{1.3} \sqrt{1+x^3}\,dx$ as accurately as possible. You must justify the precision quoted. **[1]**

4 The Newton-Raphson method is to be used to find the positive root of the equation

$\tanh x - x^2 + 4 = 0$.

The diagram shows part of the graph of $y = \tanh x - x^2 + 4$.

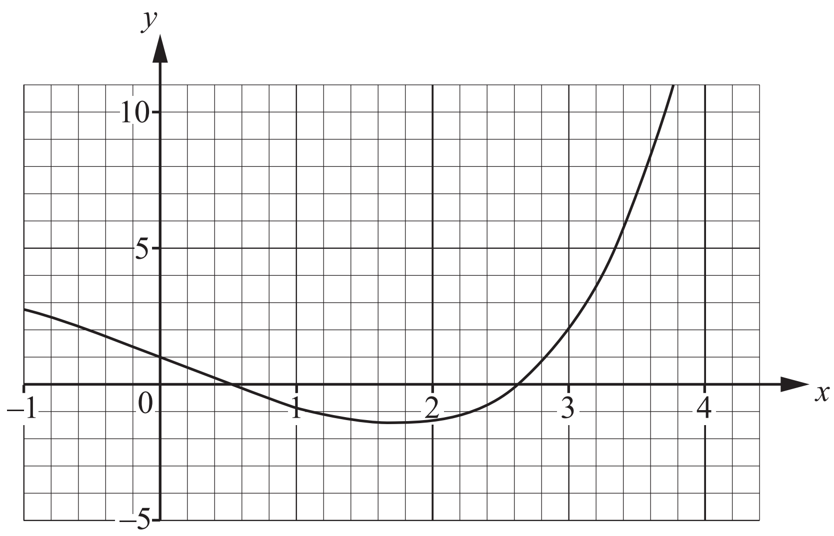


(a) On the copy of the diagram in the **Printed Answer Booklet**, show how the Newton-Raphson method works to find x_1 using the starting value $x_0 = 1$. **[1]**

(b) Use the Newton-Raphson method using the starting value $x_0 = 1$ to determine the values of x_1 and x_2 correct to **8** decimal places. **[4]**

(c) Continue the iteration to determine the value of the positive root of the equation $\tanh x - x^2 + 4 = 0$ correct to **7** decimal places. **[2]**

5 The diagram shows part of the graph of $y = \sinh x - 3x + 1$.



The largest positive root, α, of the equation $\sinh x - 3x + 1 = 0$ is to be found using the secant method with $x_0 = 2$ and $x_1 = 3$.

Table 5.1 shows the associated spreadsheet output.

Table 5.1

	A	B	C	D	E
2	r	x_r	$f(x_r)$	x_{r+1}	$f(x_{r+1})$
3	0	2	–1.37314	3	2.01787
4	1	3	2.017875	2.404935	–0.7211
5	2	2.404935	–0.72109	2.561598	–0.2451
6	3	2.561598	–0.24513	2.642285	0.06018
7	4	2.642285	0.060175	2.626382	–0.0035
8	5	2.626382	–0.00348	2.627252	–5E-05
9	6	2.627252	–4.5E-05	2.627264	3.5E-08

(a) Write down a suitable formula for cell D4. **[2]**

(b) State the most accurate approximation to α in the spreadsheet output. **[1]**

(c) Determine whether this approximation is accurate to **6** decimal places. **[2]**

(d) Explain what would happen if the initial entry in cell D3 were to be changed from 3 to 1. **[1]**

Table 5.2 shows some further analysis of the sequence of approximations to α.

Table 5.2

A	B	C	D	E
r	x_r	$x_{r+1}-x_r$	$\frac{x_{r+2}-x_{r+1}}{x_{r+1}-x_r}$	$\frac{x_{r+2}-x_{r+1}}{(x_{r+1}-x_r)^2}$
1	3	–0.5951	–0.2633	0.44242
2	2.404935	0.15666	0.51504	3.28756
3	2.561598	0.08069	–0.1971	–2.4427
4	2.642285	–0.0159	–0.0547	3.44193
5	2.626382	0.00087	0.01315	15.1113
6	2.627252	1.1E-05		
7	2.627264			

(e) Explain what may be inferred from the entries in column D about the order of convergence of this sequence of approximations to α. **[2]**

(f) Explain what may be inferred from the entries in column E about the order of convergence of this sequence of approximations to α. **[2]**

6 The midpoint rule is used to find a sequence of approximations to $\int_0^1 \sqrt{\sin x}\,dx$.

The associated spreadsheet output, together with some further analysis, is shown in the table.

n	M_n	difference	ratio
1	0.6924056		
2	0.6615057	–0.0308999	
4	0.6498274	–0.0116782	0.37794
8	0.6454773	–0.0043501	0.3725
16	0.6438811	–0.0015962	0.36692
32	0.643302	–0.0005791	0.36281
64	0.6430936	–0.0002085	0.35996
128	0.6430189	–7.463E-05	0.35801

(a) Explain what the entries in the ratio column tell you about the order of the midpoint rule in this case. **[2]**

(b) Use the information in the table to determine the most accurate estimate of $\int_0^1 \sqrt{\sin x}\,dx$ possible. You must justify the precision quoted. **[5]**

7 The equation $8\ln(x+2)-3x+1=0$ has a positive root, γ, and a negative root, δ.

It is proposed that the iterative formula

$x_{n+1}=\mathrm{g}(x_n)$, where $\mathrm{g}(x)=8\ln(x+2)-2x+1$,

should be used to find γ.

It is found that $\mathrm{g}'(\gamma)\approx -0.977$.

(a) Explain what this tells you about the speed of convergence of the sequence of approximations to γ which would be generated by $x_{n+1}=\mathrm{g}(x_n)$. **[2]**

The relaxed iteration

$x_{n+1}=(1-\lambda)x_n+\lambda\mathrm{g}(x_n)$ with $\lambda=0.506$

is used to find a sequence of approximations to γ.

The spreadsheet output is shown in the table, together with the values of $\mathrm{f}(x_r)=8\ln(x_r+2)-3x_r+1$ for $r=0,1,2,3,4$.

	E	F	G
1	r	x_r	$\mathrm{f}(x_r)$
2	0	5	1.567281192
3	1	5.7930443	0.046719768
4	2	5.8166845	3.04288E-05
5	3	5.8166999	–4.07401E-09
6	4	5.8166999	5.47118E-13

(b) State the value of γ as accurately as you can, justifying the precision quoted. **[1]**

(c) Explain why the values displayed in cells F5 and F6 are the same as each other, but the values in cells G5 and G6 are different to each other. **[2]**

The negative root of the equation $8\ln(x+2)-3x+1=0$, δ, is close to –1.

It is found that $\mathrm{g}'(\delta)\approx 13.9$.

(d) Explain why the iterative formula

$x_{n+1}=\mathrm{g}(x_n)$ with $x_0=-1$

may **not** be used to find δ. **[1]**

(e) **(i)** Use the relaxed iteration

$$x_{n+1} = (1-\lambda)x_n + \lambda g(x_n) \text{ with } \lambda = 0.506 \text{ and } x_0 = -1$$

to find x_1, x_2, x_3 and x_4, giving your answers correct to **3** decimal places. **[2]**

(ii) State what would happen if the iteration in part **(e)(i)** were to be continued. **[1]**

(f) Use the relaxed iteration

$$x_{n+1} = (1-\lambda)x_n + \lambda g(x_n) \text{ with } \lambda = -0.078 \text{ and } x_0 = -1$$

to determine the value of δ correct to **7** decimal places. **[2]**

8 Following an injury an athlete returns to training in preparation for a regional competition. At the end of each week the athlete's trainer records the time, t seconds, the athlete takes to run 100 metres.

Table 8.1 shows some of the times recorded at the end of week W.

Table 8.1

W	0	2	3
t	18.00	17.68	16.83

The trainer uses the information in the table to construct a polynomial of degree 2 to model these data.

(a) Use Lagrange's interpolating polynomial of degree 2 to obtain the trainer's model. **[3]**

Subsequently it is found that when $W = 4$, $t = 15.44$.

(b) Determine whether the model is a good fit for these values. **[1]**

The trainer decides to use the three most recent data points, when $W = 2$, 3 and 4, to construct a new model using Newton's forward difference interpolation formula.

Table 8.2 shows the difference table for these data.

Table 8.2

W	t		
2	17.68		
		–0.85	
3	16.83		–0.54
		–1.39	
4	15.44		

(c) The trainer obtains the interpolation formula

$t = -0.27W^2 + BW + C$, where B and C are constants.

- Determine the value of B.
- Determine the value of C. **[3]**

(d) Explain why the new model is a refinement of the model found in part **(a)**. **[1]**

At the regional competition, the athlete needs to run 100 metres in less than 11 seconds in order to qualify for a national competition. The regional competition takes place when $W = 6$.

(e) Determine whether, according to the new model, the athlete will qualify for the national competition. **[1]**

(f) Explain whether it would be appropriate to use the new model to predict the athlete's times for $W \geqslant 7$. **[1]**

END OF QUESTION PAPER

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