OCR
Oxford Cambridge and RSA

Tuesday 3 June 2025 – Afternoon

A Level Further Mathematics B (MEI)

Y433/01 Modelling with Algorithms

Time allowed: 1 hour 15 minutes

You must have:
- the Printed Answer Booklet
- the Formulae Booklet for Further Mathematics B (MEI)
- a scientific or graphical calculator

QP

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined page at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give your final answers to a degree of accuracy that is appropriate to the context.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION
- The total mark for this paper is **60**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **8** pages.

ADVICE
- Read each question carefully before you start your answer.

1 Four workers, Ali (A), Beth (B), Casey (C) and Dev (D), are available to complete three tasks, P, Q and R. Each worker can only be assigned to at most one task, and each task must be done by at most one worker.

The table in **Fig. 1.1** shows the average time, in minutes, that each worker takes to complete each task.

Fig. 1.1

	P	Q	R
A	32	39	37
B	29	28	32
C	35	36	39
D	38	39	34

The four workers need to know, based on the times in the table, who should be allocated to each task so that the total time to complete all three tasks is minimised.

(a) Formulate this allocation problem as an LP. **[4]**

The LP was run in an LP solver and the output is shown in the table in **Fig. 1.2**.

Fig. 1.2

Variable	**Value**
AP	1.000 000
AQ	0.000 000
AR	0.000 000
BP	0.000 000
BQ	1.000 000
BR	0.000 000
CP	0.000 000
CQ	0.000 000
CR	0.000 000
DP	0.000 000
DQ	0.000 000
DR	1.000 000

(b) (i) State which worker will **not** be assigned to a task according to the output given in **Fig. 1.2**. **[1]**

(ii) Find the predicted total time for the **three** assigned workers to complete the **three** tasks, according to the output given in **Fig. 1.2**. **[1]**

2 Consider the following algorithm.

Line 10	Let $A = 5,\ B = 6,\ C = 150$
Line 20	Calculate $D = (A + B) \div 2$
Line 30	Calculate $E = C - D^3$
Line 40	If $E^2 < 0.1$ go to Line 100
Line 50	If $E > 0$ go to Line 80
Line 60	Let $B = D$
Line 70	Go to Line 20
Line 80	Let $A = D$
Line 90	Go to Line 20
Line 100	Output D
Line 110	Stop

(a) Work through the algorithm, recording the values of A, B, D and E every time they change. You should record the exact values of A, B and D, and the value of E correct to **3** decimal places. Give the final output to **2** decimal places. **[4]**

The algorithm gives the cube root of 150 correct to 2 decimal places.

A student adapts the algorithm to try to find the cube root of 1500 correct to 2 decimal places by changing only Line 10 to ‘Let $A = 5,\ B = 6,\ C = 1500$’.

(b) Explain why this change will **not** give the output of the cube root of 1500 correct to 2 decimal places. **[1]**

3 The table lists the duration (in hours) and immediate predecessors for each activity in a project.

Activity	**Duration (hours)**	**Immediate predecessors**
A	3	–
B	5	A
C	6	–
D	6	B
E	4	B, C
F	2	B
G	5	B
H	4	C
I	7	C
J	4	E, H
K	2	G, I, J
L	4	I

(a) Draw an activity network, using activity on arc, to represent the project. **[3]**

(b) **(i)** Carry out a forward pass and a backward pass through the activity network, showing the early event time and the late event time at each vertex of your network. **[3]**

(ii) State the minimum project completion time. **[1]**

(iii) List the critical activities. **[1]**

Each activity requires one worker. When an activity is started it must be completed without interruption.

(c) Use the diagram in the Printed Answer Booklet to show how **four** workers can complete the project in the minimum time found in part **(b)(ii)**. Each column in the diagram represents 1 hour. For each worker, write the letter of the activity they are doing in each box, or leave the box blank if the person is resting for that 1 hour. **[3]**

(d) Explain why it is not possible for only **three** people to complete the project in the minimum time found in part **(b)(ii)**. **[1]**

4 The following LP problem in x, y and z is as follows.

Maximise $P = 2x + y + 4z$

Subject to

$$x - 3y + z \leqslant 40$$
$$3x + y + z \leqslant 56$$
$$-2x + 4y + z \leqslant k$$
$$x \geqslant 0,\ y \geqslant 0,\ z \geqslant 0$$

where k is a non-negative constant.

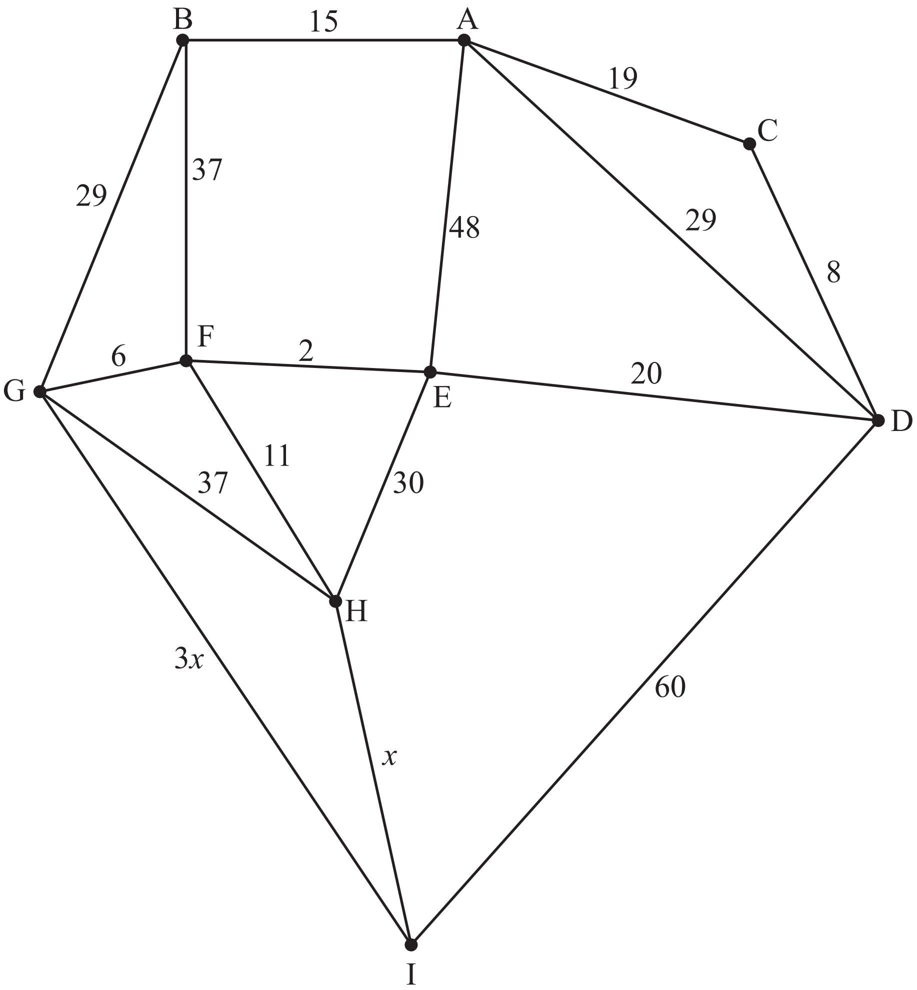
(a) Explain why k must be non-negative if the simplex method is to be used to solve this problem. **[1]**

(b) Complete the initial tableau in the Printed Answer Booklet so that the simplex method may be used to solve this problem. **[2]**

If $k = 0$, the maximum value of P is 112. When the value of k is increased then the maximum value of P also increases until k reaches some value k_1. After reaching this value of k, the maximum value of P increases no further.

(c) By first using an entry in the z column as the pivot element, use the simplex method to determine the value of k_1 and the corresponding optimal values of P, x, y and z. **[7]**

5 The diagram shows a network. The weights on the arcs are distances.



The weight of arc HI is x and the weight of arc GI is $3x$, where x is an integer such that $6 \leqslant x \leqslant 59$.

(a) • Apply Dijkstra's algorithm to the network to find the **three** possible shortest paths from A to I.

• State the length of each path, leaving your answer in terms of x where necessary. **[7]**

(b) **(i)** Apply Prim's algorithm, **starting at A**, to find the minimum spanning tree for the network in the diagram. You must state the order in which the arcs were included in the tree. **[2]**

(ii) State, in terms of x, the weight of the minimum spanning tree. **[1]**

For the network in the diagram, you are given that the shortest path from A to I passes through F. Furthermore, the weight of the minimum spanning tree is greater than the weight of the shortest path from A to I that passes through G.

(c) Determine the range of possible values of x. **[3]**

6 An incomplete initial tableau for a two-stage simplex solution for a maximisation linear programming problem, in x, y and z where $x, y, z \geqslant 0$, is shown below.

Q	P	x	y	z	s_1	s_2	s_3	s_4	a_1	a_2	RHS
0	1	−2	−1	−3	0	0	0	0	0	0	0
0	0	2	−3	4	1	0	0	0	0	0	55
0	0	3	1	1	0	−1	0	0	1	0	23
0	0	2	4	1	0	0	−1	0	0	1	23
0	0	2	4	1	0	0	0	1	0	0	23

(a) Write down the equality constraint for the LP problem. **[1]**

(b) In the **Printed Answer Booklet**, complete the top row of the initial tableau for the LP problem. **[2]**

A 2-D graphical method is to be used to solve the LP problem.

(c) Explain why P is maximised when $4x + 11y$ is minimised. **[3]**

(d) Use a 2-D graphical method, with axes representing x and y, to determine the following.

- The maximum value of P
- The corresponding values of x, y and z **[8]**

END OF QUESTION PAPER