OCR
Oxford Cambridge and RSA

Friday 6 June 2025 – Afternoon

A Level Further Mathematics B (MEI)

Y421/01 Mechanics Major

Time allowed: 2 hours 15 minutes

You must have:
- the Printed Answer Booklet
- the Formulae Booklet for Further Mathematics B (MEI)
- a scientific or graphical calculator

QP

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give your final answers to a degree of accuracy that is appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\text{m}\,\text{s}^{-2}$. When a numerical value is needed use $g = 9.8$ unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION
- The total mark for this paper is **120**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **12** pages.

ADVICE
- Read each question carefully before you start your answer.

Section A (33 marks)

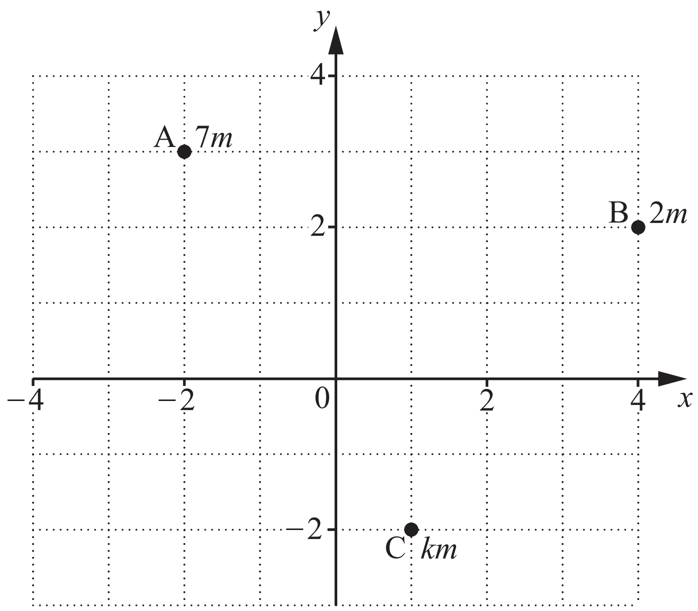
1 A small box of mass 3 kg is pulled along a horizontal floor by a constant force of magnitude 35 N. The force acts at an angle of θ above the horizontal.

The total resistance to the motion of the box has magnitude 25 N.

The box starts from rest at a point O, and passes through a point 4 m from O with speed $1.5\,\text{m}\,\text{s}^{-1}$.

Use an energy method to determine the value of θ. **[3]**

2



The diagram shows a system of three particles of masses $7m$, $2m$ and km situated in the x-y plane at the points A$(-2, 3)$, B$(4, 2)$ and C$(1, -2)$ respectively.

The centre of mass of the three particles is at the point with coordinates $(0, d)$.

(a) Find the value of k. **[2]**

(b) Find the value of d. **[2]**

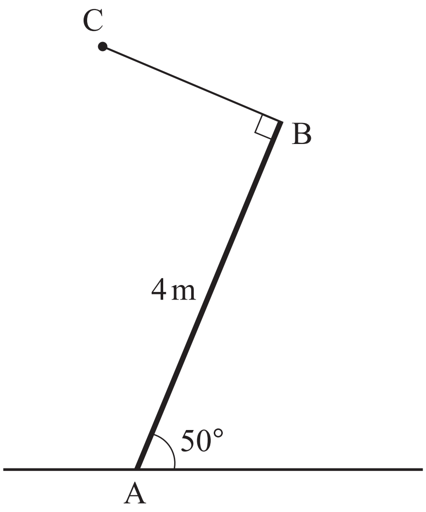
3 At time t seconds, the total force acting on a particle P of mass 4 kg is $\begin{pmatrix} 32\cos 2t \\ -8\sin t \\ 0 \end{pmatrix}$ N.

When $t = 0$ the velocity of P is $\begin{pmatrix} 0 \\ 5 \\ 6 \end{pmatrix}$ $\mathrm{m\,s^{-1}}$.

(a) Find, in column vector form, the acceleration of P at time t seconds. **[1]**

(b) Determine the speed of P when $t = \frac{1}{3}\pi$. **[5]**

4



A uniform rod AB of mass 3 kg and length 4 m rests with one end A on rough horizontal ground. A light inextensible string has one end attached to the rod at B and the other end attached to a fixed point C.

The string is perpendicular to the rod and lies in the same vertical plane as the rod. The rod is in equilibrium, inclined at 50° to the ground, as shown in the diagram.

(a) Complete the diagram in the **Printed Answer Booklet** to show all the forces acting on the rod. **[1]**

(b) By taking moments about A, find the tension in the string. **[2]**

(c) Determine the magnitude of the total contact force between the rod and the ground. **[4]**

5 In this question you may assume that if r and s are any physical quantities then $\left[\frac{\mathrm{d}r}{\mathrm{d}s}\right] = \left[\frac{r}{s}\right]$.

You may also assume that all given numerical constants are dimensionless.

(a) Find the dimensions of power. **[1]**

A car of mass m moves horizontally in a straight line. When the car is a distance x from a point O, it is moving away from O with speed v.

The power developed by the car is P. The maximum possible speed of the car during its motion is U.

The differential equation for the motion of the car is given by

$$m^{\alpha} v^{\beta} \frac{\mathrm{d}v}{\mathrm{d}x} = P^{\gamma}\left(1 - \frac{v^{\delta}}{U^3}\right).$$

(b) Explain why $\delta = 3$. **[1]**

(c) Show that the dimensions of $\frac{\mathrm{d}v}{\mathrm{d}x}$ are T^{-1}. **[1]**

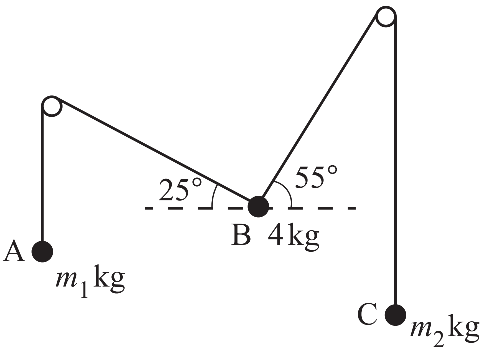
(d) Use dimensional analysis to determine the values of α, β and γ. **[3]**

Once the car attains a speed of $\frac{1}{2}U$, the power developed by the car is immediately reduced to zero.

A student calculates that the time taken, t, for the speed of the car to reduce from $\frac{1}{2}U$ to $\frac{1}{4}U$ is given by $t = \frac{2mU^2}{P}$.

(e) Determine whether this expression for t is dimensionally consistent. **[2]**

6



One end of a light inextensible string is attached to a particle A of mass m_1 kg. The other end of the string is attached to a second particle B of mass 4 kg.

A second light inextensible string is attached to B. The other end of this second string is attached to a third particle C of mass m_2 kg. Both strings are taut and pass over small smooth fixed pulleys.

The diagram shows the three particles A, B and C, and the two strings. The diagram also shows the angles that the non-vertical portions of the strings make with the horizontal.

The system is in equilibrium with all three particles at rest in the same vertical plane.

(a) Draw a closed figure to represent the three forces acting on B. **[1]**

(b) Hence, or otherwise, find the following.

- The value of m_1
- The value of m_2 **[4]**

Section B (87 marks)

7 Two uniform smooth spheres, A and B, have the same radius. The mass of A is 4 kg, and the mass of B is 6 kg.

The spheres are travelling in the same direction in a straight line on a smooth horizontal surface, A with speed $6\,\text{m}\,\text{s}^{-1}$ and B with speed $3\,\text{m}\,\text{s}^{-1}$.

Sphere A collides directly with B and, immediately after the collision, the speed of B is $5\,\text{m}\,\text{s}^{-1}$.

(a) Determine the exact value of the coefficient of restitution between A and B. **[3]**

Sphere B subsequently collides directly with a uniform smooth stationary sphere C. The mass of C is 2 kg, and C has the same radius as A and B. The coefficient of restitution between B and C is 0.6.

(b) Show that there will be **no** further collisions between any of the three spheres. **[4]**

(c) Show that the impulse of B on C is equal in magnitude to the impulse of A on B. **[2]**

(d) Determine the total kinetic energy lost due to the **two** collisions. **[3]**

8 A box of mass m kg rests on a horizontal platform. The platform moves vertically with simple harmonic motion of period T seconds and amplitude a m. At time t seconds, the upward displacement of the platform from its mean position is x m.

(a) Express $\ddot{x}$ in terms of and π, T and x. **[2]**

(b) Show that the box will maintain contact with the platform throughout its motion if $T > 2\pi\sqrt{\frac{a}{g}}$. **[4]**

(c) Given that $T = 4\pi\sqrt{\frac{a}{g}}$, determine the ratio of the magnitudes of the greatest and least forces exerted by the platform on the box. Give your answer in the form $p : q$ where p and q are integers to be found. **[2]**

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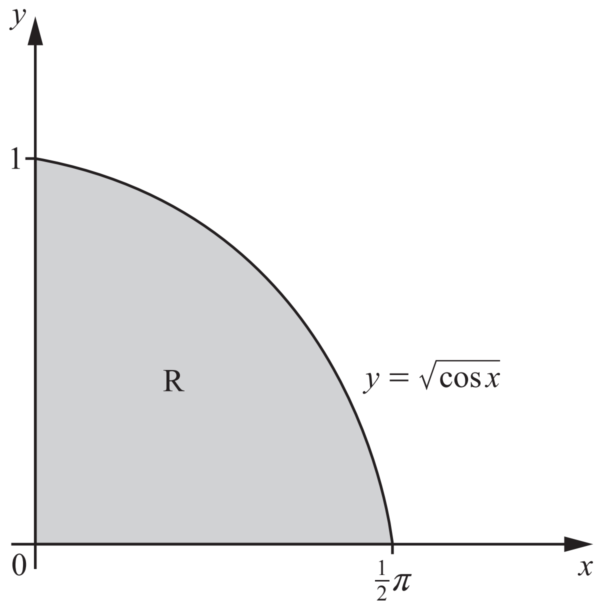


The diagram shows the shaded region R bounded by the curve $y = \sqrt{\cos x}$, the x-axis and the y-axis.

The region R is rotated through 2π radians about the x-axis to form a uniform solid of revolution S.

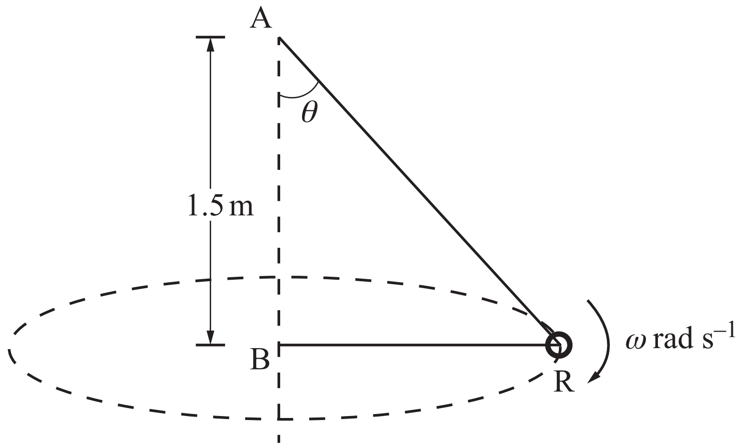
(a) Determine the exact x-coordinate of the centre of mass of S. **[6]**

The uniform solid S is placed with its plane face on an inclined plane. The inclined plane makes an angle θ with the horizontal.

(b) Given that the inclined plane is sufficiently rough to prevent S from sliding and that S is on the point of toppling when $\theta = \alpha$, find the angle α. **[2]**

(c) Given instead that S is on the point of sliding down the plane when $\theta = \beta$ and the coefficient of friction between S and the plane is 0.4, find the angle β. **[2]**

10



One end of a light inextensible string is attached to a fixed point A. The other end of the string is attached to a fixed point B, vertically below A, where AB = 1.5 m.

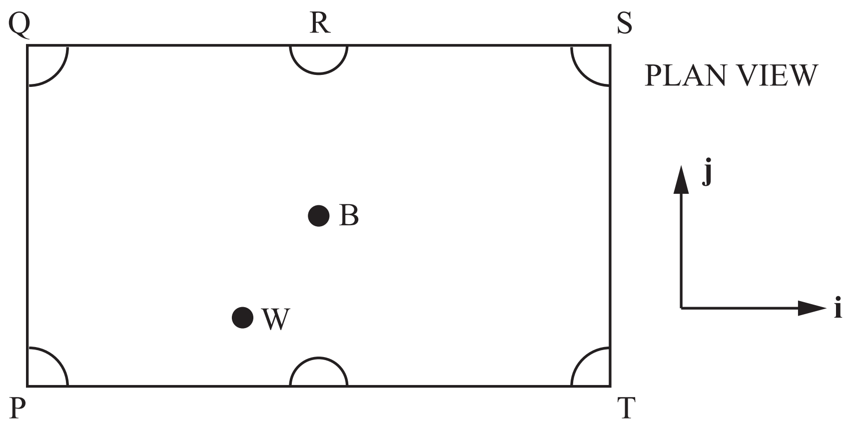
A small smooth ring R of mass 2 kg is threaded on the string. The ring R moves in a horizontal circle with centre B.

The upper part of the string makes an angle θ with the downward vertical and R moves with constant angular speed ω rad s^{-1} (see diagram).

(a) By finding an expression for ω^2 in terms of g and $\sin\theta$, show that $\omega > 2\sqrt{\frac{1}{3}g}$. **[6]**

(b) Given that $\omega = \sqrt{2g}$, determine, in terms of g, the exact tension in the string. **[3]**

11



A student is playing snooker. The snooker table, PQST, is rectangular and the unit vectors, **i** and **j**, are parallel to PT and PQ respectively (see diagram).

The surface of the table is horizontal and smooth. The balls on the table are all uniform smooth spheres with the same radius and mass. The coefficient of restitution between the balls is 0.6. The blue ball, B, lies at rest at the centre of the table.

The student strikes the white ball, W, so that it collides obliquely with B. Immediately before the collision with B, W is travelling with velocity $(6\mathbf{i}+8\mathbf{j})\,\mathrm{m\,s^{-1}}$. As a result of the collision, B travels directly towards R, where R is the mid-point of QS.

(a) Explain why, at the moment of the collision, the line of centres of B and W must be parallel to **j**. **[1]**

(b) Determine, correct to the nearest degree, the angle through which the direction of motion of W is deflected by the collision. **[7]**

12 One end of a light elastic string of natural length a m and modulus of elasticity $4mg$ N is attached to a fixed point O. The other end of the string is attached to a particle P of mass m kg. The particle is projected vertically downwards from O with speed $\sqrt{4ga}\,\mathrm{m\,s^{-1}}$.

(a) When P is at a distance of h m below O, where $h > a$, it has speed, $v\,\mathrm{m\,s^{-1}}$.

By using the principle of conservation of energy, show that $v = \sqrt{10gh - \frac{4gh^2}{a}}$. **[4]**

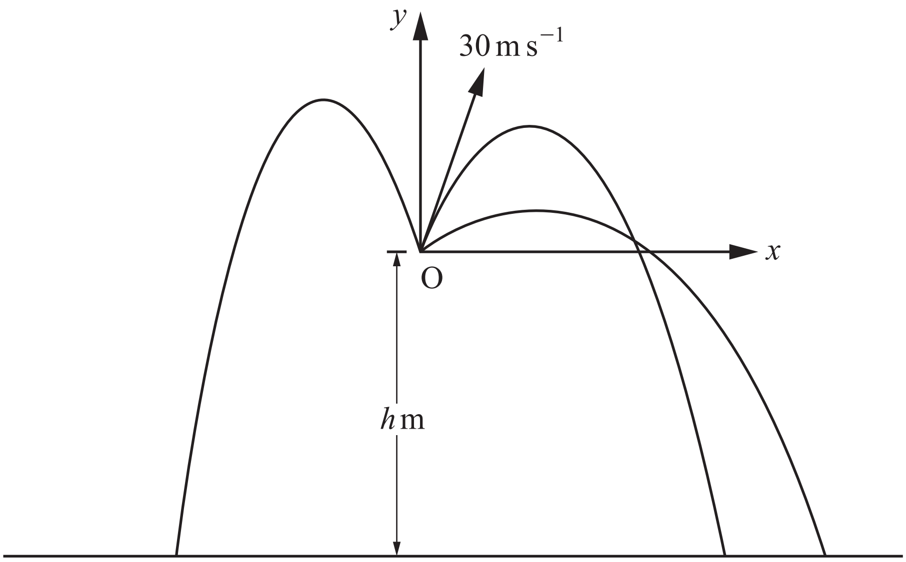
(b) Determine, in terms of a, the greatest distance below O attained by P. **[2]**

(c) Determine, in terms of g and a, the maximum speed of P during its downward motion. **[4]**

(d) Find, in terms of a, the greatest height above O attained by P. **[2]**

(e) Explain why the greatest distance below O attained by P may **not** in practice be given by your answer to part **(b)**. **[1]**

13



In this question you should take the acceleration due to gravity to be $10\,\mathrm{m\,s^{-2}}$.

A machine can launch balls with speed $30\,\mathrm{m\,s^{-1}}$ from a height h m above horizontal ground. The point of projection O is the origin for a horizontal x-axis and a vertical y-axis which define the vertical plane in which the balls move (see diagram). You should assume that the balls, as they leave the machine, can be modelled as particles moving freely under gravity.

At time $t = 0$, a ball B is projected at an angle θ above the horizontal in the x-y plane. Before B hits the ground, the horizontal and vertically upwards displacements of B from O at time t seconds are denoted by x m and y m respectively.

You are given that B hits the ground at the point $(X, -h)$.

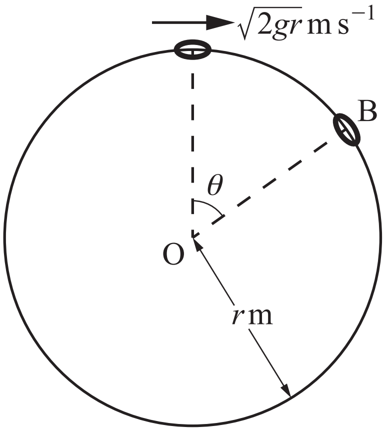
(a) By expressing x and y in terms of t, show that

$$X^2 \tan^2\theta - 180X\tan\theta + X^2 - 180h = 0.$$ **[5]**

(b) Determine an expression, as $\tan\theta$ varies, for the maximum value of X^2 at the instant when B hits the ground. Give your answer in the form $X^2 = p + qh$, where p and q are integers to be determined. **[3]**

(c) Hence find the equation of the curve that bounds the region in the x-y plane through which B can pass. Give your answer in the form $y = k_1x^2 + k_2$, where k_1 and k_2 are constants to be determined. **[2]**

14



A smooth thin wire is formed into a circle of radius r m and centre O and is fixed in a vertical plane. A small bead B of mass m kg is threaded on the wire.

The bead is initially at the highest point of the wire and is set in motion along the wire with speed $\sqrt{2gr}$ m s^{-1} (see diagram).

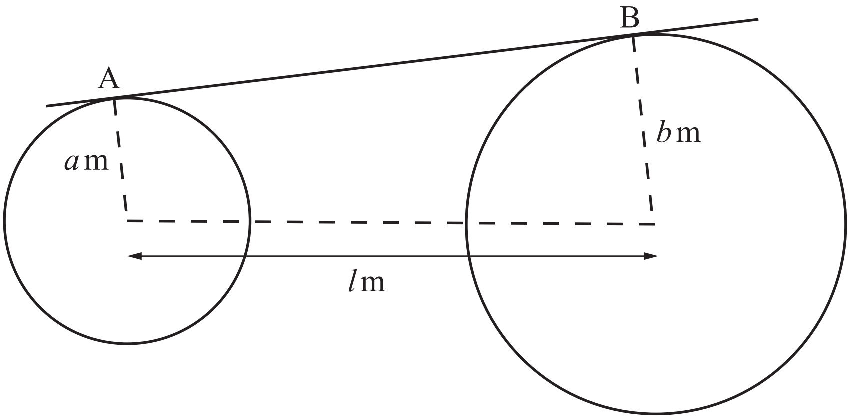
(a) Show that, when the radius from O to B is inclined at an angle θ to the upward vertical, the downward vertical component of the acceleration of B is

$(1 + 4\cos\theta - 3\cos^2\theta)g$. **[5]**

(b) Determine, in terms of m and g, the magnitude of the force of the bead on the wire when the downward vertical component of the acceleration is a maximum. **[6]**

Turn over for question 15

15



The diagram shows two fixed rough circular cylinders of radii a m and b m, where $b > a$. The axes of the cylinders are parallel and in the same horizontal plane at a distance l m apart. A heavy non-uniform straight rod rests in limiting equilibrium on the cylinders at A and B. The centre of mass of the rod lies between A and B but is not at the mid-point of AB. The coefficient of friction between the rod and each cylinder is μ.

Determine, in terms of a, b and l, an expression for μ. **[6]**

END OF QUESTION PAPER