OCR
Oxford Cambridge and RSA

Friday 6 June 2025 – Afternoon

AS Level Further Mathematics B (MEI)

Y414/01 Numerical Methods

Time allowed: 1 hour 15 minutes

You must have:
- the Printed Answer Booklet
- the Formulae Booklet for Further Mathematics B (MEI)
- a scientific or graphical calculator

QP

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined page at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give your final answers to a degree of accuracy that is appropriate to the context.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION
- The total mark for this paper is **60**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **8** pages.

ADVICE
- Read each question carefully before you start your answer.

1 The method of interval bisection is used to find a sequence of approximations to one of the roots of the equation $e^x - x^2 - 3x = 0$.

The table shows the associated spreadsheet output.

	A	B	C	D	E	F
1	a	$f(a)$	b	$f(b)$	c	$f(c)$
2	2	−2.61094	3	2.085537	2.5	−1.56751
3	2.5	−1.56751	3	2.085537	2.75	−0.16987
4	2.75	−0.16987	3	2.085537	2.875	0.834799
5	2.75	−0.16987	2.875	0.834799	2.8125	0.303839
6						

(a) Write down a suitable formula for cell E2. **[1]**

The formula in cell A3 is = IF(F2 < 0,E2,A2) .

(b) Write down a similar formula for cell C3. **[1]**

(c) Complete row 6 of the table in the **Printed Answer Booklet**. **[2]**

(d) **Without** doing any more calculations, write down the value of the root correct to the maximum number of decimal places which seems justified. You must explain the precision quoted. **[1]**

2 The table gives three values of x and the associated values of y.

x	−1	2	3
y	−3.39	0.18	0.45

Use Lagrange's form of the interpolating polynomial to construct a polynomial of degree 2 for the values in the table. Give your answer in the form

$y = ax^2 + bx + c,$

where a, b and c are constants to be determined. **[4]**

3 **(a)** Find the relative error when π is **chopped** to **3** decimal places. **[2]**

(b) Find the relative error when π is **rounded** to **3** decimal places. **[2]**

You are given that $y = \pi^2 - 5$ and $z = (\pi - 5)^2$.

You are also given the following information.

- Y is an approximation to y.
- Z is an approximation to z.
- Y and Z are found by using $\pi = 3.14$.

A student states that the relative error in using Y to approximate y is exactly the same as the relative error in using Z to approximate z, because in each case the calculation involves squaring and subtracting 5.

(c) **Without** doing any calculations, explain whether the student is correct. **[1]**

4 A student is using a spreadsheet to investigate the curve $y = \mathrm{f}(x)$ at the point where $x = 2$. Some of the spreadsheet output is shown below.

Table 4.1 shows some values of x and the associated values of $\mathrm{f}(x)$.

Table 4.2 shows **two** approximations to $\mathrm{f}'(2)$ found using the same method with $h = 0.001$ and $h = 0.01$.

	E	F	G	H	I	J
2						
3		**Table 4.1**				
4		x	2	2.001	2.01	
5		$\mathrm{f}(x)$	0.30103	0.30125	0.3032	
6						
7						
8		**Table 4.2**				
9		h	0.001	0.01		
10		$\mathrm{f}'(2)$	0.21709	0.21661		
11						

(a) The formula in cell G10 is $= (\text{H5} - \text{G5})/\text{G9}$.

The formula in cell H10 is $= (\text{I5} - \text{G5})/\text{H9}$.

State the method being used to find these approximations to $\mathrm{f}'(2)$. **[1]**

(b) • Calculate $\dfrac{0.30125 - 0.30103}{0.001}$.

• Explain why your answer is not the same as the value displayed in cell G10. **[3]**

(c) State the value of $\mathrm{f}'(2)$ as accurately as you can. You must justify the precision quoted. **[1]**

(d) Hence determine an estimate for the error when $\mathrm{f}(2)$ is used as an approximation to $\mathrm{f}(2.1)$. **[2]**

(e) Explain whether your answer to part **(d)** is likely to be an over-estimate or an under-estimate. **[1]**

5 The spreadsheet output shows a sequence of approximations to $\int_{0.5}^{1.5} f(x)\,dx$ using the trapezium rule and Simpson's rule, together with some further analysis of the approximations found using Simpson's rule.

	F	G	H	I	J
21	n	T_n	S_n	difference	ratio
22	1	0.84557616			
23	2	0.88144672	0.89340357		
24	4	0.89066651	0.89373977	0.00034	
25	8	0.89299453	0.89377054	3.1E-05	0.092
26	16	0.89357822	0.89377279	2.2E-06	0.073
27	32	0.89372426	0.89377294	1.5E-07	0.066
28	64	0.89376077	0.89377295	9.4E-09	0.063
29	128	0.8937699	0.89377295	5.9E-10	0.063

(a) Write down the value displayed in cell I25 using standard mathematical notation. **[1]**

(b) State the value of $\int_{0.5}^{1.5} f(x)\,dx$ as accurately as you can. You must justify the precision quoted. **[1]**

(c) Write down a suitable formula for cell H23. **[2]**

(d) (i) Explain what the values in column J tell you about the order of Simpson's rule in this case. **[2]**

(ii) Explain whether this is usual. **[1]**

6 The equation $0.5\ln x + x^2 - x - 2 = 0$ has a root α, such that $1 < \alpha < 2$.

A student attempts to find α using the iterative formula

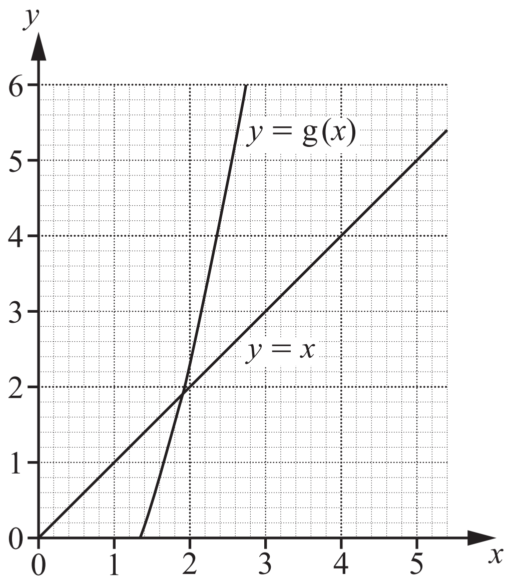
$x_{n+1} = \mathrm{g}(x_n)$ with $x_0 = 1$, where $\mathrm{g}(x) = 0.5\ln x + x^2 - 2$.

The associated spreadsheet output is shown in the table.

	K	L
1	n	x_n
2	0	1
3	1	−1
4	2	#NUM!

(a) Explain why #NUM! is displayed in cell L4. **[1]**

The graph shows part of the graphs of $y = x$ and $y = \mathrm{g}(x)$.



(b) On the copy of the graph in the **Printed Answer Booklet** show how the iterative formula $x_{n+1} = \mathrm{g}(x_n)$ with $x_0 = 2$ finds x_1 and x_2. **[1]**

(c) Explain why the iterative formula $x_{n+1} = \mathrm{g}(x_n)$ may **not** be used to successfully find α, whatever the value of x_0. **[1]**

(d) Use the relaxed iteration

$$x_{n+1} = (1-\lambda)x_n + \lambda \mathrm{g}(x_n),$$

with $\lambda = -0.328$ and $x_0 = 1$, to determine the value of α correct to **6** decimal places. **[3]**

7 A student makes a cup of tea and investigates how long it takes to reach room temperature. The student places the cup of tea on a table in the kitchen and records the temperature T°C of the tea t minutes after it was made, at **5** minute intervals starting with $t = 5$. The data collected are shown in the table.

Time after tea was made t (minutes)	5	10	15	20
Temperature of tea T (°C)	78.5	68.1	58.7	50.3

(a) Show that a quadratic polynomial would be a reasonable model for these data. **[3]**

(b) Use Newton's forward difference interpolation method to determine a suitable quadratic polynomial for these data. **[4]**

(c) The temperature in the kitchen is 19°C. It takes one hour for the tea to cool to room temperature.

(i) Use the model to find an estimate of the temperature of the tea after 1 hour. **[1]**

(ii) Use the model to find an estimate of the temperature of the tea after an hour and a half. **[1]**

(iii) Hence identify a limitation of the model. **[1]**

8 **Table 8.1** shows some values of x and the associated values of $f(x)$.

Table 8.1

x	0.5	0.9	1.3
$f(x)$	1.414 21	1.099 47	0.711 01

(a) Use the central difference method to determine an approximation to $f'(0.9)$. **[2]**

Table 8.2 shows a sequence of approximations to $f'(0.9)$ for different values of the step length, h, together with some further analysis.

Table 8.2

h	0.2	0.1	0.05	0.025	0.0125
$f'(0.9)$	−0.957 844	−0.977 203	−0.982 021	−0.983 225	−0.983 525
difference		−0.019 360	−0.004 818	−0.001 203	−0.000 301
ratio			0.248 882	0.249 719	0.249 930

(b) Use extrapolation to determine the value of $f'(0.9)$ as accurately as you can. You must justify the precision quoted. **[5]**

Turn over for question 9

9 In this question you must show detailed reasoning

The equation $0.2x^4 + 0.8x - 0.2 = 0$ has a root α such that $0 < \alpha < 1$.

(a) Use the Newton-Raphson method with $x_0 = 1$ to determine the value of α correct to **6** decimal places. **[5]**

The equation $0.2x^4 + 0.8x - 0.2 = 0$ has another root β such that $-2 < \beta < -1$.

The Newton-Raphson method is used to find a sequence of approximations to β.
The associated spreadsheet output is shown in the table below, together with some further analysis.

A	B	C	D
r	x_r	difference	ratio
0	−2		
1	−1.75	0.25	
2	−1.670923	0.079077	0.3163082
3	−1.663319	0.007604	0.096159
4	−1.663252	6.7E-05	0.0088147
5	−1.663252	5.18E-09	7.724E-05

(b) (i) Explain what the values in column D suggest about the order of convergence of this sequence of approximations. **[2]**

(ii) Explain whether this is usual. **[1]**

END OF QUESTION PAPER